Robust Hough Transform with Edge Strength

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***Abstract*—The Hough transform (HT) is a well-known method for detecting analytical shapes represented by a number of free parameters. However the basic property of the HT, the one-to-many mapping from an image space to a Hough space, causes noise sensitivity and therefore distortion. In this paper, the Edge Strength Hough Transform (ESHT) is proposed to decrease the distortion due to noise sensitivity and applied to line detection. The ESHT is different from the HT in that it uses broadened edge with edge strength based on gradients, which makes it possible to decrease the effect of noise and achieve robust detection. Though the ESHT requires two crucial parameters – broadening parameter and decreasing parameter – to broaden the traditional 1-pixel-wide edge into arbitrary width and to set the exponentially decreasing strength, they can be decided with the information given before the transform. Formal analyses and examples show that the proposed method can decrease distortion.**

Keywords— Hough transform; edge strength; gradient

I. INTRODUCTION

The detection of geometric primitives from a digital image is one of the basic tasks in computer vision area. Generally the primitive is defined by a set of parameters and the detection is a process of conversion from the information in an image to the estimated parameter set. The Hough transform (HT) is a well-known method for detecting analytical shapes [1] using the correspondence between a point in an image and parameter set describing object. The HT was originally proposed to fit straight lines by Hough and then extended to analytical curves [2][3][4]. Though the HT has been used widely as its original or modified form [5], the basic process of it, one-to-many mapping from an image space to a Hough space gives rise to several problems: (1) the HT might find an object which is caused by noise and not apparent to human, (2) it might find multiple objects with similar parameter sets and actually there exists only one real object, (3) the parameter set found might be different from the actual one. All of them are defined as a kind of distortion in this paper. Discretization of an image, size of a Hough space, and noise can cause the distortion. The effect of discretization [6] is not treated in this paper and the size of a Hough space neither. In this paper, the Edge Strength Hough Transform (ESHT), a modified HT with broadened edge and exponentially decreasing strength, is introduced to reduce the distortion due to noise. It tries to find one parameter set per one actual object under the current discretization and Hough space setting.

There have been several groups of methods to make the HT robust though each group has their own meaning of robustness. The first group of methods tries to find lines with arbitrary thickness from gray-scale images. The grayscale Hough transform [7] and the vector-gradient Hough transform [8] belong to this group. The former assumes that the pixels on a line have equal gray level but the latter does not. They use the gray level as the evidence of line existence but the gray level does not have direct relationship with the degree of line existence and they describe a line with four parameters to consider the thickness and gray level. The second group puts a focus on the calculation of end points or length of a line [9][10][11]. To calculate them, an augmented three dimensional Hough space is used in [9] and an additional post-processing routine is employed in [10] and [11]. The coordinates of end points are useful, but they are useful only for some applications and do not have connection with the quality of detected lines, which is the main concern of the ESHT. The objective of the third group is to reduce distortion defined in this paper with the introduction of pixel weight [12][13]. They uses the output value of edge detection algorithm, i.e. the magnitude of gradients, as a degree of edge-ness with a traditional two dimensional Hough space and one-pixel-wide edge. Though they are effective in reducing distortion, it is shown later that the ESHT reduces more distortion by edge broadening which smoothes the Hough space. There are also several other methods to reduce distortion though they are quite different from the traditional HT. Kim et al. proposed a robust HT (RHT) with several modifications of the original HT [14]. The core difference of the RHT from the HT lies in the analog HT and robust clustering. The RHT does not digitize the Hough space and records the calculated parameter set in a list. The list is clustered with a robust clustering method to group the parameter sets with similar values. Voting based on multiple points and validity measure are also introduced to reduce distortion. Aggarwal et al. proposed another method to detect lines in a robust way [15]. They formulate the task of finding a transformed space, which corresponds to the Hough space in the HT, as an inverse problem based on the inverse Radon transform. This approach allows the method to incorporate prior constraints within a regularization framework. The constraints correspond to the manipulation of the Hough space, which is not intuitive and simple to understand. All of the previous methods have some properties of the ESHT, but to our knowledge, the ESHT is the first systematic approach in which line distortion by noise is formally analyzed to produce a solution. Furthermore the ESHT uses the same parameter set with the HT, which makes it easy to plug-in the ESHT in the place of the HT or its variants.

This paper is organized as follows. In section 2, the HT is reviewed and distortion caused by noise is shown. In section 3, the ESHT is proposed to reduce distortion and actual reduction is proved. Section 4 is devoted to derive two parameter values which play an important role in the ESHT. Section 5 summarizes experimental results and a discussion section ends the paper.

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II. HOUGH TRANSFORM

The Hough transform (HT) is a technique which can be used to extract a particular shape within an image. The HT generally requires that the desired shape be specified in some parametric form, so it is most commonly used for the detection of regular curves such as lines, circles, ellipses, etc. The Generalized Hough Transform (GHT) [16] can be employed theoretically in applications where a simple analytic description of a shape is not possible. However, due to the computational complexity of the GHT, the focus of the HT is restricted to the curves having parametric description. This paper only concerns about lines but there is no restriction on the form or the number of parameters.

A line can be described in a number of ways. However, a widely used one in the HT is parametric or normal form in (1):

$$x\cos\theta + y\sin\theta = r \tag{1}$$

For any point (x, y) on the same line, r and θ are constants. In an image, the coordinates of the points are known and serve as constants in the parametric line, while r and θ are the unknown variables. If we plot the possible (r, θ) values defined by a point (x, y), we can get a curve in a $r-\theta$ space. The $\mathrm{r}-\theta$ space is called a Hough space and the point-to-curve transform is the HT for lines. Each parameterized line corresponds to a specific position in a Hough space. The specific position will be referred to as a cell and the accumulated evidence in a cell will be referred to as cell value.

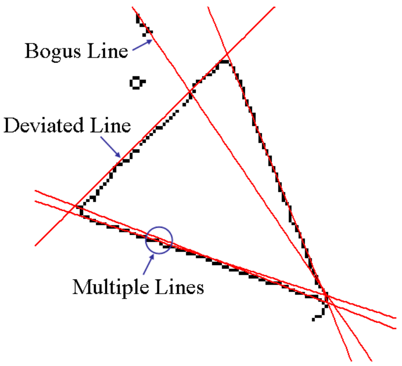

Fig. 1. Line distortion in the Hough transform

Though the HT is a simple and well-known method to detect lines from an image, the basic process of the HT, one-to-many mapping, makes it noise-sensitive. Generally the HT is assumed to have noise immunity but it stresses different aspect from that mentioned in this paper. The well-known noise immunity means that the HT can detect partially occluded or degraded objects by noise. The noise sensitivity mentioned in this paper stresses the quality of lines detected. As shown in fig. 1, the detected line may deviate from the real line. Also several lines for one real line may be detected and some bogus lines may occur. All of these problems are collectively called distortion in this paper. All of them are mainly caused by noise and multiple lines are partially due to improper setting of a Hough space.

III. EDGE STRENGTH HOUGH TRANSFORM (ESHT)

In this section, a modified Hough transform is proposed to reduce distortion, which is partially supported by an experiment in psychology. There is a well-known result in psychology stated as human can recognize objects from line drawings as well as they can from color images [17]. This result is often pointed out as evidence that object recognition systems can be constructed using edge detection as the only low-level image processing [18]. However it is also well-known that the result does not hold when edge maps are substituted for line drawings. One explanation of this is that line drawings used in psychology are renderings made with the object identity already known, whereas edge detection in computer vision is a bottom-up process prior to object recognition. Experimental result in [19] and [20] shows that some portion of the recognition gap between edge maps and line drawings can be closed by using edge maps coded with edge strength. Though this does not support directly the fact that edge strength can relieve the noise burden in the HT, we can have an idea that the usefulness of edge strength in the recognition process by human might also be true in the recognition process by a machine.

A. Edge strength

Edge strength can be thought as a kind of weight that represents the importance of an edge pixel in a recognition process following the experiments conducted in psychology, or the membership of a pixel to the real boundary of an object. In general, the pixels on the boundary of an object have large edge strength compared to others, which serves as the evidence of an object in extracting or recognizing an object. Most of the edge detection algorithms use the magnitude of gradients as a measure of edge-ness because it satisfies the above condition: the magnitude of gradients of an edge pixel is generally greater than that of a non-edge pixel. So the magnitude of gradients is also used as the strength of a pixel in this paper.

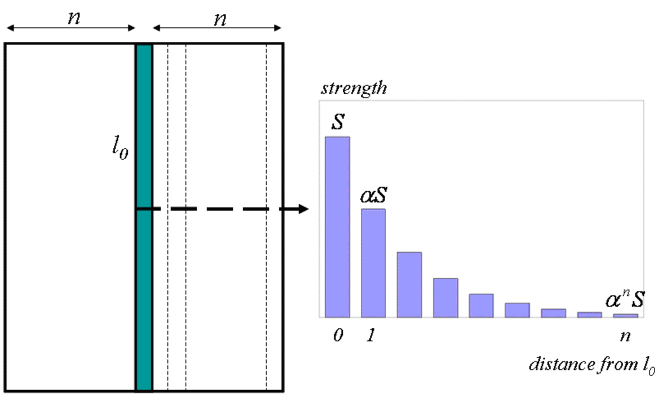

Fig. 2. Edge broadening and exponentially decreasing strength assignment

The HT uses a binary edge map generated by an edge detector, but the proposed method uses an edge strength map in which the 1-pixel-wide edge is broadened into width ($2n$+1) and exponentially decreasing strength based on gradients is assigned on each pixel. Fig. 2 shows the basic way of assigning edge strength, where S represents the magnitude of gradients of pixels on line l_0. It is assumed that all the pixels on l_0 have the same magnitude of S. α is a decreasing parameter which governs the decreasing rate of strength

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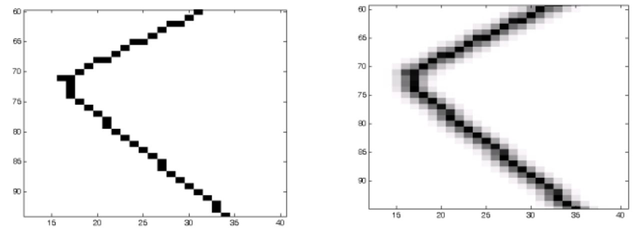
according to the distance from l_0 and n is a broadening parameter which tells the width of an edge after broadening. In an actual image, the magnitudes of gradients of pixels on a line do not equal and the actual strength of broadened pixel is calculated as the average strength of neighboring pixels multiplied by α iteratively. Fig. 3 shows an example of edge strength map generated in this way.



(a) (b)

Fig. 3. (a) A binary edge map and (b) an edge strength map

The decreasing and broadening parameters play an important role in the ESHT, which are described in detail later. In fig. 3, α and n are set as 0.4 and 2 respectively.

B. Edge strength Hough transform

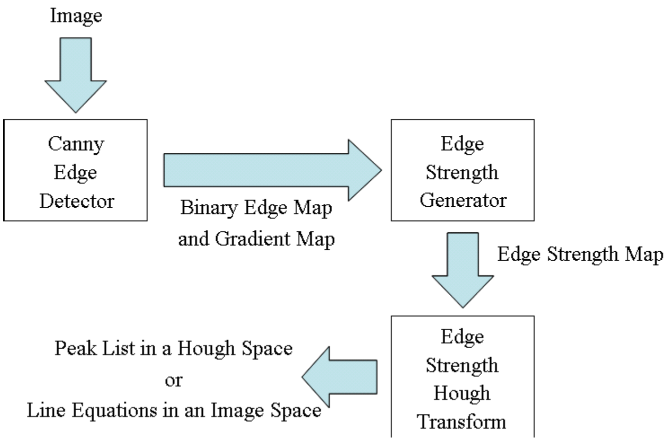
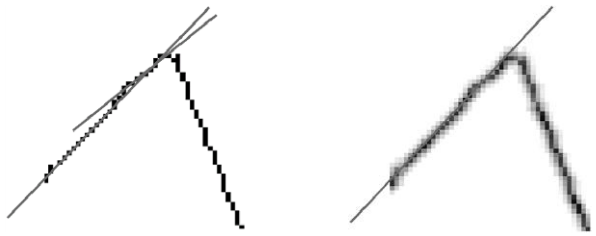


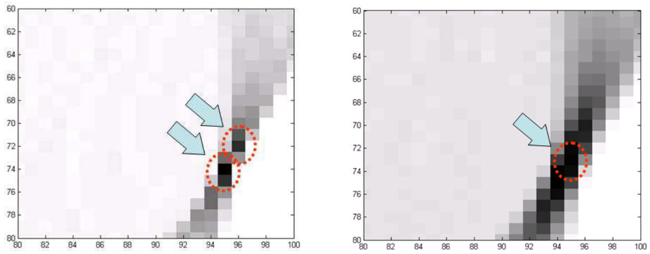
Fig. 4. Edge strength Hough transform

Fig. 4 shows the overall process of the ESHT. First of all, a binary edge map and a gradient map are generated using Canny edge detector [21]. Gradient map contains the magnitudes of gradients of edge pixels. The two maps are combined to generate an edge strength map as described in the previous section. The edge strength map serves as an input to the ESHT and finally a peak list in a Hough space or a set of line equations in an image space is obtained.



(a) (b)

Fig. 5. Lines found using (a) the HT and (b) the ESHT



(a) (b)

Fig. 6. Hough space images of (a) the HT and (b) the ESHT

The edge strength map plays a role of suppressing spurious lines due to noise. It forces one and only one local maximum in a Hough space around the real one. As is clear from fig. 5, the HT finds two lines but the ESHT finds only one. In the HT, the edge pixel whose position is changed by noise can not affect the cell corresponding to a real line. In the ESHT, however, it can affect the cell with a certain degree, which smoothes the Hough space; suppresses noise-deriving local maxima and reinforces real maxima. Fig. 6 shows the Hough space images corresponding to the lines found in fig. 5. The HT forms two local maxima but the ESHT does one in the Hough space. In this case, however, the number of local maxima can be reduced to one with a large search window. Nevertheless it is not desirable because a large search window makes it impossible to find nearing lines in parallel.

C. Noise robustness

In this section, it is shown that under a wide range of conditions that are likely to hold in practice, the ESHT discriminates real peaks from noise peaks better than the HT. Assume an image containing evidence of two lines: one due to a real line in the imaged scene and the other due to noise. The corresponding Hough space will have 2 local maxima, referred to as the real peak (P_{real}) and noisy peak (P_{noisy}). Assume further that there are a pixels from the true line and b pixels from the noise line that provide evidence for P_{real}, and their edge strengths are N and M respectively. Then P_{real} can be written as,

$$P_{real} = aN + bM \tag{2}$$

Similarly P_{noisy} can be written as:

$$P_{noisy} = cN + dM \tag{3}$$

It is assumed here that all the edge and noise pixels have the same strength. Moreover the following 4 assumptions hold generally: (1) the strength of an edge pixel is greater than that of a noise pixel ($N > M$). (2) The number of edge pixels that affect P_{real} is greater than the number of noise pixels that affect P_{real} ($a > b$). (3) The number of noise pixels that affect P_{real} is roughly equal to those that affect P_{noisy} ($b \approx d$). (4) The number of edge pixels that affect P_{real} is greater than those that affect P_{noisy} ($a > c$). When the reliability of the HT is measured based on the ability to discriminate between real and noisy peaks, the ratio between two peaks can be a good measure.

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$$R = P_{real} / P_{noisy} \quad (4)$$

In the HT, all the edge and noise pixels have the same strength, i.e. $N_{HT} = M_{HT} = 1$. Therefore the ratio of the HT and the ESHT can be written as:

$$R_{HT} = \frac{P_{real,HT}}{P_{noisy,HT}} = \frac{aN_{HT}+bM_{HT}}{cN_{HT}+dM_{HT}} \approx \frac{a+b}{c+b} \quad (5)$$

$$R_{ESHT} = \frac{P_{real,ESHT}}{P_{noisy,ESHT}} = \frac{aN+bM}{cN+dM} \approx \frac{aN+bM}{cN+bM} \quad (6)$$

The ratio difference, $R_{ESHT} - R_{HT}$, takes a value greater than zero as shown in (7), which means that in the ESHT, P_{real} has relatively larger value compared to P_{noisy} than in the HT i.e. the ESHT has more power to discriminate real peaks from noisy peaks than the HT and lowers the probability of noise-deriving line detected.

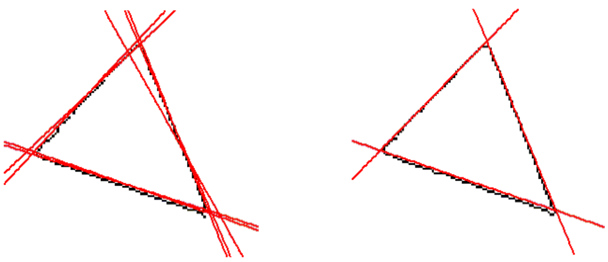
$$\begin{aligned} D &= R_{ESHT} - R_{HT} \\ &\approx \frac{aN+bM}{cN+bM} - \frac{a+b}{c+b} \\ &= \frac{b(N-M)(a-c)}{(cN+bM)(c+b)} > 0 \end{aligned} \quad (7)$$

IV. BROADENING AND DECREASING PARAMETER

There are two important parameters in the ESHT: broadening parameter and decreasing parameter. The former is related to the width of edges and the latter is related to the strength of edge pixels. In this section, we derived the optimal value of them to minimize distortion only using the information given before the transform.

A. *Broadening parameter*

In the HT, the size of a Hough space for lines is determined by angle resolution (r_θ) and length resolution (r_l). When a line is described in a parametric form in (1), the size of a Hough space is given by $L_\theta \times L_l$. L_θ is the number of digitized slopes and L_l is the number of digitized distances from the origin. Generally length resolution is set as 1 and L_l as the diagonal length of an image. Angle resolution is commonly set as 1.0° and satisfies $L_\theta = 180/ r_\theta$. Angle resolution has a great influence on the quality of a detected line. On the other hand, length resolution limits the range of line position but does not have direct relation with the quality of a detected line. Generally the smaller angle resolution, the less line distortion. However too small angle resolution may give rise to multiple line detection. Fig. 7 shows an example of detected lines with respect to angle resolution. With r_θ = 0.5°, one real line can be described with multiple slopes and multiple local maxima with similar parameter sets appear. The Hough space is not the primary concern of this paper so we followed the most common setting: r_θ = 1.0° and L_l is the diagonal length of an image.



(a) r_θ = 0.5° (b) r_θ = 1.0°

Fig. 7. Detected lines with different angle resolution

The slope of a real line generally does not have a corresponding digitized slope and this disagreement can cause missing or multiple lines. To detect one and only one line for one real line, the slope of a real line should have one corresponding parameterized line though it might be an approximate one. Edge broadening is hired to solve this problem. Edge should be broadened to cover at least one parameterized line, and fig. 8 shows the worst case in which a real line is put in the middle of two consecutive parameterized lines. So the 1-pixel wide line should be broadened into the width with which 2 consecutive parameterized lines are covered.

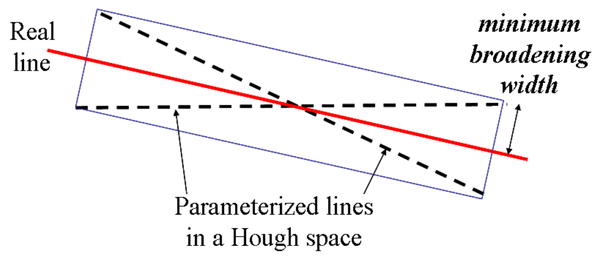


Fig. 8. Minimum broadening width

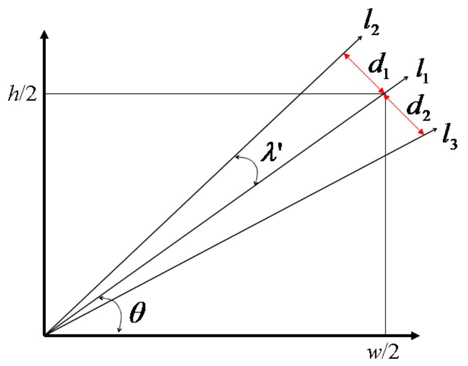


Fig. 9. Maximum distance between two consecutive parameterized lines

Generally the larger broadening width, the less noise sensitivity. However large broadening requires more computation and might smear two adjacent lines. So we need minimum broadening width minimizing line distortion, which is the half of maximum distance between two consecutive parameterized lines. Fig. 9 shows the maximum distance in a w by h size image. In that figure, l_2 and l_3 are two consecutive lines can be positioned with the maximum distance and λ' = 0.5°. l_1 and l_2 can be represented as $y = hx/w$ and $y = \tan(\theta + \lambda')x = \beta x$ respectively, and the slope of l_2 as:

$$\beta = \tan(\theta + \lambda') = \frac{\tan\theta + \tan\lambda'}{1 - \tan\theta \cdot \tan\lambda'} \quad (8)$$

The minimum broadening width should be greater than the half of the maximum distance between l_2 and l_3 ($n \geq d_1 = d_2$), and can be formulated as:

$$ceiling\left(\left|\frac{w}{2}\beta - \frac{h}{2}\right| / \sqrt{\beta^2+1}\right) \leq n \quad (9)$$

where n should be an integer and $ceiling(m)$ takes the smallest integer greater than or equal to m. Broadening parameter, n, increases as the size of an image increases under the same

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angle resolution, due to the increase of the maximum distance. Fig. 10 shows the decrease of multiple line detection with edge broadening where all other parameters have the same values.

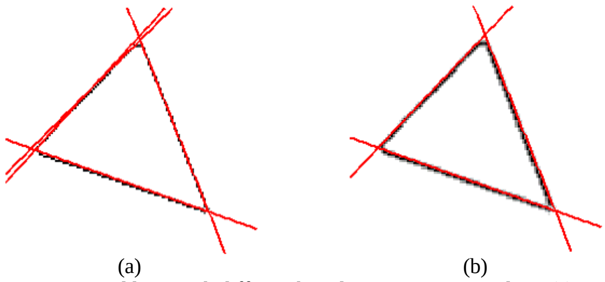

Fig. 10. Detected lines with different broadening parameter values, (a) $n = 0$ and (b) $n = 1$

B. Decreasing parameter

The purpose of edge broadening is to reduce multiple lines but it may generate other redundant ones. To suppress the redundant lines, the cell of a redundant line should be placed next to the cell corresponding to the real line and the cell value should be smaller than that of the real line. Or the cell value should not form a local maximum at least. That is why we set the strength of broadened edge pixels decreasing exponentially. Fig. 11 shows some possible redundant lines according to edge broadening. When l_0 is the real line detected, there are $2n$ slanted lines after broadening. Only n lines are suggested in fig. 11 and the other symmetric n lines are not shown. There are also $2n$ parallel lines with l_0. With decreasing strength setting, however, it is clear that the cell values for them have smaller value than that of l_0 and are adjacent to that, which means none of them can be a local maximum. It is also possible that there are some perpendicular lines to l_0. It is generally assumed that the length of a line is much larger than the width of it so they are not considered. As a result, the only redundant lines considered are $2n$ slanted lines in fig. 11 and the cell values of them should be smaller than that of l_0.

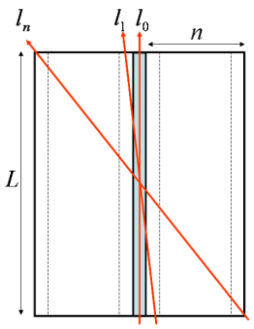

Fig. 11. Redundant lines according to edge broadening

Assume that the real line l_0 has a uniform edge strength S, the decreasing parameter is α, the broadening parameter is n, and the length of the line is L. The cell value corresponding to l_0 is $Acc_0 = LS$ accordingly. Also assume that the cell value corresponding to the 1st slanted line, l_1, is Acc_1, which can be calculated as:

$$Acc_1 = \frac{1}{3}L_1 S + \frac{2}{3}L_1 \alpha S \tag{10}$$

where L_1 is the length of line l_1 and $L_1 = \sqrt{L^2 + 3^2}$. To ensure $Acc_0 > Acc_1$, the following equation should hold.

$$\frac{Acc_1}{Acc_0} = \frac{\frac{1}{3}L_1 S + \frac{2}{3}L_1 \alpha S}{LS} = \frac{L_1}{3L}(1 + 2\alpha) < 1 \tag{11}$$

The ratio in (11) is a function of the length ratio L_1/L and decreasing parameter α. Among them, the length ratio can be represented in general,

$$\frac{L_k}{L} = \frac{\sqrt{L^2 + (2k+1)^2}}{L} \tag{12}$$

and this value is inversely proportional to L and proportional to k. The smallest value of L is 3 and the corresponding largest value of k is 1. Therefore the maximum value of the length ratio is

$$\frac{L_k}{L} = \frac{\sqrt{3^2 + 3^2}}{3} \leq \sqrt{2} \tag{13}$$

Using this, (11) can be simplified as,

$$\frac{Acc_1}{Acc_0} = \frac{L_1}{3L}(1 + 2\alpha) \leq \frac{\sqrt{2}}{3}(1 + 2\alpha) < 1 \tag{14}$$

and $\alpha \approx 0.56$ is the largest value satisfying (14) which ensures $Acc_0 > Acc_1$. Similarly, the ratio between l_0 and l_k ($k = 1, \dots, n$) can be written as:

$$\frac{Acc_k}{Acc_0} \leq \frac{\sqrt{2}}{2k+1}\left(1 + 2\sum_{i=1}^{k} \alpha^i\right) < 1 \tag{15}$$

This value is proportional to α and inversely proportional to k. Therefore the α calculated above holds for all other k (= 2, … , n), which means that $\alpha \approx 0.56$ is the maximum value can be set without redundant line detection.

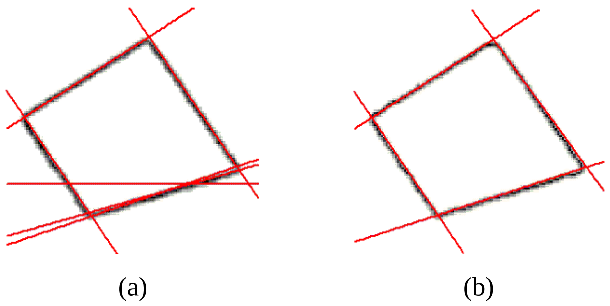

Fig. 12. Detected lines with different decreasing parameter values, (a) $\alpha = 0.8$ and (b) $\alpha = 0.6$

Fig. 12 shows the results according to α. As is clear from fig. 12, small α value decreases the probability of detecting redundant lines. Too small value however makes the broadened edge to be looked like 1-pixel-wide edge and noise sensitive. One thing that should be mentioned is that $\alpha = 0.6$ is greater than the derived value (≈ 0.56). In the derivation, we assumed that the minimum length of a line is 3, which is impractical. So a little larger value works well also in this example. If some prior knowledge about the length of a line is given, different α can be derived using the same procedure.

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V. EXPERIMENTAL RESULTS

In the previous sections, the edge strength Hough transform is introduced and the usefulness of it is shown. In this section, the effectiveness of the formulae for the decreasing and broadening parameters is shown, and the experimental results on some test images are summarized. To check the effectiveness of the derived parameter values, two experiments are conducted. Fig. 13 shows the number of detected lines as a function of the decreasing parameter α. The number is an averaged one over 100 runs and each run uses the 100×100 rectangular image in fig. 12. Gaussian white noise of mean zero and variance 0.1 is added to the image before edge detection. Broadening parameter is set as one following (9).

In fig. 13, $0.3 < \alpha < 0.5$ shows good results. With the value smaller than 0.3 the result is noise sensitive and there are too many redundant lines with the value larger than 0.5. The traditional HT found 5.9 lines on average, which is not included for clear comparison. There are so many factors to be considered in deciding optimal α, however, in a series of experiments $\alpha = 0.4$ shows consistent result with different image size and different number of lines in an image though it is smaller than the maximum value derived. This is mainly due to the line smearing through broadening and noise. The value of α might be different with different types of image but the derived value can put an upper limit of α.

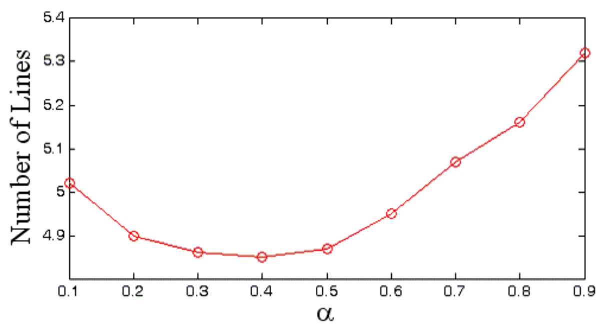

Fig. 13. Number of average detected lines with respect to α

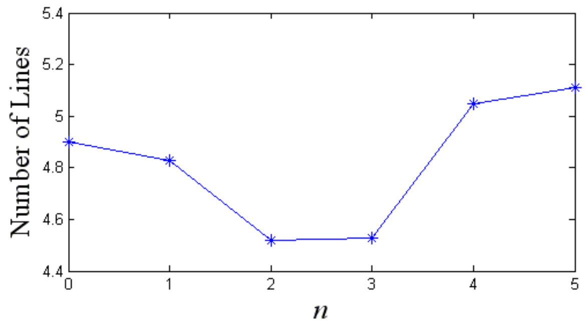

Fig. 14. Number of average detected lines with respect to n

Fig. 14 shows the number of detected lines as a function of broadening parameter, which is also averaged over 100 runs. The same rectangular image is used except that the image size is 400×400. The decreasing parameter α is set as 0.4 following the previous experiment.

The optimal broadening parameter value calculated is 3 in this case but fig. 14 shows that the number of detected line is minimized when n = 2. This is also a smaller value than the derived one due to the same reason with the previous. In this experiment $n = 3$ also showed almost the same result with the best one, and in general max(1, $n-1$) showed reasonable result. The HT found 5.6 lines on average. Experiments on different images show the similar results with the previous two experiments.

Fig. 15 compares the quality of detected lines using the HT and the ESHT. No noise is added so the HT and the ESHT detect the same number of lines. It is evident however that the detected lines using the ESHT are more close to real ones than those using the HT. When an image is noisy, the ESHT also found some bogus lines. However the number is smaller than that of the HT and the number of lines detected is summarized in table I. In table I, the numbers are averaged over 100 runs with randomly generated noise. Gaussian white noise is used as before. As is clear from table I, edge broadening and exponential weight setting make the HT robust to noise.

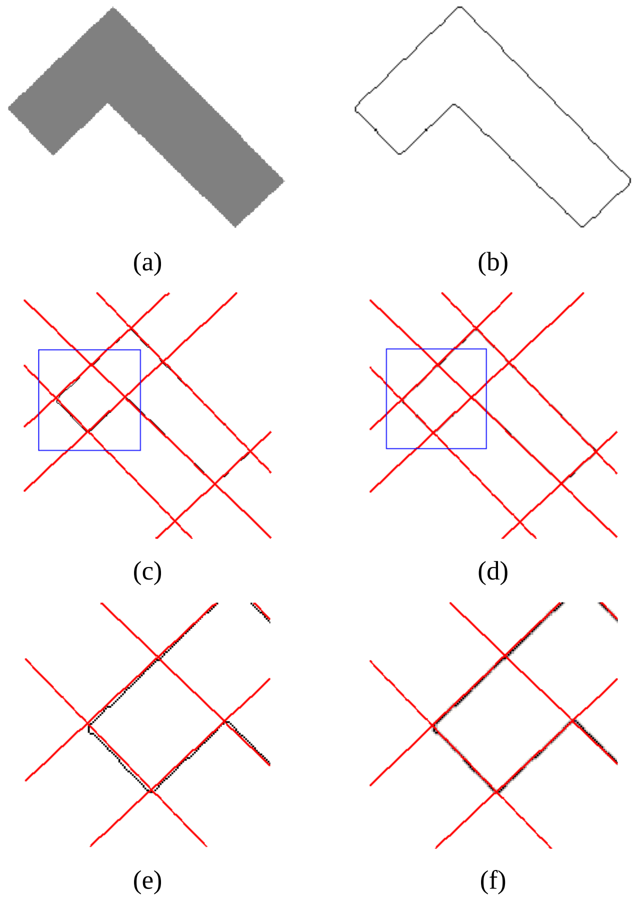

Fig. 15. Detected lines using the HT and the ESHT: (a) original image; (b) edge image from Canny edge detector; (c) result of the HT; (d) result of the ESHT; (e) detailed image of (c); and (f) detailed image of (d)

TABLE I. NUMBER OF LINES DETECTED

Image	HT	ESHT
Triangle	3.8	3.3
Rectangle	5.9	5.0
Pentagon	6.9	6.1
6 lines	7.5	6.9
10 lines	12.3	11.5

VI. DISCUSSION

Though the Hough transform is a simple and effective method to detect geometric primitives in an image, it is well known that the HT is sensitive to noise, which means that the number and quality of detected lines are affected by noise. The noise-sensitivity results in several kinds of distortion. In this paper, the Edge Strength Hough Transform is proposed to decrease the distortion and applied to line detection. The ESHT is different from the HT in that it uses broadened edges with strength proportional to the magnitude of gradients, which makes it possible its robustness to noise. Though the ESHT requires two parameters to broaden the 1-pixel-wide edge into arbitrary width and to set the exponentially decreasing strength, they can be decided only with the information given prior to the transform. Formal derivation is given and experimental results support the usefulness of the derived values.

Though the proposed method works better than the HT, it still has some problems. The most important one is that the parameters depend on the image given. In the experiments we conducted, $\alpha = 0.4$ and $\max(1, n-1)$ are the best choice but they might be different for different image. We are currently conducting some analysis to define more precise parameters based on the characteristics of an image, but the formulae still can be a good guideline in deciding parameters. In the derivation of the parameters, the strengths of edge pixels are assumed to be constant, which is not true in real-world problems and another problem under investigation.

High computational complexity is also a problem in the ESHT. As edges are broadened into width $(2n+1)$, the number of pixels considered also increases $(2n+1)$ times approximately. Furthermore the ESHT requires real number operations instead of integer operation in the HT. To reduce the burden, the number of edge pixels should be reduced, which can be done in many ways. One of them is to remove some pixels using the structural information, which is also under consideration.

The last thing left for further research is the extension of the ESHT to other geometric primitives. The ESHT does not assume any particular form of primitive or number of parameters. It is possible therefore that the ESHT can be applied to other primitives, like circles or ellipses. It is true that the 2nd order curve suffers more distortion than lines after digitization. So it is hoped that the ESHT can also decrease distortion in the 2nd order curves, which is another set of primitives widely used in computer vision area.

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