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Robust Optimal Sliding Mode Control of Twin Rotor MIMO System

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***Abstract*—The twin rotor MIMO system is a laboratory setup resembling the dynamics of a helicopter. It is a complicated non linear system with heavy cross coupling effects between the propellers commonly used for verification of control methods and observers. In this paper the robustness and tracking capabilities of the decoupled system are evaluated and compared using PID controllers, LQI controller and a robust optimal controller. It was observed that the robust optimal controller exhibited superior performance to the output feedback and optimal controllers.**

Keywords—Optimal control,Linear Quadratic Regulator with integral action(LQI),robust optimal control etc..

I. INTRODUCTION

The Twin Rotor MIMO System (TRMS) is a laboratory prototype of a helicopter developed by Feedback Instruments Ltd. for the purpose of validating new control methodologies. A lot of research work has been carried out in applying machine learning algorithm to the system to find out the optimal parameters that guarantee the desired system performance. However, these methods do not guarantee a robust performance which is very essential in aerospace applications. To ensure robustness different control strategies like deadbeat control, H infinity and sliding mode control are applied to the system.

The simplest way of controlling a plant is by using a PID controller. However, manual tuning of the PID controller requires a greater human effort which is a major drawback. In [6], Meon M.S. proposed a PID active force control method which estimated the external torque disturbances and used soft computing techniques to optimize the PID response. The method provided a smooth response but the response of the yaw subsystem was not reported.

A better but complex method is to use the optimal controllers which deal with finding the optimal solution by solving the Algebraic Riccatti Equation. These controllers require linear system and full state feedback. In 2004, A.Q. Khan used a Linear Quadratic Regulator (LQR) method to control a 3 DOF helicopter [7]. The controller provided good response but was not robust to uncertainties. In 2012, B.Pratap decoupled the system into two subsystems and designed two LQR controllers for the subsystems [5]. The Kalman gain was updated iteratively to find the optimal solution. The regulation was excellent but nothing was published on the tracking results. In 2014, Andrew Phillips proposed a sub optimal LQR controller by adding integral action [4]. Both regulation and tracking responses were excellent but no conclusion was drawn on the controller's ability to withstand disturbances.

Among the various controllers available the Sliding Mode Controller (SMC) provides superior robustness in case of internal as well as external disturbances affecting the system. The main idea of SMC is to create a switching surface and then to force the states of the system to reach this surface and then to slide along it. In 2013, D.K.Saroj designed and implemented SMC with non linear state observer was for decoupled TRMS [8]. The controller provided excellent tracking of reference signal but nothing was published about the controller's ability to attenuate input chattering. In 2012, S.Mondal et al designed an adaptive integral sliding mode controller that takes care of the unmodeled disturbances and dynamics [1]. The controller eliminated chattering in simulation.

In this paper, combining LQI and SMC, the design of a robust optimal sliding mode controller is concerned. In part two, mathematical model of the TRMS is derived. In part three, optimal control with integral action is derived. In part four, the optimal controller is robustified by combining sliding mode control action. In part five, simulation results are discussed and in part six, conclusions are drawn.

II. MATHEMATICAL MODEL OF THE TRMS

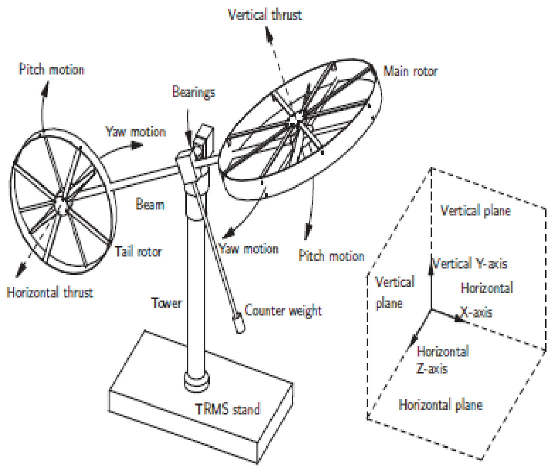

Fig. 1. The Twin Rotor MIMO System.

The Twin Rotor MIMO system consists of two rotors. One is the main rotor which is responsible for controlling the flight of the TRMS in the vertical direction and other is the tail rotor which is responsible for controlling the flight of the TRMS in the horizontal direction. A horizontal beam is

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affixed to a vertical pillar via a two dimensional pivot. The main rotor is attached to the front of the beam, parallel to the ground and the tail rotor is attached to the rear of the beam, perpendicular to the ground. A counter balance weight is attached to the beam to balance the system under steady state. The two rotors are driven by two separate DC motors.

The plant model can be derived by forming the vertical and horizontal equations of momentum. The momentum equation for vertical movement can be given as:

$$I_1\ddot{\theta}_V = M_1 - M_{FG} - M_{B\theta_V} - M_G \tag{1}$$

The momentum equation for horizontal movement is given by:

$$I_2\ddot{\theta}_H = M_2 - M_{B\theta_H} - M_R \tag{2}$$

Where, θ_V and θ_H are the pitch and yaw angles, M_1 and M_2 are the nonlinear static characteristics of the main and tail rotors defined by:

$$M_1 = a_1{\tau_1}^2 + b_1\tau_1 \tag{3}$$

$$M_2 = a_2{\tau_2}^2 + b_2\tau_2 \tag{4}$$

$M_{B\theta_X}$ is the frictional momentum given by:

$$M_{B\theta_H} = B_{1\theta_X}\dot{\theta}_X - \frac{0.0326}{2}\sin(2\theta_X)\,{\dot{\theta}_X}^2 \tag{5}$$

M_{FG} is the gravitational momentum defined by:

$$M_{FG} = M_g \sin\theta_V \tag{6}$$

M_G is the gyroscopic momentum given by:

$$M_G = K_{gy}M_1\dot{\theta}_H cos\theta_v \tag{7}$$

M_R is the cross reaction momentum given by:

$$M_R = \frac{K_c(T_0 s + 1)}{(T_P s + 1)}M_1 \tag{8}$$

Above equation can be written in state space form as:

$$\dot{M}_R = -\frac{1}{T_P}M_R + M_1 \tag{9}$$

The DC motors can be modeled as first order systems. Equations for vertical and horizontal systems are given by:

$$\tau_1 = \frac{K_1}{T_{11}s + T_{10}}U_V \tag{10}$$

$$\tau_2 = \frac{K_2}{T_{21}s + T_{20}}U_H \tag{11}$$

Combining equations (1) to (11) the complete state equations for the TRMS plant can be derived as in (12):

$$\frac{d\theta_V}{dt} = \Omega_V$$

$$\frac{d\Omega_V}{dt} = \frac{1}{I_1}\Big[a_1{\tau_1}^2 + b_1\tau_1 - M_g\sin\theta_V - B_{1\theta_V}\Omega_V + \frac{0.0326}{2}\sin(2\theta_V)\,{\Omega_V}^2 - K_{gy}a_1\cos(\theta_V)\,\Omega_H{\tau_1}^2 - K_{gy}b_1\cos(\theta_V)\,\Omega_H\tau_1\Big]$$

$$\frac{d\theta_H}{dt} = \Omega_H$$

$$\frac{d\Omega_H}{dt} = \frac{1}{I_2}\Big[a_2{\tau_2}^2 + b_2\tau_2 - B_{1\theta_H}\Omega_H - \left(\frac{K_c}{T_P} - \frac{K_cT_P}{{T_P}^2}\right)M_R - \frac{K_cT_0}{T_P}(a_1{\tau_1}^2 + b_1\tau_1)\Big]$$

$$\dot{M}_R = -\frac{1}{T_P}M_R + a_1{\tau_1}^2 + b_1\tau_1$$

$$\frac{d\tau_1}{dt} = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{K_1}{T_{11}}U_V$$

$$\frac{d\tau_2}{dt} = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{K_2}{T_{21}}U_H \tag{12}$$

Now the state variables are defined as:

$$X = [\theta_V \quad \Omega_V \quad \theta_H \quad \Omega_H \quad M_R \quad \tau_1 \quad \tau_2]^T \tag{13}$$

TABLE I. PHYSICAL PARAMETERS OF THE TRMS

Parameter	Description	Value	Units
a_1	Main Rotor Coefficient	0.0135	N/A
b_1	Main Rotor Coefficient	0.0924	m
a_2	Tail Rotor Coefficient	0.01	m
b_2	Tail Rotor Coefficient	0.09	m
$B_{1\theta_V}$	Friction Coefficient	0.003	Nms/rad
$B_{1\theta_H}$	Friction Coefficient	0.1	Nms/rad
M_g	Moment of Gravity	0.29	Nm
I_1	Pitch Moment of Inertia	0.0535	Kgm^2
I_2	Yaw Moment of inertia	0.02	Kgm^2
K_{gy}	Gyroscopic Momentum	0.05	s/rad
T_P	Cross-Reaction Momentum Parameter	2	N/A
T_0	Cross-Reaction Momentum Parameter	3.5	N/A
K_c	Cross-Reaction Momentum Gain	-0.2	N/A
T_{10}	Main Rotor Denominator	1	N/A
T_{11}	Main Rotor Denominator	1.1	N/A
T_{20}	Tail Rotor Denominator	1	N/A
T_{21}	Tail Rotor Denominator	1	N/A

III. OPTIMAL CONTROL WITH INTEGRAL ACTION

It has been widely accepted that the method of full state feedback provides a much superior performance than output feedback. Controller design by pole placement results in an over determined system of equations. That is the number of equations are more than the number of variables to be solved

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for. Since there is more than one solution, one will be better than others in a quantifiable manner and this gave birth to the concept of optimal control theory.

To implement optimal control, the plant model given by (12) is linearized about the origin to form a continuous time linear system is given by:

$$\dot{X} = AX + BU \tag{14}$$

The optimal control input can be obtained as

$$U = -KX \tag{15}$$

Where, K is the state feedback gain or kalman gain,

$$K = R^{-1}B^{T}P \tag{16}$$

Obtained by solving the Algebraic Riccatti Equation for P given by:

$$-\dot{P} = A^{T}P + PA - PBR^{-1}B^{T}P + Q \tag{17}$$

Here P is a positive definite time varying matrix that weights the final states of the system, Q is a positive definite time varying matrix that weights the states of the system over time and R is a positive semi definite matrix that weights the input of the system over time.

Equations (16) and (17) implies that the Kalman gain is time varying. The optimal Kalman gain matrix can be approximated by assuming a steady state solution to (17). This is given by (18).

$$A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0 \tag{18}$$

In order to design the controller using above equation that is by ensuring zero steady state error to a step input, integral action must be imparted on the control loop. This is imparted by restating the system with additional states that are the output errors of the system. This method releases an outstanding precise tracking response compared to the conventional state dependent riccatti equation controller.

Integral action is imparted by augmenting the state space system as:

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{19}$$

The result of this augmentation is that number of poles equal to number of outputs is placed at the origin.

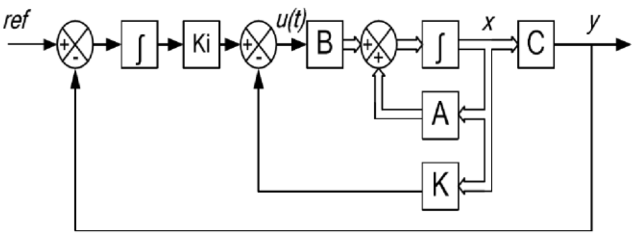

Fig. 2. Block diagram of optimal control with integral action.

IV. ROBUSTIFYING THE OPTIMAL CONTROLLER

The optimal controller design described above is based on precise mathematical models. If the controlled system is subjected to uncertainties or external disturbances, the performance criterion optimized based on nominal system might deviate from optimal value and the system might even become unstable.

To alleviate this problem, the sliding mode control which is a precise and robust control algorithm is combined with the optimal controller to generate a global robust optimal sliding mode controller(ROSMC) for the system.

The state equation for an uncertain dynamic system can be written as:

$$\dot{x}(t) = Ax(t) + Bu(t) + \delta(x,t) \tag{20}$$

Where,

$$\delta(x,t) = B\check{\delta}(x,t)$$

$$\|\check{\delta}(x,t)\| \le \gamma_0 + \gamma_1\|x(t)\| \tag{21}$$

is the uncertain extraneous disturbance acting on the system and γ_0 and γ_1 are small positive constants.

A. *Design of optimal sliding mode controller*

Considering the uncertain system in (20), an integral sliding surface is defined as:

$$s(x,t) = G[x(t) - x(0)] - G\int_0^t (A - BR^{-1}B^{T}P)x(\tau)d\tau$$

$$= 0 \tag{22}$$

Where $G \in R^{mXn}$, which satisfies GB is non singular.

Differentiating (22) with respect to t, we get,

$$\dot{s} = G[Ax + Bu + \delta] - G[A - BR^{-1}B^{T}P]x \tag{23}$$

The equivalent control law becomes,

$$u_{eq} = -[GB]^{-1}[G\delta + G(R^{-1}B^{T}P)x] \tag{24}$$

Substituting (24) into (20), the ideal sliding mode dynamics become,

$$\dot{x} = Ax - B[GB]^{-1}[G\delta + G(R^{-1}B^{T}P)x] + \delta \tag{25}$$

To ensure reachability of the sliding mode in finite time, the control law is defined as:

$$u = u_c + u_d \tag{26}$$

Where, u_c is the continuous part which is same as U given by (15) used to stabilize and optimize the nominal system and u_d is the discontinuous part which completely compensates for the uncertainties of the system.

$$u_d = -(GB)^{-1}(\eta + \gamma_0\|GB\| + \gamma_1\|GB\|\|x\|)sgn(s) \tag{27}$$

With the above control law (26) and integral sliding surface (22), the uncertain system (20) achieves global sliding mode and the performance index can be minimized.

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V. SIMULATION RESULTS

All the simulations were performed in MATLAB and SIMULINK environment using ode5 solver and a sampling time of 1 ms. A Luenberger observer was designed to provide full state feedback and for the purpose of comparison a PID controller was used. The step response and tracking response of the plant with PID controller, LQI controller and robust optimal sliding mode controller are evaluated and compared. Then a robustness evaluation of the plant with LQI and ROSMC is performed.

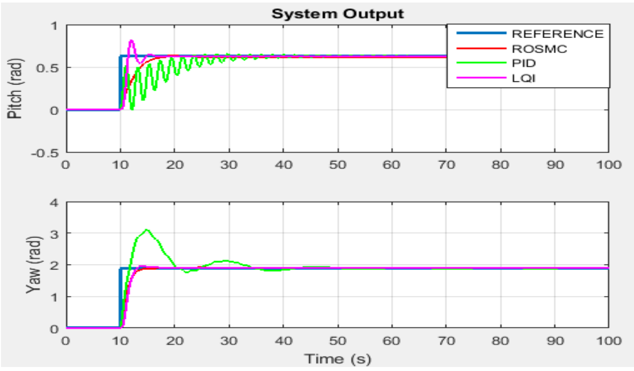

Fig. 3. Step response of TRMS with different controllers.

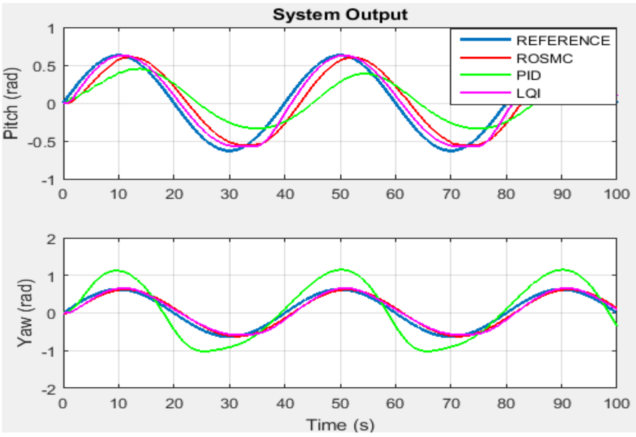

Fig. 4. Tracking response of TRMS with different controllers.

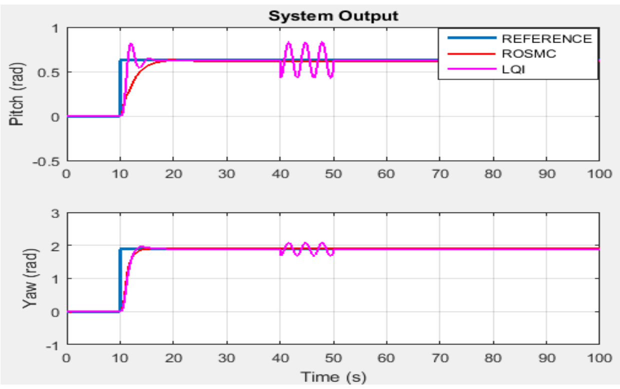

Fig. 5. Robustness evaluation of ROSMC and LQI controller.

The step response of the plant with PID controller, LQI controller and Robust Optimal Sliding Mode Controller was evaluated with a desired pitch of 0.2Π radians and desired yaw of 0.6 Π radians. It can be seen from Fig. 3. that the PID controller exhibits the worst performance with maximum overshoot. The LQI controller provides a better performance but has a slight overshoot in the pitch response. The ROSMC exhibits the best step response with no overshoot, rapid rise time and quick settling.

Fig. 4. Shows the tracking response of the plant with PID controller, LQI controller and Robust Optimal Sliding Mode Controller. A sine wave of 0.2 Π radians amplitude and 0.025Hz frequency was used to evaluate the tracking capabilities of the controllers. Here also the PID controller exhibits the worst tracking with greatest deviation from the reference signal. LQI and ROSMC provide almost similar tracking response with reduced deviation from the reference signal. Backward integration in the optimal control loop has greatly improved the tracking capabilities of the controllers.

It can be seen that, in the absence of external disturbances the LQI controller and ROSMC exhibits almost similar response. In order to evaluate the robustness properties of the designed controllers, an external disturbance, d(t)= 0.2sin2t was applied to the system in the time interval 40s to 50s. It can be seen from Fig. 5 that the response of system with LQI controller is affected by external disturbance while the system with ROSMC remains unaffected. Thus the designed ROSMC guarantees complete robustness to the system in case of external disturbances.

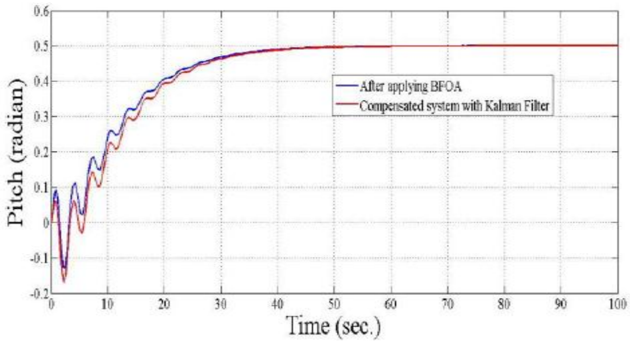

Fig. 6. Simulated pitch step response by A.K. Agrawal [3]

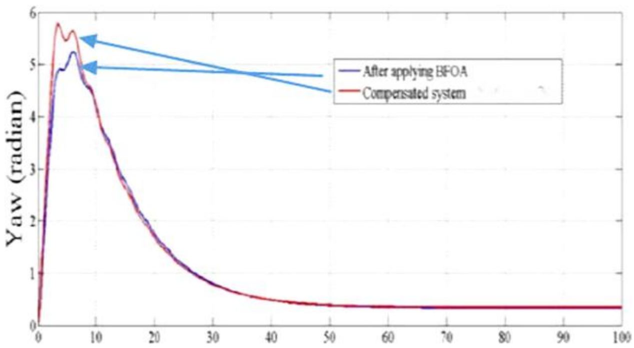

Fig. 7. Simulated yaw step response by A.K. Agrawal[3]

Fig.6 and Fig.7 show the results obtained by A.K Agrawal by optimizing Q and R matrices of Linear Quadratic Gaussian controller using Bacteria Forging Algorithm. On comparing these results with Fig. 3 we can see that the ROSMC provides much better performance than the optimized controller implemented by A.K.Agrawal due to reduced rise time and settling time and reduced overshoot.

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VI. CONCLUSION

A Linear Quadratic Regulator with integral action was designed and combined with a robust sliding mode controller. The combined robust optimal sliding mode controller was simulated and compared with output feedback and optimal controllers. It was found that the designed controller exhibits a much superior performance and robustness compared to other simulated controllers and existing controllers in the literature.

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