Robust Controller Design for Position Tracking Control of Permanent Magnet Stepper Motor Under Load Uncertainty

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Abstract: **The persistent problem of position control of stepper motors have always been looked out for betterment in terms of its efficiency and performance. Accurate estimation, tight control and close tracking of velocity and current vector are very difficult for conventional controllers. In this paper, a Robust Backstepping Controller is designed to track the position, virtual velocity and current continuously in a Permanent Magnet Stepper Motor (PMSM) in a MIMO (Multiple Input Multiple Output) System and is compared with the conventional PID controller to show its significance. Another important characteristic and advantage of this controller is that the DQ transformation (Direct Quadrature Transformation) is not required which reduces the complexity in conversion to the DQ frame. Finally an observer is also designed in case the complete information about the load torque is not known where an ideal sinusoidal flux distribution is assumed.**

Key Words: ***Permanent magnet stepper motor, Backstepping Controller, PID Controller, Observer***

INTRODUCTION

The precision is the basis for positioning and control of permanent magnet stepper motor. Over the years, various control methodologies and techniques were evolved. Marc Nonlinear state feedback control of permanent magnet stepper motor was proposed by [1]. Adaptive control of the stepper motor through backstepping control was discussed in [2]. Position control of stepper motor using exact linearization was implemented in [3]. The PWM technique with current feedback to control stepper motor was shown in [4]. Feedback linearization control technique was used in [5] and [6]. Lyapnov based control was done in [7]. An observer based design was analysed in [8]. Simple field weakening methods for position control of permanent magnet stepper motor combined with backstepping control was discussed in [9]. The papers [10-12] discussed extensively various robust control schemes while ensuring the stability of the system dynamics. In this paper, robust control scheme for position control of permanent magnet stepper motor is developed using backstepping control and also nonlinear observer is implemented under unknown load torque variations. The paper is organized as follows: section I discuss about dynamical model of Permanent Magnet Stepper Motor and its controller part. Section II discuss about the Observer part. Section III discuss about the comparison of this controller with PID controller.

A stepper motor is an electromechanical device which converts electrical pulses into discrete mechanical movements. When electrical pulses are given to it, the shaft rotates in discrete step increment. The direction of rotation is directly proportional to the sequence of pulses. There are many advantages in using a stepper motor like (i) It has excellent starting and stopping response; (ii) Rotational speed has wide range, (iii) Precise positioning and repeatability with accuracy of 3-5 percent etc.

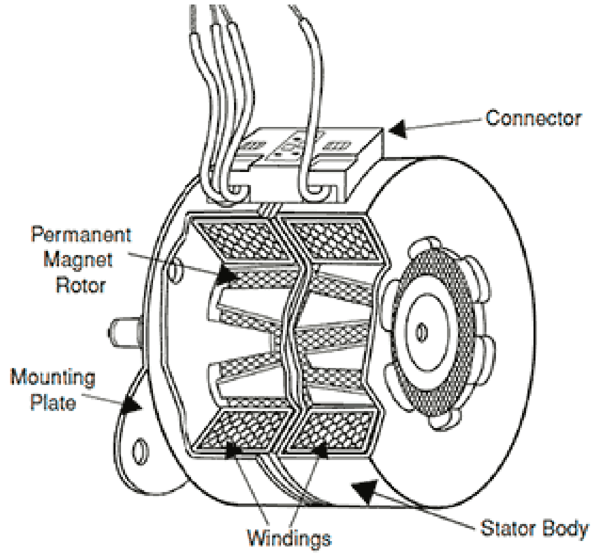

Fig.1. Permanent Magnet Stepper Motor

A) INTRODUCTION TO BACKSTEPPPING CONTROL

Backstepping technique refers to the recursive way in which stabilization is done from the origin and progressively stepped backwards. This sequential way can be done if the state space dynamics or equation is represented in strict or semi-strict feedback form as in [13] and [15]. The Lyapnov function is ensured for the stable system. Strict feedback form is expressed in Control System as follows:

$$\dot{x} = P_0(x) + Q_0(x)\, z_1$$
$$\dot{z}_1 = P_1(x,z_1) + Q_1(x,z_1)\, z_2$$
$$\dot{z}_2 = P_2(x,z_1,z_2) + Q_2(x,z_1,z_2)\, z_3$$
$$\vdots \qquad (1)$$
$$\dot{z}_i = P_i(x,z_1,z_2,\ldots,z_{i-1},z_i) + Q_i(x,z_1,z_2,\ldots,z_{i-1},z_i)\, z_{i+1}$$
$$\vdots$$
$$\dot{z}_{k-1} = P_{k-1}(x,z_1,z_2,\ldots,z_{k-1}) + Q_{k-1}(x,z_1,z_2,\ldots,z_{k-1})\, z_k$$
$$\dot{z}_k = P_k(x,z_1,z_2,\ldots,z_{k-1},z_k) + Q_k(x,z_1,z_2,\ldots,z_{k-1},z_k)\, u$$

Where $z_1, z_2, \ldots, z_i, \ldots, z_{k-1}, z_k$ are the different states, u is a scalar input to the system, the 'P' functions becomes zero when the origin is considered whereas the 'Q' function is a non zero function in the range from 1 to k. In (1), all the nonlinear functions represented in this form (the *strict feedback*) means that either of the function P or Q at the k^{th} state will be non linear dependent on the states starting from x, z_1, up to z_k which are given back as feedback. When permanent magnet stepper motor is considered, the position tracking is done first using this repetitive control algorithm and then the virtual velocity is made to track the rotor angular position and finally the current vector is made to track the virtual velocity. The stability criterion is analysed using root locus of the system with and without controller. All this are done and verified using MATLAB SIMUINK software where both the controller design is made and the simulation results are taken.

SECTION I

B) MATHEMATICAL MODEL OF PERMANENT MAGNET STEPPER MOTOR

The electromachanical dynamics of two phase permanent magnet stepper motor (PMSM) can be written as follows:

$$\dot{\theta} = \omega$$
$$\dot{\omega} = 1/J\ (-KI_a\sin(N\theta) + KI_b\cos(N\theta) - B\omega - \tau_L) \quad (2)$$
$$\dot{I}_a = 1/L\ (V_a - RI_a + K\omega\sin(N\theta))$$
$$\dot{I}_b = 1/L(V_b - RI_b - K\omega\cos(N\theta))$$

Where $x = [\Theta, w, Ia, Ib]^T$ being the state and *Va*, *Vb* being the input. Also *Ia*, *Ib* and *Va*, *Vb* are the currents in Ampere and voltages in the two phases. τ_L is the unknown load torque but this is assumed to be constant, ω being the rotor angular velocity, Θ being the rotor angular position and *R* being the resistance of phase winding. *K* is the torque constant of motor, *L* is the winding inductance, *B* is the viscous friction –coefficient of motor [N·m· s/rad], *J* is the inertia of motor [kg ·m^2], and *N or* N_r is the number of rotor teeth, respectively. The variation in inductance due to magnetic saturation is neglected. The magnetic coupling between the phases and the detent torque are also ignored.

But the above dynamics (2) of permanent magnet stepper motor does not follow the strict feedback form and as such the backstepping controller technique cannot be applied to it directly. So the dynamics is slightly modified so that it is in strict feedback form as follows:

$$\dot{\theta} = \omega$$
$$\dot{\omega} = K/J\,[\sin(N_r\theta)\ \cos(N_r\theta)]I - \tau_L/J - B\omega/J \qquad (3)$$
$$\dot{I} = AI + 1/L\begin{pmatrix} V_a \\ V_b \end{pmatrix} + K/L\begin{pmatrix} \sin(N_r\theta) \\ -\cos(N_r\theta) \end{pmatrix}$$

Where A is a 2x2 diagonal matrix with the diagonal elements being –R/L, the states are given as Θ, ω, I and the input Va, Vb. This modified equation (3) is in the form of strict feedback and hence backstepping method can be applied.

C) CONTROLLER DESIGN:

For designing the controller, some terms are required to be named. Let P_1, P_2, P_3 be the gains required to magnify the errors. The controller must satisfy the condition that if the actual angular position of the rotor is greater than the desired position then the controller must make the velocity lower than desired velocity so that the actual position comes to the desired value. Similarly if the actual position of rotor is less than the desired position then the controller must make the actual velocity more than the desired velocity so that the actual position rises to the desired position. In order to accomplish this, the difference between the actual and desired position is taken, magnified by the gain and added to the desired velocity. In this the virtual velocity is generated.

Let ω_1 be virtual velocity, I_1 be the desired current vector. Then $I_1 = [I_{a1}\ I_{b1}]^T$. The control equations are given as follows:

$$\omega_1 = \omega_d + P_1 E_1$$
$$I_1 = (\dot{\omega}_1 + E_1 + \frac{B}{J}\omega + \frac{1}{J}\tau_L + P_2 E_2)\frac{J}{K_m}\begin{pmatrix} -\sin(N_r\theta) \\ \cos(N_r\theta) \end{pmatrix} \qquad (4)$$
$$V_a = RI_a - K\omega\,[\sin(N\theta)] + L[\dot{I}_{a1} + P_3 E_{3a} - (K/J)\,E_2\,\sin(N\theta)]$$
$$V_b = RI_a + K\omega[\cos(N\theta)] + L[\dot{I}_{b1} + P_3 E_{3b} - (K/J)\,E_2\,\cos(N\theta)]$$

Where E_1, E_2, E_3 denotes the errors in corresponding positions, velocity (between virtual velocity and the actual velocity) and the current (desired current vector and actual current vector) respectively. With these specifications the system globally exponentially converges to the desired state. Here the direct current is a constant (zero) and the Quadrature current is calculated as follows:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \cos(N\theta) & \sin(N\theta) \\ -\sin(N\theta) & \cos(N\theta) \end{bmatrix} I_1 \qquad (5)$$

$$= \begin{bmatrix} 0 \\ \left(\dot{\omega_1} + E_1 + \frac{B}{J}\omega + \frac{\tau_L}{J} + P_2 E_2\right)\frac{J}{K} \end{bmatrix}$$

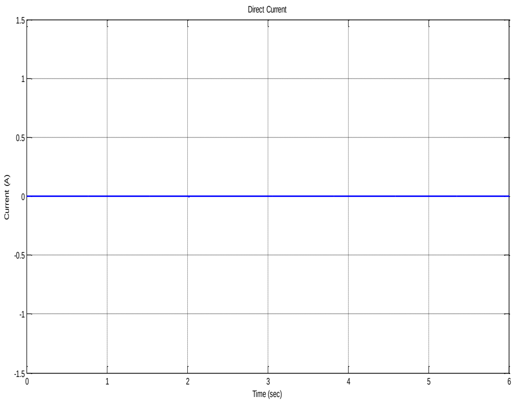

Fig.2. Direct current Id

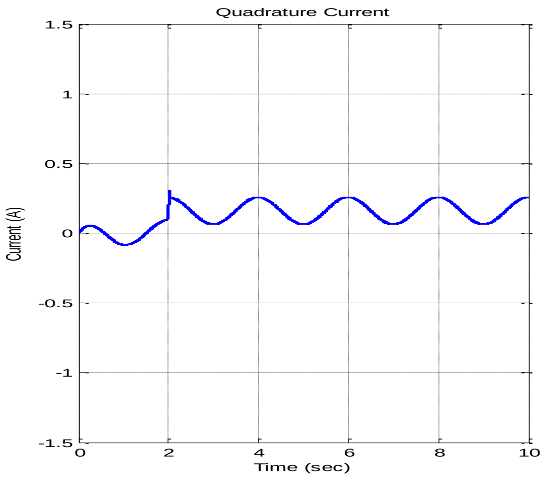

Fig.3. Quadrature current Iq

SECTION II

D) OBSERVER DESIGN

One problem with the controller was that the complete information about the motor should be known, i.e. it is assumed that the load torque is known fully. But practically this may not happen as we may not know the load torque. Also at low speeds, the proposed controller does not work too efficiently due to approximations as Euler's law are computationally not very accurate. So an observer which estimates the load torque is designed and then it is used to control the motor. The equations for observer states are slightly modified as follows:

$$\dot{\theta}_m = \omega + G_1 E_{12}$$

$$\dot{\omega}_m = \frac{K}{J}\,[\,\sin(N\theta)\ \ \cos(N\theta)\,]^T I^T - \frac{\tau_L}{J} - \frac{B\omega}{J} + G_2 E_{12} \qquad (6)$$

$$\dot{\tau}_{lm} = G_3 E_{12}$$

Where G_1, G_2 and G_3 are observer gains and E_{12} is the estimated positional error i.e. difference between the actual and modified rotor angular position (Θ- Θm). The load torque is directly proportional to the estimated positional error. Similarly the control equations also slightly change as follows:

$$I_1 = (\dot{\omega_1} + E_1 + \frac{B}{J}\,\omega + \frac{1}{J}\,\tau_L + P_2\,(E_2 + E_{21})\,\frac{J}{K_m}\begin{pmatrix} -\sin(N_r\theta) \\ \cos(N_r\theta) \end{pmatrix} \qquad (4)$$

$V_a = RI_a - K\omega[\sin(N\theta)] + L[\dot{I}_{a1} + P_3E_{3a} - (K/J)\,(E_2 + E_{21})\sin(N\theta)]$

$V_b = RI_a + K\omega\,[\cos(N\theta)] + L\,[\dot{I}_{b1} + P_3E_{3b} - (K/J)\,(E_2 + E_{21})\,\cos(N\theta)]$

Where E_2 denotes the error in rotor angular velocity i.e. difference between virtual and actual velocity, E_{21} denotes estimation error in velocity i.e. difference between actual and modified velocities (ω-ω_m). When these equations are slightly modified, the resultant backstepping observer tracks the desired position and desired velocity. In a class of electromechanical systems like permanent magnet stepper motor, angular velocity or electrical dynamics while reaching upper bound, leads to non-linear behaviour of the machine. Designing a control law from known-stable position recursively helps in full control over the system dynamics. From the stabilization perspective, the backstepping controller designed, for rotor position control, has an edge over other controllers under unknown load variations and ensures robustness of the controller performance and system stability.

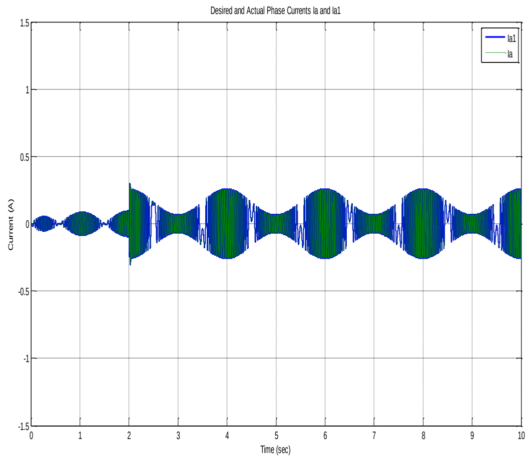

Fig.4. Desired and Actual Phase Current Ia and Ia1

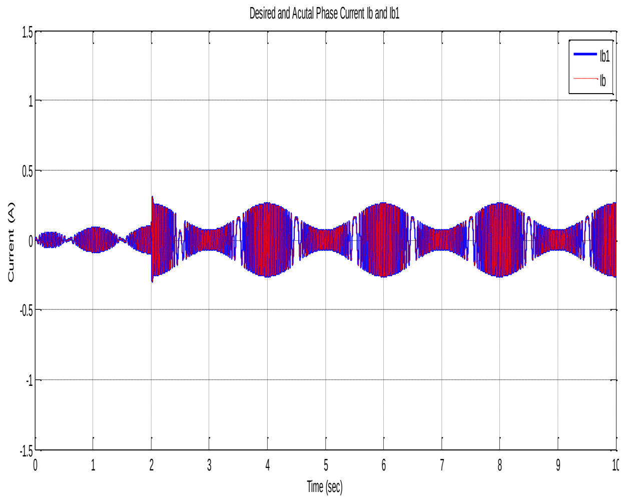

Fig.5. Desired and actual phase current Ib and Ib1

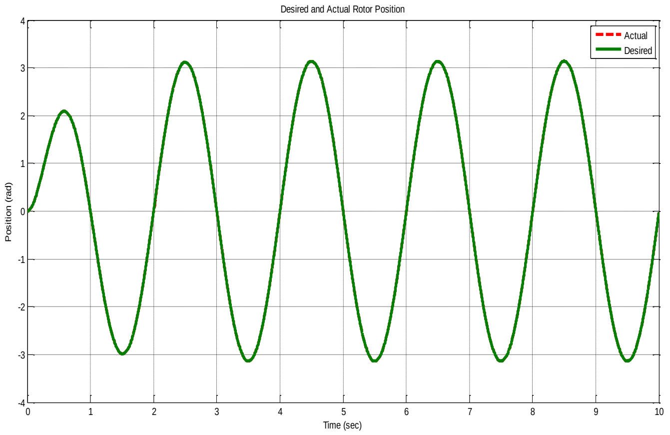

Fig.6. Desired Vs Actual Rotor position of Backstepping Controller

SECTION III

E) PID CONTROLLER:

The PID controller consists of proportional integral and derivative action taking place simultaneously in order to track the set point and reduce the error. This is a conventional controller which is used in many industrial processes with lots of modification in it. In this system, for controlling the position of stepper motor, the initial dynamics of the motor was taken and the error signal was given to the controller. By choosing suitable gains the steady state position was reached.

TABLE I
COMPARISON BETWEEN PID AND BACKSTEPPING CONTROLLERS

PARAMETER	PID CONTROLLER	BACKSTEPPING CONTROLLER
ISE	18.87	0.087
IAE	7.49	0.053
ITAE	11.15	0.004

The performance criterions for evaluating the accomplishments of a controller are Integral Square Error (ISE), Integral Absolute value of magnitude of error (IAE) and Integral Time multiplexed Absolute value of Error (ITAE). The formulae for calculating them are as follows:

$$ISE = \int e^2\, dt$$

$$IAE = \int |e| dt \qquad (8)$$

$$ITAE = \int |e| t dt$$

The PID output is given as follows:

$$z = k_p e + k_i \int e\, dt + k_d \frac{de}{dt} + z_o$$

Here, Kp is proportional gain, Ki is integral gain, Kd is differential gain, Zo is the offset and Z is the current value of PID output.

SIMULATION RESULTS:

The backstepping controller was designed using MATLAB SIMULINK software and the simulation results are shown. The input to the system i.e. the equation for desired angular position is ($(1-e^{-2t}) * \pi * \sin(\pi t)$). The controller tracked the desired position continuously with negligible error. To check the efficient performance of the controller, the torque was given as a step input after two seconds. Even then it is found that the position and velocity is tracked with a small disturbance at the 2^{nd} second. Due to the intervention of this torque it is found that the current vector tracks the desired current vector but the amplitudes of both the phases increased when the load torque is introduced. It is also found that the virtual velocity gets settled to the desired velocity. The tracking errors are also zero.

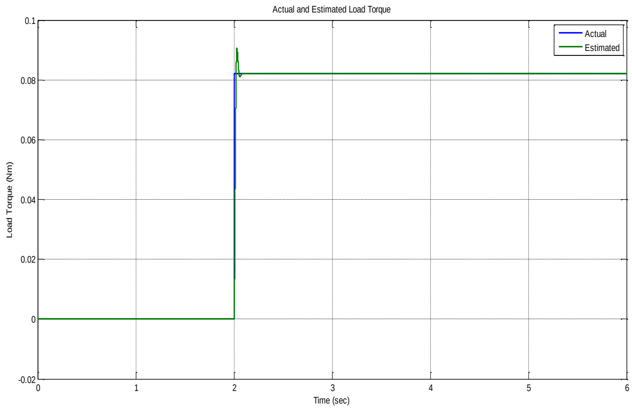

Fig.7. Actual and Estimated Load Torque

Later for the comparison of PID controller and the backstepping controller, the desired position was taken constant (as 5) and it is found that the backstepping controller tracks it much faster than the PID controller. When PID controller takes 8 seconds roughly to track the set point, the backstepping controller tracks it within 0.5

seconds. The controller criterions like ISE, IAE, ITAE and Time Constant are compared for both the controllers.

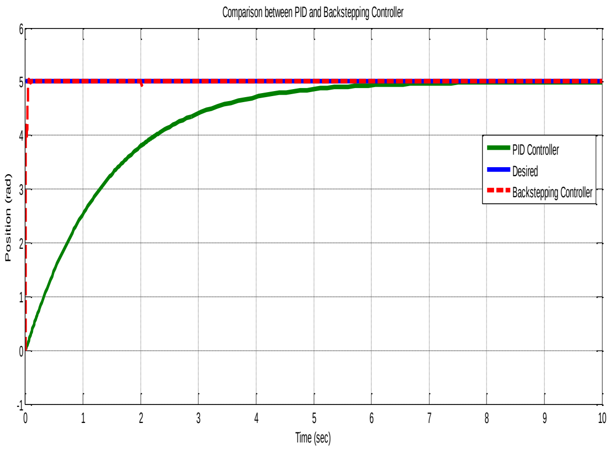

Fig.8. Comparison of PID and Backstepping Controller

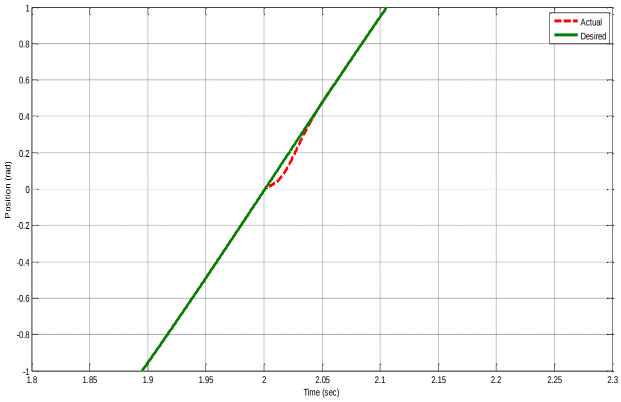

Fig.9. Desired and Actual Rotor Position of Stepper Motor showing Comparison

From that it is observed that the backstepping controller satisfies all the performance criterions 100 times better than the PID controller. The root locus for the dynamics of Permanent Magnet Stepper Motor was found in which the locus traced left half of the s-plane denoting the unstable region. But when the backstepping controller was designed, the root locus restricted to the left half of the s-plane denoting global stability. Also, the overall poles of the system with the controller were too left extreme in the root locus in compared with that of the system denoting improved stability.

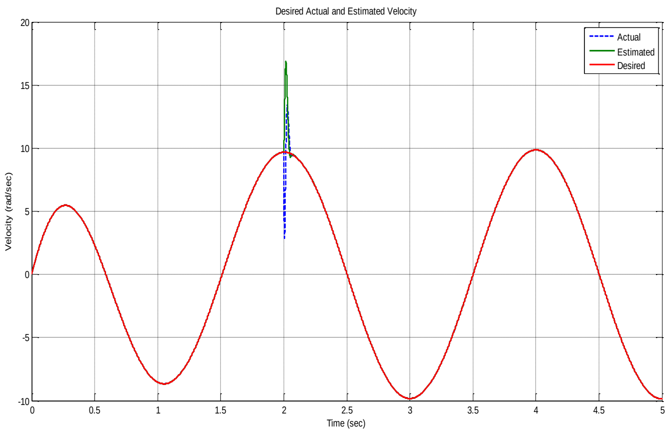

Fig.10. Actual Desired and Virtual velocity

F) MOTOR PARAMETERS:

The following motor parameters are taken for simulation [14] with suitable gains as follows:

Motor Inertia, J = 3 x 10^{-5} kg.m^2

Torque Constant, K = 0.51 N.m/A

Resistance of the phase windings, R = 14.8Ω

Viscous Friction Coefficient, B = 0.005 N.m.s/rad

Number of Rotor Teeth, N = 50

Inductance of Phase Winding, L = 40 mH

Input Voltage limit, Vlim = 24 V

CONCLUSION:

In this paper, a backstepping controller for MIMO system is designed to track the position of Permanent Magnet Stepper Motor. The virtual velocity tracked the desired position and the current vector tracked the virtual velocity. When information about the load torque is not known, an observer was proposed to estimate the position velocity and the torque and the tracking errors were converged to zero. The Backstepping controller is compared with the PID controller and its performance criterions are determined. Finally Backstepping controller is proved to be a better controller than the PID.

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