**Abstract**

In this paper, we introduced a new class of sc*g-homeomorphisms in Topological space.

**Key words:** Homeomorphism - strongly g-homeomorphism - sc*g-homeomorphism.

**1. Introduction**

Several mathematicians have generalized homeomorphisms in topological spaces. Biswas [18], Crossley and Hildebrand [19], Gentry and Hoyle [20] and Umehara and Maki [21] have introduced and investigated semi-homeomorphisms somewhat homeomorphisms and g-A-homeomorphisms Crossely and Hildebrand defined yet another “semi-homeomorphism” which is also a generalization of homeomorphisms. Sundaram [6] introduced g-homeomorphisms and gc-homeomorphisms in topological spaces.

In the section, we introduce the concept of sc*g-homeomorphisms and study some of their properties.

**Definition: 2.7.1** A bijection \( f : (X, \tau) \rightarrow (Y, \sigma) \) from a topological space \( X \) into a topological space \( Y \) is called a sc*g-generalized homeomorphism (sc*g-homeomorphism) if \( f \) is both sc*g-open and sc*g-continuous.

**Theorem: 2.7.2** Every homeomorphism is a sc*g-homeomorphism.

**Proof:** Since every continuous function is sc*g-continuous and every open map is sc*g-open, the proof follows.

The converse of the above theorem need not be true as seen from the following example.

**Example: 2.7.3** Consider the topological spaces \( X = Y = \{a, b, c\} \) with topologies \( \tau = \{\emptyset, X, \{a, b\}\} \) and \( \sigma = \{\emptyset, X, \{a\}, \{a, b\}\} \). Then the identity map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is a sc*g-homeomorphism but not a homeomorphism. Since for the open set \( \{a\} \) in \( Y \) is not open in \( X \).

**Theorem: 2.7.4** Every strongly g-homeomorphism is a sc*g-homeomorphism but not conversely.

**Proof:** Let \( f : X \rightarrow Y \) be a strongly g-homeomorphism. Then \( f \) is strongly g-continuous and strongly g-open. Since every strongly g-continuous function is sc*g-continuous and every strongly g-open map is sc*g-open, \( f \) is sc*g-continuous and sc*g-open. Hence \( f \) is a sc*g-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

**Example: 2.7.5** Let \( X = Y = \{a, b, c\} \) with \( \tau = \{\emptyset, X, \{a\}\} \) and \( \sigma = \{\emptyset, Y, \{a, b\}\} \) respectively. Then the identity map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is sc*g-homeomorphism but not a strongly g-homeomorphism. Since for the open set \( \{a, b\} \) in \( Y \) is not a strongly g-open set in \( X \).

Next we shall characterize the sc*g-homeomorphism and sc*g-open maps.

**Theorem: 2.7.6** For any bijection \( f : X \rightarrow Y \) the following statements are equivalent.

(a) \( f^{-1} \) : \( Y \rightarrow X \) is sc*g-continuous.

(b) \( f \) is a sc*g-open map.

(c) \( f \) is a sc*g-closed map.

**Proof:**

(a) \( \rightarrow \) (b) Let \( G \) be any open set in \( X \). Since \( f^{-1} \) is sc*g-continuous, the inverse image of \( G \) under \( f^{-1} \), namely \( f(G) \) is sc*g-open in \( Y \) and so \( f \) is sc*g-continuous.

(b) \( \rightarrow \) (c) Let \( f \) be any closed set in \( X \). Then \( f^c \) is open in \( X \). Since \( f \) is sc*g-open, \( f(F) \) is sc*g-open in \( Y \). But \( f(F^c) = Y - f(F) \) and so \( f(F) \) is sc*g-closed in \( Y \). Therefore \( f \) is sc*g-closed map.

(c) \( \rightarrow \) (a) Let \( F \) be any closed set in \( X \). Then \( (f^{-1})^{-1}(F) = f(F) \) is sc*g-closed in \( X \). Since \( f \) is sc*g-closed map. Therefore \( f^{-1} \) is sc*g-continuous.

**Theorem: 2.7.7** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijective and sc*g-continuous map, the following statements are equivalent.

(a) \( f \) is a sc*g-open map.

(b) \( f \) is a sc*g-homeomorphism.

(c) \( f \) is a sc*g-closed map.

**Proof:** By assumption, \( f \) is bijective and sc*g-continuous and sc*g-open. Then by definition, \( f \) is sc*g-homeomorphism.
(b)→(c) By assumption, f is sc*g-open and bijective. By theorem 2.7.4 f is sc*g-closed map.
(c)→(a) By assumption, f is sc*g-closed and bijective. By theorem 2.7.4 f is sc*g-open map.

The following example shows that the composition of two sc*g-homeomorphisms need not be sc*g-homeomorphism.

**Example : 2.7.8** Consider the topological spaces $X = Y = Z = \{a, b, c\}$ with topologies $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, Y, \{a\}\}$ respectively. Let $f$ and $g$ be sc*g-homeomorphisms but their composition $g \circ f : X \rightarrow Z$ is not a sc*g-homeomorphism. For the open set $\{a, b\}$ in $X$, $g(f(a, b)) = [a, b]$ is not sc*g-open set in $Z$.

**Definition : 2.7.9** A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a (sc*g)*-homeomorphism if $f$ and it’s inverse $f^{-1}$ are sc*g-irresolute maps.

**Notation :** Let family of all (sc*g)*-homeomorphisms from $(X, \tau)$ onto itself be denoted by $(\text{sc*g})*h(X, \tau)$ and family of all sc*g-homeomorphisms from $(X, \tau)$ onto itself be denoted by $\text{sc*g} h(X, \tau)$. The family of all homeomorphisms from $(X, \tau)$ onto itself be denoted by $h(X, \tau)$.

**Theorem : 2.7.10** Let $X$ be a Topological space. Then
(i) The set $(\text{sc*g})*h(X)$ is a group under composition of maps.
(ii) $(\text{sc*g})*h(X)$ is a subgroup of $(\text{sc*g})*h(X)$.
(iii)$(\text{sc*g})*h(X) \subset \text{sc*g} h(X)$.

**Proof:** (i) Let $f, g \in (\text{sc*g})*h(X)$. Then $g \circ h \in (\text{sc*g})*h(X)$ and so $(\text{sc*g})*h(X)$ is closed under the composition of maps. The composition of maps is associative. The identity map $i : x \rightarrow x$ is a (sc*g)*-homeomorphism and so $i \in (\text{sc*g})*h(X)$. Also $f \circ i = i \circ f$ for every $f \in (\text{sc*g})*h(X)$. If $f (\text{sc*g})*h(X)$, then $f^{-1} \in (\text{sc*g})*h(X)$ and $f^{-1} \circ f = f \circ f^{-1}$. Hence $(\text{sc*g})*h(X)$ is a group under the composition of maps.

(ii) Let $f : X \rightarrow Y$ be a homeomorphism. Then by theorem 2.6.5 both $f$ and $f^{-1}$ are (sc*g)*-irresolute and so $f$ is a (sc*g)*-homeomorphism. Therefore every homeomorphism is a (sc*g)*-homeomorphism and so $h(X)$ is a subset of $(\text{sc*g})*h(X)$. Also $h(X)$ is a group under the composition of maps. Therefore $h(X)$ is a subgroup of the group $(\text{sc*g})*h(X)$.

(iii) Since every (sc*g)*-irresolute map is sc*g-continuous, $(\text{sc*g})*h(X)$ is a subset of $\text{sc*g} h(X)$.

From the above observations we get the following diagram:

- **Homeomorphism** → strongly g-homeomorphism
- sc*g - homeomorphism

**References**