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Seismic Analysis of a Multi Storey Plane Frame using Static & Dynamic Methods

Comparison of Equivalent Static Force & Response Spectrum on Frame with & without Infills

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***Abstract*—In the context of seismic analysis and design of structures, in earthquake engineering, a verity of methods are available. Standard codes provide provision for certain methods for the analysis of wide range of structures of engineering interest. In this paper an attempt is made to present the provisions of IS 1893:2002, part-1 for the analysis of structures and its suitability.**

***Keywords*— Lumped mass; infills; CQC; seismic analysis;**

I. INTRODUCTION

In the context of seismic analysis and design of structures, a variety of methods are available for seismic analysis of structures. In this paper an attempt has made to present static and dynamic methods of seismic analysis as per Indian standard code IS 1893:2002 (Part 1). The basic concepts and codal provisions of Equivalent static force method and Response spectrum method as per IS 1893:2002 (Part 1) are explained in detail.

A. Equivalent static analysis (ESA)

This approach defines a series of forces acting on a building to represent the effect of earthquake ground motion, typically defined by a seismic design response spectrum. It assumes that the building responds in its fundamental mode. For this to be true, the building must be low rise and must not twist significantly when the ground moves. The response is read from design response spectrum, given the natural frequency of the building. The applicability of this method is extended in many building codes by applying factors to account for higher buildings with some higher modes and for low levels of twisting. To account for effects due to yielding of the structure, many codes apply modification factors that reduce the design forces.de-composing as a means of reducing degrees of freedom in the analysis.

B. Response spectrum analysis (RSM)

This approach permits the multiple modes of response of a building to be taken in to account (in the frequency domain). This is required in many building codes for all except for very simple or very complex structures. The response of a structure can be defined as a combination of many special shapes (modes) that in a vibrating string correspond to the harmonics. Computer analysis can be used to determine these modes for a structure. For each mode, a response is read from the design spectrum, based on modal mass, and they are then combined to provide an estimate of the total response of the structure. In this we have to calculate the magnitude of forces in all directions i.e. X, Y & Z and then see the effects on the building. Combination methods include the following.

1. Absolute values are added together (AVS)
2. Square root of sum of the squares (SRSS)
3. Complete Quadratic Combinations (CQC).

A method that is an improvement on SRSS for closely shaped modes.

The results of a response spectrum analysis using the response spectrum from a ground motion is typically different from that which would be calculated from linear dynamic analysis using that ground motion directly, since phase information is lost in the process of generating the response spectrum.

In cases where structures are either too irregular, too tall are of significance to a community in disaster response, the response spectrum approach is no longer appropriate and more complex analysis is often required such as non-linear static analysis or dynamic analysis.

II. STEP BY STEP PROCEDURE FOR STATIC AND DYNAMIC METHODS

A. Equivalent static force method

1) Calculation of Lumped masses to various floor levels

The earthquake forces shall be calculated for the full dead load plus the percentage of imposed load as given in Table-8 of IS

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1893 (Part 1):2002. The imposed load on roof is assumed to be zero. The lumped mass of each floor are worked out as follows:

a) Mass of each floor is calculated as

Mass of columns +Mass of beams in longitudinal and transverse direction of that floor + Mass of slab+ Imposed load of that floor if permissible.

b) Roof

Mass of infill + Mass of columns + Mass of beams in longitudinal and transverse direction of that floor + Mass of slab + Imposed load of that floor if permissible

- 50% of imposed load, if imposed load is greater than 3 kN/m^2

c) Seismic weight of building

The seismic weight of each floor is its full dead load plus appropriate amount of imposed load, as specified in Clause 7.3.1 and 7.3.2 of IS 1893 (Part 1):2002. Any weight supported in between stories shall be distributed to the floors above and below in inverse proportion to its distance from the floors.

2) Determination of Fundamental Natural Period

The approximate fundamental natural period of a vibration (T_a), in seconds, of a moment resisting frame building without brick infill panels may be estimated by the empirical expression (Clause 7.6.1)

$$T_a = 0.075xh^{0.75} \quad \text{For RC frame building}$$

$$T_a = 0.085xh^{0.75} \quad \text{For Steel frame building}$$

Where h is the height of the building, in meters.

If the structure considered with infills (Clause 7.6.2)

$$T_a = 0.09H/\sqrt{D}$$

3) Determination of Design Base Shear

Design seismic base shear, (Clause 7.5.3)

$$V_B = A_h W$$

4) Vertical Distribution of Base Shear

The design base shear (V_B) computed shall be distributed along the height of the building as per the expression, (Clause 7.7.1)

$$Q_i = V_B \frac{W_i {h_i}^2}{\sum_{i=1}^{n} W_i {h_i}^2}$$

Where

Q_i = Design lateral forces at floor i
W_i= Seismic weights of the floor i
h_i = Height of the floor i measured from base and
n = Number of stories

5) Calculation of moment due to design lateral forces

$$M = \sum_{i=1}^{n} F_i h_i$$

B. Response Spectrum Method

1) Determination of Eigen values and Eigen vectors

The mass and stiffness matrices at each floor level are calculated in S.I units and then modeled in the form of matrix as

$$M = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix}$$

Column stiffness of storey, $k = \frac{12EI_c}{L^3}$

The stiffness of infill is determined by modeling the infill as an equivalent diagonal strut

$$\alpha_h = \frac{\pi}{2}\left[\frac{E_f I_c h}{2E_m t \sin 2\theta}\right]^{\frac{1}{4}}$$

$$\alpha_l = \pi\left[\frac{E_f I_b l}{E_m t \sin 2\theta}\right]^{\frac{1}{4}}$$

Width of the strut

$$W = \frac{1}{2}\sqrt{\alpha_h^2 + \alpha_l^2}$$

Where

E_f = Young's modulus of the concrete

E_m = Young's modulus of the masonry infill

h = Height of infill wall

l = length of wall

t = Thickness of wall

I_c=Moment of inertia of columns

I_b =Moment of inertia of beam

A=cross sectional area of diagonal stiffness

l_d =Diagonal length of strut = $\sqrt{h^2 + l^2}$

Therefore, stiffness of infill is

$$k_d = \frac{AE_m}{l_d}\cos^2\theta$$

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Taking $k/m = \omega_n^{\ 2}$

$$\left|K - \omega^2 M\right| = \begin{bmatrix} 2K - \omega^2 m & -k_2 & 0 & 0 \\ -k_2 & 2K - \omega^2 m & -k_3 & 0 \\ 0 & -k_3 & 2K - \omega^2 m & -k_4 \\ 0 & 0 & -k_4 & k - 0.575\,\omega^2 m \end{bmatrix}$$

The solution for the above equation is given as

Eigen values

$$\left[\omega^2\right] = \begin{bmatrix} \omega_1^2 & 0 & 0 & 0 \\ 0 & \omega_2^2 & 0 & 0 \\ 0 & 0 & \omega_3^2 & 0 \\ 0 & 0 & 0 & \omega_4^2 \end{bmatrix}$$

Eigen vectors

$$[\Phi] = [\Phi_1\ \Phi_2\ \Phi_3\ \Phi_4] = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix}.$$

Natural frequency in various modes

$$[\omega] = \begin{bmatrix} \omega_1 & 0 & 0 & 0 \\ 0 & \omega_2 & 0 & 0 \\ 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & \omega_4 \end{bmatrix}$$

Natural time period

$$[T] = \begin{bmatrix} T_1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_3 & 0 \\ 0 & 0 & 0 & T_4 \end{bmatrix}$$

2) Determination of Modal Participation factor

$$\{P\} = \frac{\sum_{i=1}^{4} W_i \Phi_{ik}}{\sum_{i=1}^{4} W_i (\Phi_{ik})^2}$$

3) Determination of model mass

$$M_i = \frac{\left[\sum_{i=1}^{4} W_i \Phi_{i1}\right]^2}{g\left[\sum_{i=1}^{4} W_i (\Phi_{i1})^2\right]}$$

4) Determination of lateral force on each floor

The design lateral force (Q_{ik}) at floor i in mode k is given by,

$$Q_{ik} = (A_k\ P_k\ \Phi_{ik}\ W_k)$$

The design horizontal seismic coefficient A_h for various modes are,

$$A_{h1} = \frac{Z}{2}\frac{I}{R}\frac{S_{a1}}{g}$$

For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.5; & 0.10 \le T \le 0.40 \\ 1.00/T; & 0.40 \le T \le 4.0 \end{cases}$$

For medium soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.5; & 0.10 \le T \le 0.55 \\ 1.36/T; & 0.55 \le T \le 4.0 \end{cases}$$

For soft soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.5; & 0.10 \le T \le 0.67 \\ 1.67/T; & 0.67 \le T \le 4.0 \end{cases}$$

5) Determination of storey shear in each mode

The peak shear force will be obtained by

$$V_{ik} = \sum_{j=i+1}^{n} Q_{ik}$$

6) Determination of storey shear force due to all modes

The peak storey shear force (V_i) in storey i due to all modes considering is obtained by combining those due to each mode in accordance with modal combination SRSS (Square Root of Sum of Squares) or CQC (Complete Quadratic Combination) methods.

a) Maximum Absolute Value Sum (AVS)

$$V_i = \sum_{k=1}^{n} \left|V_{ik}\right| \qquad 3.15$$

b) Square Root of Sum of Squares (SRSS)

If the building does not have closely spaced modes, the peak response quantity (λ) due to all modes considered shall be obtained as,

$$\lambda = \sqrt{\sum_{k=1}^{r} (\lambda_k)^2}$$

Where

λ_k = Absolute value of quantity in mode 'k', and r is the numbers of modes being considered.

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c) Complete Quadratic Combination (CQC)

$$\lambda = \sqrt{\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i \rho_{ij} \lambda_j} \qquad \rho_{ij} = \frac{8\zeta^2 (1+\beta_{ij})\beta^{1.5}}{(1-\beta_{ij})^2 + 4\zeta^2 \beta_{ij}(1+\beta_{ij})^2}$$

Where
r = Number of modes being considered
ρ_{ij} = Cross modal coefficient
λ_i = Response quantity in mode i (including sign)
λ_j = Response quantity in mode j (including sign)
ζ = Modal damping ration (in fraction)
β_{ij} = Frequency ratio ω_j/ω_i
ω_i = Circular frequency in i^{th} mode, and ω_j = Circular frequency in j^{th} mode.

7) Determination of lateral forces at each storey
The design lateral forces F_{roof} and F_i, at roof and at i^{th} floor, are calculated as,

$F_{roof} = V_{roof}$, and $F_i = V_i - V_{i+1}$

H: Calculation of moment due to design lateral forces

$$M = \sum_{i=1}^{n} F_i h_i$$

III. ANALYSIS OF MULTI STOREY PLANE FRAME

A. Equivalent static force method

1) Frame Without infills

a) Calculation of Lumped masses to various floor levels

Roof
= Mass of infill + Mass of columns + Mass of beams in longitudinal and transverse direction of that floor + Mass of slab + Imposed load of that floor if permissible

={((0.25x10x(3.5/2)+0.15x15x(3.5/2))20}+{(0.25x10x0.40+0.25x15x0.35)25}+{0.10x5x10x25}+{(0.25x0.45x(0.35/2)x3)x25}
= 363.82 kN (weight) = 37.087 ton (mass)

3rd, 2nd, 1st Floors

={((0.25x10x3.5)+(0.15x15x3.5))20}+{(0.25x10x0.40+0.25x15x0.35)25}+{0.10x5x10x25}+{0.25x0.45x3.5x3x25}+{5x10x3.5x0.5}
= 632.43 kN (weight) = 64.45 ton (mass)

- 50% of imposed load, if imposed load is greater than 3 kN/m^2

Seismic weight of building

= Seismic weight of all floors = $M_1+M_2+M_3+M_4$
= 64.46+64.46+64.46+37.09 = 230.43

b) Determination of Fundamental Natural Period

The approximate fundamental natural period of a vibration (T_a), in seconds, of a moment resisting frame building without brick infill panels may be estimated by the empirical expression

$T_a = 0.075xh^{0.75}$

$= 0.075x14^{0.75} = 0.5423$ s

Where h is the height of the building, in meters.

c) Determination of Design Base Shear
Design seismic base shear, $V_B = A_h W$

$$A_h = \frac{Z}{2}\frac{I}{R}\frac{S_a}{g} = \frac{0.24}{2}\frac{1}{5}1.842 = 0.0443$$

For $T_a = 0.5423 \Rightarrow \frac{S_a}{g} = \frac{1}{T_a} = 1.842$, for rock site from Fig 2 of IS 1893 (part 1): 2002

Design seismic base shear, V_B = 0.0443 x (230.43x9.81) = 99.933 kN

d) Vertical Distribution of Base Shear

$$Q_i = V_B\left(\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2 + W_4 h_4^2}\right)$$

$$=99.93\left(\frac{632.25x3.5^2}{632.25x3.5^2 + 632.25x7^2 + 632.25x10.5^2 + 363x14^2}\right)$$

=4.307kN

Similarly,
Q_2 = 0.1724 x 99.933 = 17.23 kN
Q_3 = 0.3872 x 99.933 = 38.768kN
Q_4 = 0.3967 x 99.933 = 39.654kN

Maximum overturning moment=1097.9kN-m

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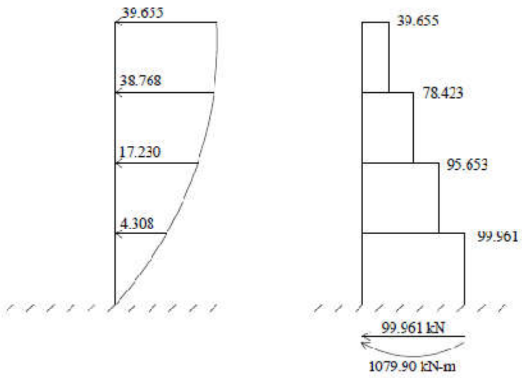

Fig.1. Distribution of lateral forces and shear by Equivalent static force method on frame without infill

2) Frame With infills

a) Calculation of Lumped masses to various floor levels

Roof

Mass of infill + Mass of columns + Mass of beams in longitudinal and transverse direction of that floor + Mass of slab + Imposed load of that floor if permissible

= 363.82 kN (weight) = 37.087 ton (mass)

3^{rd}, 2^{nd}, 1^{st} Floors

={((0.25x10x3.5)+(0.15x15x3.5))20}+{(0.25x10x0.40+0.25x 15x0.35)25}+{0.10x5x10x25}+
{0.25x0.45x3.5x3x25}+{5x10x3.5x0.5}
= 632.43 kN (weight) = 64.45 ton (mass)

- 50% of imposed load, if imposed load is greater than 3 kN/m^2

Seismic weight of building

= Seismic weight of all floors = M_1+M_2+M_3+M_4
= 64.46+64.46+64.46+37.09 = 230.47

b) Determination of Fundamental Natural Period

$$T_a = 0.09H/\sqrt{D}\sqrt{D}$$
$$= 0.09x14/\sqrt{10}\sqrt{10} = 0.3984 \text{ sec}$$

c) Determination of Design Bash Shear

$$A_h = \frac{Z}{2}\frac{I}{R}\frac{S_a}{g} = \frac{0.24}{2}\frac{1}{5}x2.5 = 0.06$$

For T_a = 0.3984 => $\frac{S_a}{g} = \frac{1}{T_a}$ = 2.5, for rock site from Figure 2 of IS 1893 (part 1): 2002

Design seismic base shear, V_B = 0.06 x (230.43x9.81) = 134.68kN

d) Vertical Distribution of Base Shear

$$Q_i = V_B\left(\frac{W_1h_1^2}{W_1h_1^2 + W_2h_2^2 + W_3h_3^2 + W_4h_4^2}\right)$$
$$=135.6$$
$$\left(\frac{632.25x3.5^2}{632.25x3.5^2 + 632.25x7^2 + 632.25x10.5^2 + 363x14^2}\right)$$

= 5.8kN

Similarly,

Q_2 = 0.1724 x 135.63 = 23.215kN
Q_3 = 0.3872 x 135.63 = 52.234kN
Q_4 = 0.3967 x 135.63 = 53.429kN

Maximum overturning moment=1479.3kN-m

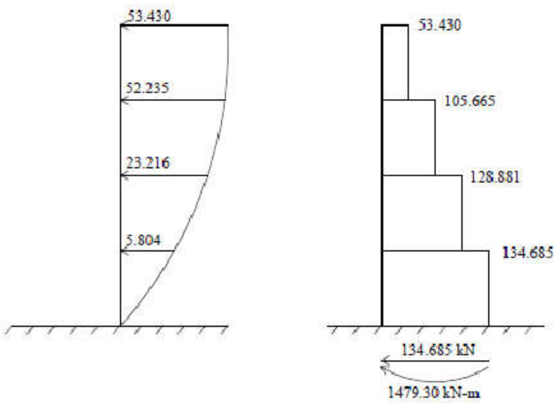

Fig.2. Distributions of lateral forces and shear by Equivalent static force method on frame with in fills

B. Response Spectrum Method

1) Frame without stiffness of infills

a) Determination of Eigen values and Eigen vectors

The mass and stiffness matrices at each floor level are calculated in S.I units and then modeled in the form of matrix as

$$M = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix}$$

$$M = 10^3 \begin{bmatrix} 64.46 & 0 & 0 & 0 \\ 0 & 64.46 & 0 & 0 \\ 0 & 0 & 64.46 & 0 \\ 0 & 0 & 0 & 37.09 \end{bmatrix} \text{kg}$$

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$$K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix}$$

Column stiffness of storey,

$$k = \frac{12EI}{L^3} = \frac{12x22360x10^3\left(\frac{0.25x0.45^3}{12}\right)}{3.5^3} = 11880.78 \text{ kN/m}$$

Total lateral stiffness of each storey,

$k_1 = k_2 = k_3 = k_4 = 3x11880.78 = 35642.36$ kN/m

$$K = \begin{bmatrix} 71284.72 & -3564.36 & 0 & 0 \\ -3564.36 & 71284.72 & -3564.36 & 0 \\ 0 & -3564.36 & 71284.72 & -35642.36 \\ 0 & 0 & -35642.36 & 35642.36 \end{bmatrix} \text{kN/m}$$

Eigen values

$$\left[\omega^2\right] = \begin{bmatrix} 81 & 0 & 0 & 0 \\ 0 & 657 & 0 & 0 \\ 0 & 0 & 1475 & 0 \\ 0 & 0 & 0 & 2065 \end{bmatrix}$$

Eigen vectors

$$[\Phi] = [\Phi_1\ \Phi_2\ \Phi_3\ \Phi_4] = \begin{bmatrix} -0.0328 & 0.0795 & 0.0808 & -0.0397 \\ -0.0608 & 0.0644 & -0.0540 & 0.0690 \\ -0.0798 & -0.0273 & -0.0448 & -0.0799 \\ -0.0872 & -0.0865 & 0.0839 & 0.0696 \end{bmatrix}.$$

Natural frequency in various modes

$$[\omega] = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 25.63 & 0 & 0 \\ 0 & 0 & 38.41 & 0 \\ 0 & 0 & 0 & 45.44 \end{bmatrix}$$

Natural time period

$$[T] = \begin{bmatrix} 0.6977 & 0 & 0 & 0 \\ 0 & 0.2450 & 0 & 0 \\ 0 & 0 & 0.1636 & 0 \\ 0 & 0 & 0 & 0.1383 \end{bmatrix}$$

b) Modal participation factor

$$\{P\} = \frac{\sum_{i=1}^{4} W_i \Phi_{ik}}{\sum_{i=1}^{4} W_i (\Phi_{ik})^2} = \frac{(W_1\Phi_{1k} + W_2\Phi_{2k} + ...+ W_4\Phi_{4k})}{\left[W_1(\Phi_{1k})^2 + W_2(\Phi_{2k})^2 + ...+ W_4(\Phi_{4k})^2\right]} = \begin{Bmatrix} -14.40 \\ 4.30 \\ 1.95 \\ -0.68 \end{Bmatrix}$$

c) Determination of model mass

$$M_1 = \frac{[9.81(64.45(-0.0328) + ... + 37.08(-0.0872))]^2}{9.81[9.81(64.45(-0.0328)^2 + ... + 37.08(-0.0872)^2)]}$$

$= 207.60$

Similarly, $M_2 = 18.54$, $M_3 = 3.82$, $M_4 = 0.47$

Modal contributions of various modes

For mode 1, $\frac{M_1}{M} = \frac{207.60}{230.43} = 0.90 = 90\%$

For mode 2, $\frac{M_2}{M} = \frac{18.54}{230.43} = 0.0804 = 8.04\%$

For mode 3, $\frac{M_3}{M} = \frac{3.82}{230.43} = 0.0165 = 1.65\%$

For mode 4, $\frac{M_4}{M} = \frac{0.47}{230.43} = 0.0020 = 0.20\%$

d) Determination of lateral force on each floor

The design lateral force (Q_{ik}) at floor i in mode k is given by,

$$Q_{ik} = A_k \Phi_{ik} P_k W_i$$

The design horizontal seismic coefficient A_h for various modes are,

$$A_{h1} = \frac{Z}{2}\frac{I}{R}\frac{S_{a1}}{g} = \frac{0.24}{2}\frac{1}{5}1.433 = 0.0343$$

$$A_{h2} = \frac{Z}{2}\frac{I}{R}\frac{S_{a2}}{g} = \frac{0.24}{2}\frac{1}{5}2.5 = 0.060$$

Similarly,

$A_{h3} = 0.060$, $A_{h4} = 0.060$.

For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.5; & 0.10 \le T \le 0.40 \\ 1.00/T; & 0.40 \le T \le 4.0 \end{cases}$$

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For $T_1 = 0.6978 \Rightarrow \dfrac{S_{a1}}{g} = 1.433$

For $T_2 = 0.2450 \Rightarrow \dfrac{S_{a2}}{g} = 2.5$

For $T_3 = 0.1636 \Rightarrow \dfrac{S_{a3}}{g} = 2.5$

For $T_4 = 0.1382 \Rightarrow \dfrac{S_{a4}}{g} = 2.5$

Design lateral force

$$[Q_{i1}] = \begin{bmatrix} (A_{h1}\ P_1\ \Phi_{11}\ W_1) \\ (A_{h2}\ P_1\ \Phi_{21}\ W_2) \\ (A_{h3}\ P_1\ \Phi_{31}\ W_3) \\ (A_{h4}\ P_1\ \Phi_{41}\ W_4) \end{bmatrix}$$

$$= \begin{bmatrix} ((0.0343)(-14.40)(-0.0328)(64.45\times 9.81)) \\ ((0.0343)(-14.40)(-0.0608)(64.45\times 9.81)) \\ ((0.0343)(-14.40)(-0.0798)(64.45\times 9.81)) \\ ((0.0343)(-14.40)(-0.0872)(37.08\times 9.81)) \end{bmatrix} = \begin{bmatrix} (10.275) \\ (19.043) \\ (25.018) \\ (15.720) \end{bmatrix} kN$$

Similarly,

$$[Q_{i2}] = \begin{bmatrix} -43.44 \\ -35.199 \\ 14.920 \\ 27.207 \end{bmatrix}, [Q_{i3}] = \begin{bmatrix} -44.204 \\ 29.499 \\ 24.517 \\ -26.385 \end{bmatrix}, [Q_{i2}] = \begin{bmatrix} 21.749 \\ -37.722 \\ 43.675 \\ -21.878 \end{bmatrix}$$

e) Determination of storey shear in each mode

The storey shear forces for the first mode is,

$$V_{i1} = \sum_{j=i+1}^{n} Q_{i1} = \begin{bmatrix} (Q_{11}+Q_{21}+Q_{31}+Q_{41}) \\ (Q_{21}+Q_{31}+Q_{41}) \\ (Q_{31}+Q_{41}) \\ (Q_{41}) \end{bmatrix} = \begin{bmatrix} 70.056 \\ 59.925 \\ 40.738 \\ 15.720 \end{bmatrix} kN$$

$$V_{i2} = \begin{bmatrix} V_{12} \\ V_{22} \\ V_{32} \\ V_{42} \end{bmatrix} = \begin{bmatrix} -36.514 \\ 6.927 \\ 42.127 \\ 27.207 \end{bmatrix}, V_{i3} = \begin{bmatrix} V_{13} \\ V_{23} \\ V_{33} \\ V_{43} \end{bmatrix} = \begin{bmatrix} -16.572 \\ 27.632 \\ -1.867 \\ -26.385 \end{bmatrix},$$

$$V_{i4} = \begin{bmatrix} V_{14} \\ V_{24} \\ V_{34} \\ V_{44} \end{bmatrix} = \begin{bmatrix} 5.824 \\ -15.925 \\ 21.796 \\ -21.878 \end{bmatrix}$$

f) Determination of storey shear force due to all modes

Maximum Absolute Value Sum (AVS)

$$V_i = \sum_{k=1}^{n} |v_{ik}| = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \begin{bmatrix} 70.056+36.514+16.572+5.824 \\ 59.781+6.927+27.632+15.925 \\ 40.738+42.127+1.867+21.796 \\ 15.720+27.207+26.385+21.878 \end{bmatrix} = \begin{bmatrix} 128.986 \\ 110.282 \\ 106.544 \\ 91.207 \end{bmatrix}$$

Square Root of Sum of Squares (SRSS)

$V_1 = [(V_{11})^2 + (V_{12})^2 + (V_{13})^2 + (V_{14})^2]^{1/2}$
$= [(70.056)^2 + (-36.514)^2 + (-16.572)^2 + (5.824)^2]^{1/2}$
$= 80.930kN$
Similarly,
$V_2 = 68.110kN$
$V_3 = 62.553kN$
$V_4 = 46.507kN$

Complete Quadratic Combination (CQC)

$V_1 =$

$$\sqrt{\left\langle [70.056 - 36.514 - 16.572 + 5.824] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 70.056 \\ -36.51 \\ -16.57 \\ 5.82 \end{bmatrix} \right\rangle}$$

$V_1 = [80.70]$

$V_2 =$

$$\sqrt{\left\langle [59.781 + 6.927 + 27.632 - 15.925] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 59.78 \\ 6.92 \\ 27.63 \\ -15.92 \end{bmatrix} \right\rangle}$$

$V_2 = [66.61]$

$V_3 =$

$$\sqrt{\left\langle [40.738 + 42.127 - 1.867 + 21.796] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 40.738 \\ 42.127 \\ -1.867 \\ 21.796 \end{bmatrix} \right\rangle}$$

$V_3 = [62.95]$

$V_4 =$

$$\sqrt{\left\langle [15.720 + 27.207 - 26.385 - 21.878] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 15.720 \\ 27.207 \\ -26.385 \\ -21.878 \end{bmatrix} \right\rangle}$$

$V_4 = [48.48]$

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g) Determination of Lateral Forces at Each Storey

Maximum Absolute Value Sum (AVS)

$F_{roof} = V_4 = 91.207$ kN
$F_{floor\ 3} = F_3 = V_3 - V_4 = 106.544 - 91.207 = 15.337$ kN
$F_{floor\ 2} = F_2 = V_2 - V_3 = 110.2815 - 106.544 = 3.738$ kN
$F_{floor\ 1} = F_1 = V_1 - V_2 = 128.986 - 110.282 = 18.705$ kN

Square Root of Sum of Squares (SRSS)

$F_{roof} = F_4 = V_4 = 46.508$kN
$F_{floor\ 3} = F_3 = V_3 - V_4 = 62.562 - 46.508 = 16.055$kN
$F_{floor\ 2} = F_2 = V_2 - V_3 = 68.119 - 62.562 = 5.56$kN
$F_{floor\ 1} = F_1 = V_1 - V_2 = 80.941 - 68.119 = 12.823$kN

Complete Quadratic Combination (CQC)

$F_{roof} = F_4 = V_4 = 48.492$kN
$F_{floor\ 3} = F_3 = V_3 - V_4 = 62.957 - 48.492 = 14.465$kN
$F_{floor\ 2} = F_2 = V_2 - V_3 = 66.625 - 62.957 = 3.67$kN
$F_{floor\ 1} = F_1 = V_1 - V_2 = 80.714 - 66.625 = 14.089$kN

h) Calculation of moment due to design lateral forces

Absolute Value Sum (AVS)
Overturning moment = 1529.6 kN-m

Square Root of Sum of Squares (SRSS)
Overturning moment = 903.456 kN-m

Complete Quadratic Combination (CQC)
Overturning moment = 905.758 kN-m

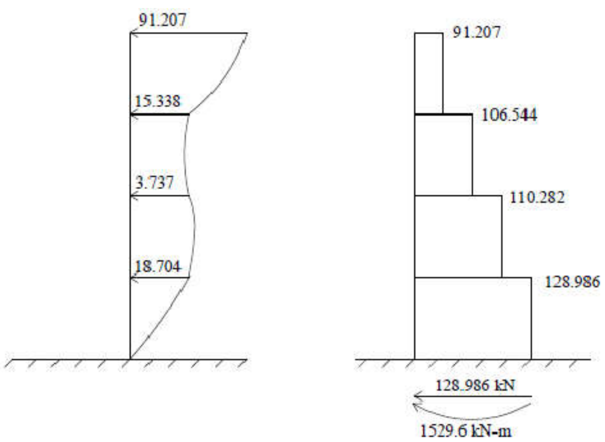

Fig.3. Distribution of lateral forces and shear by Absolute value sum method on frame without infills

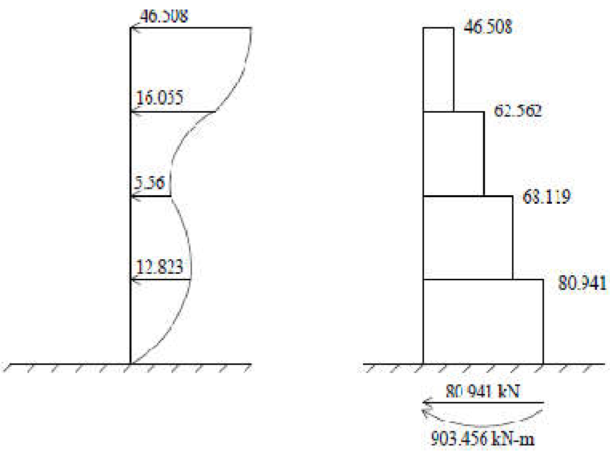

Fig.4. Distributions of lateral forces and shear by SRSS method on frame without infills

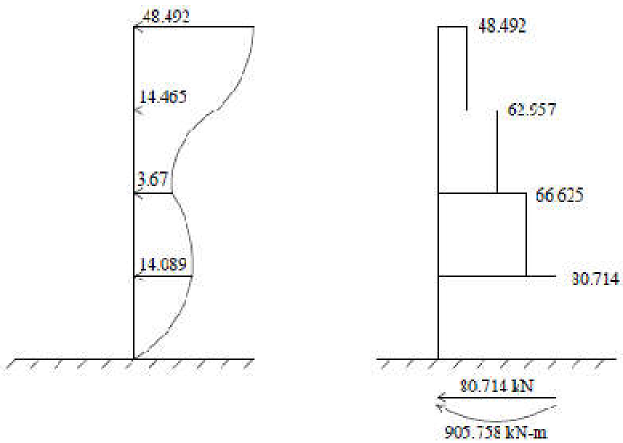

Fig.5. Distribution of lateral forces and shear by CQC method on frame without infills

2) Frame with the stiffness of Infills

a) Determination of Eigen values and Eigen vectors

$$M = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix}$$

$$M = 10^3 \begin{bmatrix} 64.46 & 0 & 0 & 0 \\ 0 & 64.46 & 0 & 0 \\ 0 & 0 & 64.46 & 0 \\ 0 & 0 & 0 & 37.09 \end{bmatrix} \text{kg}$$

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Column stiffness of storey,

$$k = \frac{12EI}{L^3} = \frac{12x22360x10^3(0.001893)}{3.5^3} = 11846.758 \text{ kN/m}$$

E_f = Young's modulus of the concrete = 22360 N/m^2

E_m = Young's modulus of the masonry infill = 13800 N/m^2

h = Height of infill wall = 3.5 m

l = length of wall = 5 m

t = Thickness of wall = 250 mm

I_c =Moment of inertia of columns = $\frac{1}{12}$x (0.25 x 0.45^3)

= 0.001893 m^4

I_b =Moment of inertia of beam = $\frac{1}{12}$x (0.25 x 0.40^3)

= 0.001333 m^4

$W = \frac{1}{2} x \sqrt{\alpha_h^2 + \alpha_l^2} = 0.7885$

A=cross sectional area of diagonal stiffness

= W*t = 0.7885 x 0.25 = 0.1972 m^2

l_d =Diagonal length of strut = $\sqrt{h^2 + l^2}$ = 6.103 m

$$\alpha_h = \frac{\pi}{2}\left[\frac{22360 \text{ x } 0.001893 \text{ x } 3.5}{2 \text{ x } 13800 \text{ x } 0.25 \text{ x } \sin 2 \text{ x } 35}\right]^{1/4} = 0.611\,\text{m}$$

$$\alpha_l = \pi\left[\frac{22360 \text{ x } 0.001333 \text{ x } 5.0}{13800 \text{ x } 0.25 \text{ x } \sin 2 \text{ x } 35}\right]^{1/4} = 1.45\,\text{m}$$

Therefore, stiffness of infill is

$$\frac{AE_m}{l_d}\cos^2\theta = \frac{0.1972 \text{ x } 13800 \text{ x } 10^6}{6.103}\ 0.819^2$$
$$= 299086.078 \text{ N/m}$$

For the frame with two bays there are two struts participating in one direction, total lateral stiffness of each storey

$k_1 = k_2 = k_3 = k_4$ = 3x11846.758 + 2x299086078

= 633712.430 kN/m

$$K = 10^9\begin{bmatrix} 1.2686 & -0.6343 & 0 & 0 \\ -0.6343 & 1.2686 & -0.6343 & 0 \\ 0 & -0.6343 & 1.2686 & -0.6343 \\ 0 & 0 & -0.6343 & 0.6343 \end{bmatrix} \text{ N/m}$$

Eigen values

$$[\omega^2] = \begin{bmatrix} 1443 & 0 & 0 & 0 \\ 0 & 11707 & 0 & 0 \\ 0 & 0 & 26247 & 0 \\ 0 & 0 & 0 & 36746 \end{bmatrix}$$

Eigen vectors

$$[\Phi] = [\Phi_1\ \Phi_2\ \Phi_3\ \Phi_4] = \begin{bmatrix} -0.0010 & 0.0025 & 0.0025 & -0.0013 \\ -0.0019 & 0.0020 & -0.0017 & 0.0022 \\ -0.0025 & -0.0009 & -0.0014 & -0.0025 \\ -0.0028 & -0.0027 & 0.0027 & 0.0022 \end{bmatrix}.$$

Natural frequency in various modes

$$[\omega] = \begin{bmatrix} 37.99 & 0 & 0 & 0 \\ 0 & 108.198 & 0 & 0 \\ 0 & 0 & 162.008 & 0 \\ 0 & 0 & 0 & 191.693 \end{bmatrix}$$

Natural time period

$$[T] = \begin{bmatrix} 0.1654 & 0 & 0 & 0 \\ 0 & 0.0581 & 0 & 0 \\ 0 & 0 & 0.0388 & 0 \\ 0 & 0 & 0 & 0.0328 \end{bmatrix}$$

b) Determination of model participation factor

$$\{P\} = \frac{\sum_{i=1}^{4} W_i \Phi_{ik}}{\sum_{i=1}^{4} W_i (\Phi_{ik})^2}$$

$$= \frac{(W_1\Phi_{1k} + W_2\Phi_{2k} + \ldots + W_4\Phi_{4k})}{\left[W_1(\Phi_{1k})^2 + W_2(\Phi_{2k})^2 + \ldots + W_4(\Phi_{4k})^2\right]} = \begin{Bmatrix} -14.40 \\ 4.30 \\ 1.95 \\ -0.68 \end{Bmatrix}$$

c) Determination of model mass

$$M_1 = \frac{\left[\sum_{i=1}^{4} W_i \Phi_{i1}\right]^2}{g\left[\sum_{i=1}^{4} W_i (\Phi_{i1})^2\right]}$$

$$M_1 = \frac{[9.81(64.45(-0.0328) + \ldots + 37.08(-0.0872))]^2}{9.81[9.81(64.45(-0.0328)^2 + \ldots + 37.08(-0.0872)^2)]}$$

Similarly, M2 = 18.54, M3 = 3.82, M4 = 0.47.

Modal contributions of various modes

For mode 1, $\frac{M_1}{M} = \frac{207.60}{230.43} = 0.90 = 90\%$

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For mode 2, $\frac{M_2}{M} = \frac{18.54}{230.43} = 0.0804 = 8.04\%$

For mode 3, $\frac{M_3}{M} = \frac{3.82}{230.43} = 0.0165 = 1.65\%$

For mode 4, $\frac{M_4}{M} = \frac{0.47}{230.43} = 0.0020 = 0.20\%$

d) Determination of lateral force on each floor

The design lateral force (Q_{ik}) at floor i in mode k is given by

$$Q_{ik} = A_k \Phi_{ik} P_k W_i$$

The design horizontal seismic coefficients Ah for various modes are,

$$A_{h1} = \frac{Z}{2}\frac{I}{R}\frac{S_{a1}}{g} = \frac{0.24}{2}\frac{1}{5}2.5 = 0.060$$

$$A_{h2} = \frac{Z}{2}\frac{I}{R}\frac{S_{a2}}{g} = \frac{0.24}{2}\frac{1}{5}1.871 = 0.045$$

Similarly,

$$A_{h3} = 0.037, A_{h4} = 0.035$$

For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T; & 0.00 \le T \le 0.10 \\ 2.5; & 0.10 \le T \le 0.40 \\ 1.00/T; & 0.40 \le T \le 4.0 \end{cases}$$

For $T_1 = 0.1655 \Rightarrow \frac{S_{a1}}{g} = 2.5$

For $T_2 = 0.0581 \Rightarrow \frac{S_{a2}}{g} = 1+15T = 1.871$

For $T_3 = 0.0388 \Rightarrow \frac{S_{a3}}{g} = 1+15T = 1.582$

For $T_4 = 0.0382 \Rightarrow \frac{S_{a4}}{g} = 1+15T = 1.492$

Design lateral force

$$[Q_{i1}] = \begin{bmatrix} (A_{h1}\ P_1\ \Phi_{11}\ W_1) \\ (A_{h2}\ P_1\ \Phi_{21}\ W_2) \\ (A_{h3}\ P_1\ \Phi_{31}\ W_3) \\ (A_{h4}\ P_1\ \Phi_{41}\ W_4) \end{bmatrix}$$

$$= \begin{bmatrix} ((0.060)(-14.40)(-0.0328)(64.45\times 9.81)) \\ ((0.060)(-14.40)(-0.0608)(64.45\times 9.81)) \\ ((0.060)(-14.40)(-0.0798)(64.45\times 9.81)) \\ ((0.060)(-14.40)(-0.0872)(64.45\times 9.81)) \end{bmatrix} = \begin{bmatrix} (17.922) \\ (33.215) \\ (43.637) \\ (27.419) \end{bmatrix} \text{kN}$$

Similarly,

$$[Q_{i2}] = \begin{bmatrix} -32.512 \\ -26.343 \\ 11.166 \\ 20.361 \end{bmatrix}, [Q_{i3}] = \begin{bmatrix} -27.972 \\ 18.667 \\ 15.514 \\ -16.696 \end{bmatrix}, [Q_{i2}] = \begin{bmatrix} 12.980 \\ -22.512 \\ 26.065 \\ -13.057 \end{bmatrix}$$

e) Determination of storey shear in each mode

The peak shear force

$$V_{ik} = \sum_{j=i+1}^{n} Q_{ik}$$

The storey shear forces for the first mode is,

$$V_{i1} = \sum_{j=i+1}^{n} Q_{i1} = \begin{bmatrix} (Q_{11} + Q_{21} + Q_{31} + Q_{41}) \\ (Q_{21} + Q_{31} + Q_{41}) \\ (Q_{31} + Q_{41}) \\ (Q_{41}) \end{bmatrix} = \begin{bmatrix} 70.056 \\ 59.925 \\ 40.738 \\ 15.720 \end{bmatrix} \text{kN}$$

Similarly,

$$V_{i2} = \begin{bmatrix} V_{12} \\ V_{22} \\ V_{32} \\ V_{42} \end{bmatrix} = \begin{bmatrix} -27.327 \\ 5.184 \\ 31.528 \\ 20.361 \end{bmatrix}, V_{i3} = \begin{bmatrix} V_{13} \\ V_{23} \\ V_{33} \\ V_{43} \end{bmatrix} = \begin{bmatrix} -10.487 \\ 17.485 \\ -1.182 \\ -16.696 \end{bmatrix},$$

$$V_{i2} = \begin{bmatrix} V_{14} \\ V_{24} \\ V_{34} \\ V_{44} \end{bmatrix} = \begin{bmatrix} 3.476 \\ -9.5042 \\ 13.008 \\ -13.057 \end{bmatrix}$$

f) Determination of storey shear force due to all modes

Maximum Absolute Value Sum (AVS)

$$V_i = \sum_{k=1}^{n} |V_{ik}| = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \begin{bmatrix} 122.194 + 27.327 + 10.487 + 3.476 \\ 104.272 + 5.184 + 17.485 + 9.5042 \\ 71.057 + 31.528 + 1.182 + 13.008 \\ 27.419 + 20.361 + 16.696 + 13.057 \end{bmatrix}$$

$$= \begin{bmatrix} 163.508 \\ 136.463 \\ 116.791 \\ 77.544 \end{bmatrix} \text{kN}$$

Square Root of Sum of Squares (SRSS)

$$\lambda = \sqrt{\sum_{k=1}^{r} (\lambda_k)^2}$$

$V_1 = [(V_{11})^2 + (V_{12})^2 + (V_{13})^2 + (V_{14})^2]^{1/2}$
$= [(122.194)^2+(-27.327)^2+(-10.487)^2+(3.475)^2]^{1/2}$
$= 125.699$ kN
Similarly,
$V_2 = 106.281$kN
$V_3 = 78.827$kN
$V_4 = 40.196$kN

Complete Quadratic Combination (CQC)

$$\lambda = \sqrt{\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i \rho_{ij} \lambda_j}$$

$$V_1 = \sqrt{\left\langle [122.194 - 27.327 - 10.487 + 3.475] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 122.272 \\ -27.327 \\ -10.487 \\ 3.475 \end{bmatrix} \right\rangle}$$

$V_1 = [125.512]$

$$V_2 = \sqrt{\left\langle [104.272 + 5.184 + 17.485 - 9.504] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 104.272 \\ 5.184 \\ 17.485 \\ -9.504 \end{bmatrix} \right\rangle}$$

$V_2 = [105.977]$

$$V_3 = \sqrt{\left\langle [71.057 + 31.528 - 1.182 + 13.008] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 71.057 \\ 31.528 \\ -1.182 \\ 13.008 \end{bmatrix} \right\rangle}$$

$V_3 = [79.125]$

$$V_4 = \sqrt{\left\langle [27.419 + 20.362 - 16.696 - 13.057] \begin{bmatrix} 1 & 0.0073 & 0.0031 & 0.0023 \\ 0.0073 & 1 & 0.0559 & 0.0278 \\ 0.0031 & 0.0559 & 1 & 0.2597 \\ 0.0023 & 0.0278 & 0.2597 & 1 \end{bmatrix} \begin{bmatrix} 27.419 \\ 20.362 \\ -16.696 \\ -13.057 \end{bmatrix} \right\rangle}$$

$V_4 = [40.964]$

g) Determination of Lateral Forces at Each Storey

Maximum Absolute Value Sum (AVS)

$F_{roof} = F_4 = V_4 = 77.544$ kN
$F_{floor\ 3} = F_3 = V_3 – V_4 = 116.791 – 77.544 = 39.247$ kN
$F_{floor\ 2} = F_2 = V_2 – V_3 = 136.463 – 116.791 = 19.672$kN
$F_{floor\ 1} = F_1 = V_1 – V_2 = 163.508 – 136.463 = 27.045$kN

Square Root of Sum of Squares (SRSS)

$F_{roof} = F_4 = V_4 = 40.202$kN
$F_{floor\ 3} = F_3 = V_3 – V_4 = 78.840 – 40.202 = 38.638$kN
$F_{floor\ 2} = F_2 = V_2 – V_3 = 106.297 – 78.840 = 27.457$kN
$F_{floor\ 1} = F_1 = V_1 – V_2 = 125.716 – 106.297 = 19.422$kN

Complete Quadratic Combination (CQC)

$F_{roof} = F_4 = V_4 = 44.990 - 0 = 44.990$kN
$F_{floor\ 3} = F_3 = V_3 – V_4 = 79.136 – 44.990 = 38.146$kN
$F_{floor\ 2} = F_2 = V_2 – V_3 = 105.995 – 79.136 = 26.859$kN
$F_{floor\ 1} = F_1 = V_1 – V_2 = 125.533 – 105.995 = 19.538$kN

h) Calculation of moment due to design lateral forces

$$M = \sum_{i=1}^{n} F_i h_i$$

Absolute Value Sum (AVS)
Overturning moment=1730.1kN-m

Square Root of Sum of Squares (SRSS)
Overturning moment=1228.7kN-m

Complete Quadratic Combination (CQC)
Overturning moment=1230.8kN-m

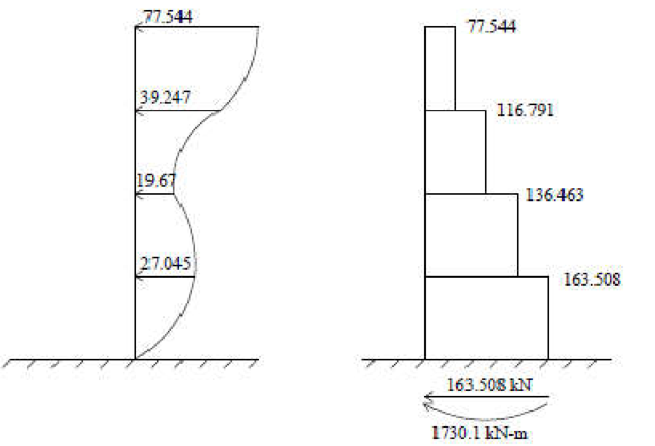

Fig.6. Distribution of lateral forces and shear by AVS method on frame with infills

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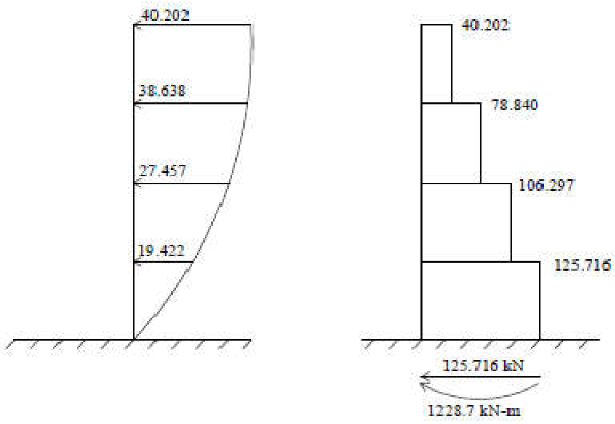

Fig.7. Distribution of lateral forces and shear by SRSS method on frame with infills

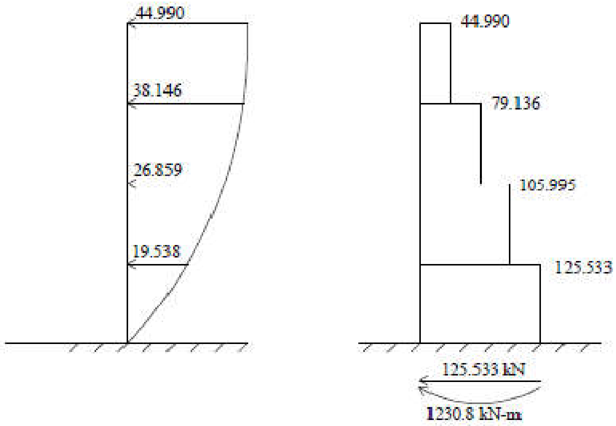

Fig.8. Distribution of lateral forces and shear by CQC method on frame with infills

CONCLUSION

In case of Equivalent Static Force method, the design base shear value is found maximum for frame with infills over the frame without infills with the seismic weight being unchanged. The increase in base shear is due to natural period of the structure according to IS 1890:2002. This method is suitable for structures with low to medium range heights.

In case of Response spectrum method, Complete Quadratic Combination (CQC) provides, reasonable accuracy over AVS and SRSS methods due to provision for modal contribution. This method is suitable for almost all kinds of engineering interest.

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