

$\tau_1\tau_2\#RG$ - Homeomorphisms in Bitopological Spaces

S.Thilaga Leevathi¹ and S.Sivanthi²

¹Associate Professor of Mathematics, Pope's College(Autonomous), Sawyerpuram,
Tamil Nadu - 627 251, India.

²Assistant Professor of Mathematics, Pope's College(Autonomous), Sawyerpuram,
Tamil Nadu - 627 251, India

Abstract - A bijection $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $\tau_1\tau_2\#$ regular generalized $\#$ -homeomorphism if f and f^{-1} are $\tau_1\tau_2\#rg$ -continuous. Also we introduce new class of maps, namely $\tau_1\tau_2\#rgc$ -homeomorphisms which form a subclass of $\tau_1\tau_2\#rg$ -homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all $\tau_1\tau_2\#rgc$ homeomorphisms forms a group under the operation composition of maps.

Mathematical Subject Classification: 54C10, 54C08, 54C05

Keywords: $\tau_1\tau_2\#rg$ -homeomorphism, $\tau_1\tau_2\#rgc$ homeomorphism

1. INTRODUCTION

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map $f: X \rightarrow Y$ when both f and f^{-1} are continuous. It is well known that as Jänich [[9], p.13] says correctly: homeomorphisms play the same role in topology that linear isomorphism play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphism in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Maki et al. [12] introduced g -homeomorphisms and gc -homeomorphisms in topological spaces.

In this paper, we introduce the concept of $\tau_1\tau_2\#rg$ -homeomorphism and study the relationship between homeomorphisms, $\tau_1\tau_2g$ homeomorphism, $g\tau_1\tau_2s$ - homeomorphism and $\tau_1\tau_2rg$ homeomorphism.

Also we introduce new class of maps $\tau_1\tau_2\#rgc$ -homeomorphism which form a subclass of $\tau_1\tau_2\#rg$ -homeomorphism. This class of maps is closed under composition of maps. We prove that the set of all $\tau_1\tau_2\#rgc$ homeomorphisms forms a group under the operation composition of maps.

Let us recall the following definition which we shall require later.

Definition 1.1. A subset A of a bitopological space (X, τ_1, τ_2) is called:

- (1) $\tau_1\tau_2$ preopen set if $A \subseteq \tau_1\text{int}\tau_2\text{cl}(A)$ and a $\tau_1\tau_2$ preclosed set if $\tau_2\text{cl}\tau_1\text{int}(A) \subseteq A$.
- (2) $\tau_1\tau_2$ semiopen set[1] if $A \subseteq \tau_2\text{cl}\tau_1\text{int}(A)$ and a $\tau_1\tau_2$ semiclosed set if $\tau_1\text{int}\tau_2\text{cl}(A) \subseteq A$.
- (3) $\tau_1\tau_2$ regular open set if $A = \tau_1\text{int}\tau_2\text{cl}(A)$ and a τ_2 regular closed set if $A = \tau_2\text{cl}\tau_1\text{int}(A)$.
- (4) $\tau_1\tau_2$ π - open set if A is a finite union of regular open sets.
- (5) $\tau_1\tau_2$ regular semi open if there is a τ_1 regular open U such $U \subseteq A \subseteq \tau_2\text{cl}(U)$.

Definition 1.2. A subset A of (X, τ_1, τ_2) is called

- (1) $\tau_1\tau_2$ generalized closed set (briefly, $\tau_1\tau_2g$ -closed) if $\tau_2\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

- (2) $\tau_1\tau_2$ regular generalized closed set (briefly, $\tau_1\tau_2$ rg-closed) if $\tau_2\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -regular open in X .
- (3) $\tau_1\tau_2$ generalized preregular closed set (briefly, $\tau_1\tau_2$ gpr-closed) if $\tau_2\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -regular open in X .
- (4) $\tau_1\tau_2$ regular weakly generalized closed set (briefly, $\tau_1\tau_2$ wg-closed) if $\tau_2\text{cl}\tau_1\text{int}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -regular open in X .
- (5) $\tau_1\tau_2$ rw-closed if $\tau_2\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 regular semi open.
- (6) $\tau_1\tau_2$ #rg-closed if $\tau_2\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 rw-open.

The complements of the above mentioned closed sets are their respective open sets.

Definition: 1.3. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called #rg-continuous if $f^{-1}(V)$ is $\tau_1\tau_2$ #rg-closed in (X, τ_1, τ_2) for every closed subset V of (Y, σ_1, σ_2) .

Definition: 1.4. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $\tau_1\tau_2$ #rg-irresolute if $f^{-1}(V)$ is $\tau_1\tau_2$ #rg-closed in X for every $\tau_1\tau_2$ #rg-closed subset V of Y .

Definition 1.5. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $\tau_1\tau_2$ #rg-closed (resp. $\tau_1\tau_2$ #rg-open) if for every #rg-closed (resp. $\tau_1\tau_2$ #rg-open) set U of X the set $f(U)$ is $\tau_1\tau_2$ #rg-closed (resp. $\tau_1\tau_2$ #rg-open) in Y .

Definition 1.6. A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be

- (i) $\tau_1\tau_2$ g homeomorphism[12] if both f and f^{-1} are $\tau_1\tau_2$ g-continuous,
- (ii) $\tau_1\tau_2$ gs- homeomorphism [6] if both f and f^{-1} are $\tau_1\tau_2$ gs-continuous,
- (iii) $\tau_1\tau_2$ rwg- homeomorphism[14] if both f and f^{-1} are $\tau_1\tau_2$ rwg-continuous,
- (iv) $\tau_1\tau_2$ gc- homeomorphism[12] if both f and f^{-1} are $\tau_1\tau_2$ gc-irresolute.

2. $\tau_1\tau_2$ #RG-homeomorphism in Bitopological Spaces

Definition 2.1. A bijection $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $\tau_1\tau_2$ #regular generalized homeomorphism (briefly, $\tau_1\tau_2$ #rg-homeomorphism) if f and f^{-1} are $\tau_1\tau_2$ #rg-continuous. We denote the family of all $\tau_1\tau_2$ #rg homeomorphisms of a topological space (X, τ_1, τ_2) onto itself by $\tau_1\tau_2$ #rg-h (X, τ_1, τ_2) .

Example 2.2. Consider $X = Y = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\sigma_1 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is bijective, $\tau_1\tau_2$ #rgcontinuous and f^{-1} is $\tau_1\tau_2$ #rg-continuous. Hence f is $\tau_1\tau_2$ #rg-homeomorphism.

Theorem 2.3. Every homeomorphism is $\tau_1\tau_2$ #rg homeomorphism, but not conversely.

Proof. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a homeomorphism. Then f and f^{-1} are continuous and f is bijection. Since every continuous function is $\tau_1\tau_2$ #rg-continuous, f and f^{-1} is $\tau_1\tau_2$ #rg-continuous. Hence f is $\tau_1\tau_2$ #rghomeomorphism. The converse of the above theorem need not be true, as seen from the following example.

Example 2.4. Consider $X = Y = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\sigma_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$ and $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $\tau_1\tau_2$ #rg-homeomorphism it is not homeomorphism, since the inverse image of closed set of $\{a,d\}$ in X is $\{a,d\}$ which is not closed in Y .

Theorem 2.5. Every $\tau_1\tau_2$ #rg-homeomorphism is g-homeomorphism.

Proof. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $\tau_1\tau_2$ #rg-homeomorphism. Then f and f^{-1} are $\tau_1\tau_2$ #rg-continuous and f is bijection. Since every $\tau_1\tau_2$ #rg-continuous function is g -continuous, f and f^{-1} are g -continuous. Hence f is $\tau_1\tau_2$ ghomeomorphism.

Corollary 2.6. Every $\tau_1\tau_2$ #rg-homeomorphism is $\tau_1\tau_2$ gs-homeomorphism.

Proof. By the fact that every $\tau_1\tau_2$ g homeomorphism is $\tau_1\tau_2$ gs-homeomorphism and by theorem 2.5.

Corollary 2.7. Every $\tau_1\tau_2$ #rg-homeomorphism is $\tau_1\tau_2$ gsp-homeomorphism.

Proof. By the fact that every gshomeomorphism is $\tau_1\tau_2$ gsp-homeomorphism and by corollary 2.6.

Theorem 2.8. Every $\tau_1\tau_2$ #rg-homeomorphism is $\tau_1\tau_2$ rg-homeomorphism.

Proof. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $\tau_1\tau_2$ #rg-homeomorphism. Then f and f^{-1} are $\tau_1\tau_2$ #rg-continuous and f is bijection. Since every #rg-continuous function is $\tau_1\tau_2$ rg-continuous, f and f^{-1} are $\tau_1\tau_2$ rg-continuous. Hence f is $\tau_1\tau_2$ rg homeomorphism.

Corollary 2.9. Every $\tau_1\tau_2$ #rg-homeomorphism is $\tau_1\tau_2$ rwg-homeomorphism and $\tau_1\tau_2$ gp rhomeomorphism.

Proof. By the fact that every $\tau_1\tau_2$ rg homeomorphism is $\tau_1\tau_2$ rwg-homeomorphism and $\tau_1\tau_2$ gpr-homeomorphism, and by theorem 2.8.

Theorem 2.10. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a bijective $\tau_1\tau_2$ #rg- continuous map. Then the following are equivalent.

- (i) f is a $\tau_1\tau_2$ #rg- open map
- (ii) f is $\tau_1\tau_2$ #rg-homeomorphism,
- (iii) f is a $\tau_1\tau_2$ #rg -closed map.

Proof. Suppose (i) holds. Let V be open in (X, τ_1, τ_2) . Then by (i), $f(V)$ is $\tau_1\tau_2$ #rg-open in (Y, σ_1, σ_2) . But $f(V) = (f^{-1})^{-1}(V)$ and so $(f^{-1})^{-1}(V)$ is $\tau_1\tau_2$ #rgopen in (Y, σ_1, σ_2) . This shows that f^{-1} is $\tau_1\tau_2$ #rg-continuous and it proves (ii).

Suppose (ii) holds. Let F be a closed set in (X, τ_1, τ_2) . By (ii), f^{-1} is $\tau_1\tau_2$ #rg-continuous and so $(f^{-1})^{-1}(F) = f(F)$ is $\tau_1\tau_2$ #rg-closed in (Y, σ_1, σ_2) . This proves (iii).

Suppose (iii) holds. Let V be open in (X, τ_1, τ_2) . Then V^c is closed in (X, τ_1, τ_2) . By (iii), $f(V^c)$ is $\tau_1\tau_2$ #rg-closed in (Y, σ_1, σ_2) . But $f(V^c) = (f(V))^c$. This implies that $(f(V))^c$ is $\tau_1\tau_2$ #rg-closed in (Y, σ_1, σ_2) and so $f(V)$ is $\tau_1\tau_2$ #rg-open in (Y, σ_1, σ_2) . This proves (i).

Definition 2.11. A bijection $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $\tau_1\tau_2$ #rgc-homeomorphism if both f and f^{-1} are $\tau_1\tau_2$ #rg-irresolute. We say that spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) are $\tau_1\tau_2$ #rgchomeomorphic if there exists a $\tau_1\tau_2$ #rgchomeomorphism from (X, τ_1, τ_2) onto (Y, σ_1, σ_2) . We denote the family of all $\tau_1\tau_2$ #rgc-homeomorphisms of a topological space (X, τ_1, τ_2) onto itself by $\tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) .

Theorem 2.12. Every $\tau_1\tau_2$ #rgc-homeomorphism is a $\tau_1\tau_2$ #rg-homeomorphism but not conversely.

Proof. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an $\tau_1\tau_2$ #rgchomeomorphism. Then f and f^{-1} are $\tau_1\tau_2$ #rg-irresolute and f is bijection. By Theorem 4.2 in [22], f and f^{-1} are $\tau_1\tau_2$ #rg-continuous. Hence f is $\tau_1\tau_2$ #rg-homeomorphism. The converse of the above theorem is not true in general as seen from the following example.

Example 2.13. Consider $X = Y = \{a, b, c, d\}$ with $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,b,c\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}, \{a,b,c\}\}$, $\sigma_1 = \{X, \emptyset, \{c\}, \{a,b\}, \{a,b,c\}\}$ and $\sigma_2 = \{Y, \emptyset, \{c\}, \{a,b\}, \{a,b,c\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow$

(Y, σ_1, σ_2) be defined by $f(a) = b, f(b) = c, f(c) = a$ and $f(d) = d$. Then f is $\tau_1\tau_2$ #rg-homeomorphism but it is not $\tau_1\tau_2$ #rgc-homeomorphism, since f is not $\tau_1\tau_2$ #rg-irresolute.

Theorem 2.14. Every $\tau_1\tau_2$ #rgc-homeomorphism is $\tau_1\tau_2$ g-homeomorphism but not conversely.

Proof. Proof follows from Theorems 2.5 and Theorem 2.12.

Remark 2.15 $\tau_1\tau_2$ #rgc-homeomorphism and $\tau_1\tau_2$ gc - homeomorphism are independent as seen from the following example.

Example 2.16 Let $X = Y = \{a,b,c,d\}$ with $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $\tau_1\tau_2$ #rgc - homeomorphism but it is not $\tau_1\tau_2$ gc-homeomorphism, since f is not $\tau_1\tau_2$ gc-irresolute.

Theorem 2.17. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \rho_1, \rho_2)$ are $\tau_1\tau_2$ #rgc-homeomorphisms, then their composition $gof: (X, \tau_1, \tau_2) \rightarrow (Z, \rho_1, \rho_2)$ is also $\tau_1\tau_2$ #rgc-homeomorphism.

Proof. Let U be a $\tau_1\tau_2$ #rg-closed set in (Z, ρ_1, ρ_2) . Since g is $\tau_1\tau_2$ #rg-homeomorphism, $g^{-1}(U)$ is $\tau_1\tau_2$ #rgclosed in (Y, σ_1, σ_2) . Since f is $\tau_1\tau_2$ #rg-homeomorphism, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is $\tau_1\tau_2$ #rg closed in (X, τ_1, τ_2) . Therefore gof is $\tau_1\tau_2$ #rgirresolute. Also for a $\tau_1\tau_2$ #rg-closed set G in (X, τ_1, τ_2) , We have $(gof)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, $f(G)$ is $\tau_1\tau_2$ #rg-closed in (Y, σ_1, σ_2) and so again by hypothesis, $g(f(G))$ is a $\tau_1\tau_2$ #rg-closed set in (Z, ρ_1, ρ_2) . That is $(gof)(G)$ is a $\tau_1\tau_2$ #rg-closed set in (Z, ρ_1, ρ_2) and therefore $(gof)^{-1}$ is $\tau_1\tau_2$ #rg-irresolute. Also gof is a bijection. Hence gof is $\tau_1\tau_2$ #rg-homeomorphism.

Theorem 2.18. The set $\tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) is a group under the composition of maps.

Proof. Define a binary operation $*$: $\tau_1\tau_2$ #rgch $(X, \tau_1, \tau_2) \times \tau_1\tau_2$ #rgc-h $(X, \tau_1, \tau_2) \rightarrow \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) by $f * g = gof$ for all $f, g \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) and o is the usual operation of composition of maps. Then by theorem 2.17, $gof \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) . We know that the composition of maps is associative and the identity map $I: (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2)$ belonging to $\tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) serves as the identity element. If $f \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) , then $f^{-1} \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of $\tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) . Therefore $(\tau_1\tau_2$ #rgch $(X, \tau_1, \tau_2), o)$ is a group under the operation of composition of maps.

Theorem 2.19. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $\tau_1\tau_2$ #rgc-homeomorphism. Then f induces an isomorphism from the group $\tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) onto the group $\tau_1\tau_2$ #rgc-h (Y, σ_1, σ_2) .

Proof. Using the map f , we define a map $\varphi_f: \tau_1\tau_2$ #rgc-h $(X, \tau_1, \tau_2) \rightarrow \tau_1\tau_2$ #rgc-h (Y, σ_1, σ_2) by $\varphi_f(h) = f \circ h \circ f^{-1}$ for every $h \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) . Then φ_f is a bijection. Further, for all $h_1, h_2 \in \tau_1\tau_2$ #rgc-h (X, τ_1, τ_2) ; $\varphi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \varphi_f(h_1) \circ \varphi_f(h_2)$. Hence φ_f is a homomorphism and so it is an isomorphism induced by f .

REFERENCES

- [1] Andrijevic.D, Semi-preopen Sets, Mat. Vesnik, 38(1986), 24-32.
- [2] Balachandran. K, Sundram .P and Maki. P, On generalized continuous maps in topological spaces, Mem. Fac.Sci.Kochi Univ.(Math) 12(1991), 5-13.
- [3] Benchalli. S.S.,and Wali. R.S.,On RWClosed sets in topological spaces, Bull. Malays. Math. Sci. Soc(2) 30(2) (2007), 99 – 110.
- [4] Cameron. D.E., Properties of S-closed spaces, Proc. Amer Math. Soc. 72(1978), 581–586.

- [5] Crossley.S.G. and Hildebrand.S.K., Semi-topological Properties, *Fund. Math.*, 74(1972), 233-254.
- [6]Devi. R, Balachandran. K and Maki. H Semi generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces, *Indian J.Pure.Appl.Math.* 26(3) (1995), 271-284.
- [7] Dontchev. J, On generalizing semi-preopen sets, *Mem.Fac.Sci. Kochi Univ.ser.A Math.* 16(1995), 35-48.
- [8]Gnanambal. Y., On generalized preregular closed sets in topological spaces, *Indian J. Pure App. Math.* 28(1997), 351–360.
- [9] J nich K, *Topologie*, Springer-Verlag, Berlin, 1980 (English translation).
- [10]Levine. N., Semi-open sets and semicontinuity in topological spaces, *Amer. Math. Monthly*,70(1963), 36–41.
- [11] Levine. N., Generalized closed sets in topology, *Rend. Circ. Mat. Palermo* 19(1970), 89–96.
- [12] Maki H, Sundram P and Balachandran K, On generalized homeomorphisms in topological spaces, *Bull. Fukuoka Univ. Ed. Part III*, 40(1991), pp. 13-21
- [13] Mashhour. A.S., Abd. El-Monsef. M. E. and El-Deeb S.N., On pre continuous mappings and weak pre-continuous mappings, *Proc Math, Phys. Soc. Egypt* 53(1982), 47– 53.
- [14] Nagaveni. N., *Studies on Generalizations of Homeomorphisms in Topological Spaces*, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999.
- [15] Palaniappan. N., and Rao. K. C., Regular generalized closed sets, *Kyungpook Math. J.* 33(1993), 211–219.
- [16] Stone. M., Application of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.* 41(1937), 374–481.
- [17] S.Sivanthi and S.Thilaga Leevathi, On #Regular Generalized Closed sets in Bitopological spaces, *Journal of Emerging Technologies and Innovative Research*, volume 6, Issue 2 (2019), (368 -372).
- [18] S.Sivanthi, On #Regular Generalized open sets in Bitopological spaces, *International Journal of Research and Analytical Reviews*, volume 7, Issue 1, March 2020, (58 – 62).
- [19] S.Sivanthi and S.Thilaga Leevathi, On $\tau_1\tau_2$ #Rg-Continuous In Bitopological Spaces And $\tau_1\tau_2$ #Rg-Irresolute Functions, *International Journal of Creative Research Thoughts*, volume9, Issue 1,January 2021, (1164 – 1168),
- [20] Syed Ali Fathima. S and Mariasingam. M, On #regular generalized closed sets in topological spaces, *International journal of mathematical archive-2*(11), 2011, 2497 – 2502.
- [21] Syed Ali Fathima. S and Mariasingam. M, On #regular generalized open sets in topological spaces, *International journal of computer applications - 42*(7), 2012, 37 - 41.
- [22] Syed Ali Fathima. S and Mariasingam. M, On #RG-Continuous and #RG-irresolute functions (To be appear in *Journal of Advanced Studies in Topology*)
- [23] Zaitsav V, On certain classes of topological spaces and their bicompatifications. *Dokl. Akad. Nauk SSSR*(1968) 178: 778-779.