International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 12, December - 2013

sc*g-Homeomorphisms in Topological Spaces

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Abstract

*In this paper, we introduced a new class sc*g-Homeomorphisms in Topological space.*

*Key words: Homeomorphism - strongly g-homeomorphism- sc*g - homeomorphism.*

1. Introduction

Several mathematicians have generalized homeomorphisms in topological spaces. Biswas[18], Crossely and Hildebrand[19], Gentry and Hoyle[20] and Umehara and Maki[21] have introduced and investigated semi-homeomorphisms somewhat homeomorphisms and g-A-homeomorphisms Crossely and Hildebrand defined yet another "semi-homeomorphism" which is also a generalization of homeomorphisms. Sundaram[6] introduced g-homeomorphisms and gc-homeomorphisms in topological spaces.

In the section, we introduce the concept of sc*g- homeomorphisms and study some of their properties.

Definition: 2.7.1 A bijection f : (X, τ) → (Y, σ) from a topological space X into a topological space Y is called a sc*generalized homeomorphism (sc*g- homeomorphism) if f is both sc*g-open and sc*g-continuous.

Theorem: 2.7.2 Every homeomorphism is a sc*g-homeomorphism.

Proof : Since every continuous function is sc*g-continuous and every open map is sc*g-open, the proof follows.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.7.3 Consider the topological spaces X = Y= {a, b, c} with topologies τ ={ϕ ,X,{a, b}} and σ = {ϕ ,Y,{a}, {a, b}}. Then the identity map f : (X, τ) → (Y, σ) is a sc*g-homeomorphism but not a homeomorphism. Since for the open set {a} in Y is not open in X.

Theorem:2.7.4 Everystrongly g-homeomorphism is a sc*g-homeomorphism but not conversely.

Proof : Let f : X→ Y be a strongly g-homeomorphism. Then f is strongly g -continuous and strongly g -open. Since every strongly g -continuous function is sc*g-continuous and every strongly g -open map is sc*g-open, f is sc*g-continuous and sc*g-open. Hence f is a sc*g-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

Example : 2.7.5 Let X=Y={a, b, c} with τ ={ϕ,X,{a}}and σ = {ϕ ,Y,{a, b}} respectively. Then the identity map f : (X, τ) → (Y, σ) is sc*g-homeomorphism but not a strongly g-homeomorphism. Since for the open set {a, b} in Y is not a strongly g-open set in X.

Next we shall characterize the sc*g-homeomorphism and sc*g-open maps.

Theorem : 2.7.6 For any bijection f : X→ Y the following statements are equivalent.

(a)f^{-1}: Y→ X is sc*g-continuous.
(b)f is a sc*g-open map.
(c)f is a sc*g-closed map.

Proof : (a) →(b) Let G be any open set in X. Since f^{-1} is sc*g-continuous, the inverse image of G under f^{-1}, namely f(G) is sc*g-open in Y and so f is sc*g-open map.

(b) →(c) Let f be any closed set in X. Then F^c is open in X. Since f is sc*g-open, $f(F^c)$ is sc*g-open in Y. But $f(F^c)$ = Y- f(F) and so f(F) is sc*g-closed in Y. Therefore f is a sc*g-closed map.

(c) →(a) Let F be any closed set in X. Then $(f^{-1})^{-1}(F)$ = f(F) is sc*g-closed in Y . Since f is a sc*g-closed map. Therefore f^{-1} is sc*g-continuous.

Theorem : 2.7.7 Let f : (X, τ) → (Y, σ) be a bijective and sc*g-continuous map, the following statements are equivalent.

(a)f is a sc*g-open map.
(b) f is a sc*g-homeomorphism.
(c) f is a sc*g-closed map.

Proof : (a)→(b) By assumption, f is bijective and sc*g-continuous and sc*g-open. Then by definition, f is sc*g- homeomorphism.

(b)→(c) By assumption, f is sc*g-open and bijective. By theorem 2.7.4 f is sc*g-closed map.
(c)→(a) By assumption, f is sc*g-closed and bijective. By theorem 2.7.4 f is sc*g-open map.
The following example shows that the composition of two sc*g-homeomorphisms need not be sc*g-homeomorphism.

Example : 2.7.8 Consider the topological spaces X = Y= Z= {a, b, c} with topologies τ_1={φ , X, {a}, {a, b}}, τ_2={φ, Y, {a}}and τ_3={φ, Z, {b, c}} respectively. Let f and g identity maps such that f : X→ Y and g : Y→ Z. Then f and g are sc*g-homeomorphisms but their composition g f : X→ Z is not a sc*g-homeomorphism. For the open set {a, b} in X, g(f{a, b}) = {a, b} is not sc*g-open set in Z.

Definition :2.7.9 A bijection f : (X, τ) → (Y, σ) is said to be a (sc*g)*-homeomorphism if f and it's inverse f -1are sc*g-irresolute maps.

Notation : Let family of all (sc*g)*-homeomorphisms from (X,τ) onto itself be denoted by (sc*g)*h(X,τ) and family of all sc*g-homeomorphisms from (X,τ) onto itself be denoted by sc*g h(X,τ). The family of all homeomorphisms form from (X,τ) onto itself be denoted by h(X,τ).

Theorem :2.7.10 Let X be a Topological space. Then
(i)The set (sc*g)*h(X) is a group under composition of maps.
(ii)h(X) is a subgroup of (sc*g)*h(X).
(iii)(sc*g)*h(X) ⊂ sc*g h(X).

Proof: (i) Let f, g ∈ (sc*g)*h(X). Then g o h (sc*g)*h(X) and so (sc*g)*h(X) is closed under the composition of maps. The composition of maps is associative. The identity map i : x→x is a (sc*g)*-homeomorphism and so i (sc*g)*h(X). Also f . i = i. f = f for every f∈ (sc*g)*h(X). If f (sc*g)*h(X), then f^{-1} ∈ (sc*g)*h(X) and f. f^{-1} = f^{-1}. f = i.
Hence (sc*g)*h(X) is a group under the composition of maps.
(ii) Let f : X→ Y be a homeomorphism. Then by theorem 2.6.5 both f and f -1 are (sc*g)*-irresolute and so f is a (sc*g)*-homeomorphism. Therefore every homeomorphism is a (sc*g)*-homeomorphism and so h(X) is a subset of (sc*g)*h(X). Also h(X) is a group under the composition of maps. Therefore h(X) is a subgroup of the group (sc*g)*h(X).
(iii)Since every (sc*g)*-irresolute map is sc*g-continuous, (sc*g)*h(X) is a subset of sc*gh(X).

From the above observations we get the following diagram:

Homeomorphism → (↚) strongly g-homeomorphism

→ (↚) sc*g - homeomorphism

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