Revisión Sistemática: Método Para Identificar Significados Institucionales De Los Números Racionales

Systematic Review: Method for Identifying Institutional Meanings of Rational Numbers

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Resumen
El siguiente artículo tiene por objetivo identificar los significados institucionales de los números racionales mediante una revisión sistemática de la literatura. Para ello, se analizan un conjunto de 65 artículos de diferentes autores y campos (enseñanza y aprendizaje). Estos artículos son seleccionados bajo los criterios de inclusión y exclusión propuestos por el método PRISMA. Con el propósito de establecer, a su vez, las variables bibliométricas, las variables de interés sobre el contenido y las limitaciones de las investigaciones identificadas. Los resultados de la revisión sistemática muestran que los significados institucionales que emergen del objeto matemático se abordan desde diferentes registros de representación (gráfico, numérico, pictórico, lenguaje natural); además, están constituidos por diversos objetos primarios que permiten establecer definiciones de este objeto matemático de una manera más completa.

Palabras clave: Significados institucionales, Números racionales, Revisión Sistemática, PRISMA, Variables bibliométricas, Enfoque Ontosemiótico, EOS.

Abstract
The following article aims to identify the institutional meanings of rational numbers through a systematic literature review. To do this, a set of sixty-five articles by different authors and fields (teaching and learning) are analyzed. These articles are selected under the inclusion and exclusion criteria proposed by the PRISMA method. To establish bibliometric variables, variables of interest about the content, and the limitations of the identified research. The results of the systematic review show that the institutional meanings emerging from the mathematical object are addressed from different registers of representation (graphical, numerical, pictorial, natural language); they are also constituted by various primary objects that allow for more complete definitions of this mathematical object.

Keywords: Institutional meanings, Rational numbers, Systematic review, PRISMA, Bibliometric variables, Ontosemiotic Approach, OSA.

1. INTRODUCTION

Rational numbers represent one of the fundamental concepts in mathematics and their understanding is essential for learning mathematical objects of greater complexity (Ni &; Zhou, 2005). As well as essential for understanding various real-world phenomena (Moss &; Case, 1999). However, despite its importance, the teaching and learning of numbers remains a challenge within mathematics education. Numerous studies have identified recurrent difficulties in students when trying to understand and operate with rational numbers (Behr Et Al., 1984; Vamvakoussi &; Vosniadou, 2010), suggesting the need to examine and rethink current teaching practices.

The mathematical practices associated with the field of problems from which the mathematical object emerges at a given time, and which are endorsed by an institution are called institutional meaning. (Breda Et Al., 2018; Godino Et Al., 2023; Godino &; Batanero, 1994). In the case of rationals, examining their institutional meanings involves analyzing the various conceptions, representations and applications attributed to them in the community of mathematics educators and researchers. Consequently, the institutional meanings found in the literature, in turn, must be in accordance with didactic suitabilities that allow us to identify which practices are more adequate and effective. The Ontosemiotic Approach to mathematical cognition and instruction (EOS), proposed by Godino and Batanero (2007), provides a valuable theoretical framework to examine the various meanings associated with mathematical objects such as rational numbers, as well as to determine from its theoretical postulates the indicators of didactic suitability to evaluate the teaching and learning processes.

The ontosemiotic approach (EOS) is a theoretical and methodological framework in mathematics education that focuses on the study of mathematical objects and processes, as well as on the way in which students represent and understand them. It is based on the idea that mathematical knowledge is a social and cultural construct that arises from the interaction of the individual with mathematical resources, situations and objects (Godino et al., 2007). The EOS considers essential for the processes of teaching and learning mathematics contributions and contributions from different disciplines such as pedagogy, psychology, sociology, philosophy, among others. The theoretical components of these sciences help analyze and understand the mathematical nature and its various practices (Godino et al., 2007).

Due to its anthropological origin, EOS considers mathematical practices to be any action or expression (verbal or graphic) conducted by someone to solve a mathematical problem, communicate the solution obtained to others, validate it or generalize it to different contexts (Godino &; Batanero, 1994). Thus, these practices can be performed by a person or an institution, which gives rise to personal practice systems and institutional practice systems. In the construction of the theoretical framework, the term meaning is conceived.
because of personal and institutional practices, Pino-Fan (2010) defines meaning as “the system of operational and discursive practices that a person or institution performs to solve certain kinds of problem situations in which that object intervenes”.

In turn, being immersed in an institutional context, Godino &; Batanero (1994, P. 340), define institutional meaning as follows: “The meaning of an institutional object is the system of institutional practices associated with the field of problems from which the object emerges at a given time”. To understand in depth what institutional meaning represents, within EOS an object is any material or immaterial entity that intervenes in mathematical practice, supporting and regulating its realization (Godino et al., 2007). Within the EOS, diverse types of primary objects that intervene in the practice systems are proposed, such as concepts, language, situations, procedures, propositions and arguments (Godino, 2002). Furthermore, the contextual and semiotic circumstances in which objects, meanings and mathematical practices participate are complemented and enriched when the five dual dimensions (Personal/Institutional, Ostensive/Non-ostensive, Extensive/Intensive, Unitary/Systemic, Expression/Content) are taken into consideration (Godino et al., 2007).

In the search for these institutional meanings surrounding rational numbers, the following question arises: What institutional meanings emerge from the review of the literature on rational numbers considering the theoretical structures of the Ontosemiotic Approach?

2. OBJECTIVES

To gather and articulate both classical and recent research that highlights the recurring difficulties in understanding and using rational numbers.
To analyze the cognitive limitations, common errors, and inadequate problem-solving strategies documented in the specialized literature.

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To promote reflection on the integration of more effective pedagogical practices, aimed at addressing these difficulties, following the guidelines of the Ontosemiotic Approach for the teaching of rational numbers.

3. METHOD

Systematic reviews seek to answer a set of questions to identify gaps in research, allow planning and making decisions about future studies and their feasibility (Álvarez Ariza &; Olatunde-Aiyedun, 2024). For this systematic review, the phases and elements of the PRISMA model (Page et al., 2021), which is a guide used to improve transparency and quality in the elaboration of systematic reviews and meta-analyses, are considered.
Once the research question and objectives of the review were defined, we proceeded to the planning stage. Following the PRISMA recommendations, rigorous inclusion and exclusion criteria were established to guarantee the quality of the studies to be considered. Subsequently, exhaustive search was carried out in various academic databases, using precise search strategies to identify relevant evidence. The identified studies were subjected to a selection and evaluation process, and the relevant information was extracted and organized in a data matrix for subsequent analysis

1. Planning the review: a problem was identified that was related, in this case, to the processes of teaching and learning rational numbers. In the process, the research question, the general objective of the review, as well as the definition of the inclusion and exclusion criteria of the research to be reviewed were raised; in addition, some bibliometric variables and interest of the content were taken into consideration, which broadened the search parameters in the research to be developed.
2. Data search: A search for research was carried out considering Boolean operators and keywords according to the research in recognized databases (Scholar Google, Scopus, Taylor and Francis, ERIC) that meet the previously established criteria.
3. Collection of the information obtained: in this phase, the identified studies are reviewed and evaluated to determine if they meet the inclusion criteria and if they contain the variables of interest in terms of content. For the exploratory review process, the bibliographic information manager Mendeley (Henning and Reichelt, 2008) was chosen because it is free of charge and linked to the Microsoft Office Word text tool, which implies agility and efficiency in developing the bibliographic references of the texts.
4. Data extraction: relevant information is collected from the selected studies, such as bibliometric data and content interest. The information is recorded in a data matrix elaborated in Excel spreadsheet software.
5. Interpretation of results: The findings are discussed and presented in the context of the problematizing question and the general objective, considering the evidence found and the implications for future research.

4. RESULTS

The planning of the review includes the statement of the problem question, the objective of the review and the inclusion and exclusion criteria, which are summarized in Table 1. This planning is carried out by means of a protocol (Manchado et al., 2009), which is adapted to the needs of the research and to the phases of PRISMA. Because it considers the observation and identification of bibliometric variables (title, database, year of publication, journal of publication, type of publication, authors, country and language) and the variables of interest of the content of this research (educational level, qualitative methodology, mathematization of situations and ontosemiotic approach).

For the development of this phase, it was considered that the research to be selected should be published in high impact journals. This was based on the H-index found in Scimago Journal &; Country Rank (SCIMAGO).

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Table 1 Protocol for the exploration systematic review of mathematical research. Adapted from Manchado

Study Question (RESEARCH PROBLEM)	*What institutional meanings emerge from the review of the literature on rational numbers considering the theoretical structures of the ontosemiotic approach?*
TARGET	**To analyze the institutional meanings that emerge when reviewing the existing literature on rational numbers, in the light of the theoretical structures proposed by the Ontosemiotic Approach**
STUDY PERIOD	**2000-2024**
LANGUAGE	**Spanish - English**
Otros: (POPULATION, GEOGRAPHIC AREA, …)	**Primary and secondary school students**
SOURCES OF INFORMATION	**Scholar Google, ScienceDirect, Scopus, Taylor and Francis, Eric**
INCLUSION CRITERIA	**Indexed journal articles, Articles with less than 20 years of publication (with some exceptions)**
EXCLUSION CRITERIA	**Little relation to the problem question, Population with significant age difference to that of the research context, Undergraduate research, master's thesis, posters, etc.**
BIBLIOMETRIC VARIABLES	**Database, Year of publication, Journal/Congress, Type of publication, Authors, Country, and Language**
Variables of interest on content	**Educational level, Performs data analysis, Mathematization of situations, Ontosemiotic approach.**

Garabito et al. (2009) (Source: Author´s own elaboration).

The second phase of the systematic review consisted of the search for information. Five databases (Scholar Google, ScienceDirect, Scopus, Taylor and Francis and ERIC) were considered for the search, which made it possible to identify the different documents initially proposed in the PRISMA flowchart (see Table 6).

Two types of criteria were considered in this search: exclusion criteria and inclusion criteria. The exclusion criteria considered the following.

- The document had to address the concept of rational numbers. Documents that dealt with this topic in a field other than Mathematics Education were excluded. In addition, documents that were published in congresses, research papers (master's or doctoral theses) and book chapters were excluded.
- The document had to address a theoretical framework in Mathematics Education. Those that did not present coherence between the title and the content were excluded.
- In addition to addressing the mathematical object of study and a theoretical framework in Mathematics Education, the document had to be framed in the context of Secondary School. Those that addressed the context of higher education and teacher training were excluded.
- The document had to be in English.

For the specific case of this systematic review, in its search, use is made of key terms or words present in each of the categories of the title, such as:

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Mathematical object: rational numbers, understanding of rationals, fractions, decimals, proportions.
Theoretical framework: Ontosemiotic approach, EOS.
Methodological framework: Problem solving, mathematization of situations.
Contextual framework: Secondary school students, elementary school students, secondary school education.
It should be clarified that the search was carried out in both Spanish and English. Due to interest in the bibliometric variable of language. In addition, logical combinations with Boolean operators such as "OR", 'AND' and "NOT" are used to combine the categories and thus find more refined research to the research in question. The number of studies found in the databases when searching using the keywords, both in Spanish and English, are recorded in Table 2 and Table 3.

Database	*Rational Numbers*	*Ontosemiotic Approach*	*Middle School*
Scholar Google	3.660.000	2.510	2.760.000
Taylor and Francis	725.838	1	61.575
ScienceDirect	627.662	2	36.403
Scopus	41.448	59	31.909
ERIC	1.056	36	51.131

Table 2 Results of keyword search in English (Source: Author´s own elaboration).

Next, in Tables 4 and 5, we share the results of the searches with the Boolean AND inclusion operator. Table 4 shares the results in English with the acronyms: R AND O for Rational Numbers AND Ontosemiotic Approach, R AND M for Rational Numbers AND Middle School and O AND M for Ontosemiotic Approach AND Middle School. Similarly, Table 5 shares the results in Spanish with the acronyms R AND EOS for Rational Numbers AND Ontosemiotic Approach, EOS AND SECUNDARIA for Ontosemiotic Approach AND Middle School and, R AND SECUNDARIA for Rational Numbers AND Middle School.

Database	*Números Racionales*	*Enfoque Ontosemiótico*	*Escuela Secundaria*
Scholar Google	11.600	4.140	208.000
Taylor and Francis	9	4	217
ScienceDirect	2	0	244
Scopus	6	9	154
ERIC	0	0	16

Table 3 Keyword search results in Spanish (Source: Author´s own elaboration).

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Database	R AND O	R AND M	O AND M
Scholar Google	121	8.720	306
Taylor and Francis	1	193	24
ScienceDirect	0	172	14
Scopus	0	36	0
ERIC	0	1.140	0

Table 4 Results of the search with the Boolean AND operator in English (Source: Author´s own elaboration).

Database	R AND EOS	EOS AND SECUNDARIA	R AND SECUNDARIA
Scholar Google	322	333	787
Taylor and Francis	0	0	0
ScienceDirect	0	0	0
Scopus	0	0	0
ERIC	0	0	0

Table 5 Results of the search with the Boolean AND operator in Spanish (Source: Author´s own elaboration).

The difference in the amount of research in the different databases is highlighted when comparing the research in English with the research in Spanish. As well as, in the few research found in the databases regarding the Boolean AND operator.

In the third phase of the research, Mendeley was used as a bibliographic manager to organize and manage the references. The references were imported from the academic databases mentioned in the first phase, labeled with keywords and grouped by subject. The use of the bibliographic manager made it easier to describe the bibliometric variables of each of the articles after screening. In the same sense, it also allowed optimal synchronization in the cloud, allowing multi-device and efficient access.
In addition to the above, the artificial intelligence tool chatPDF is implemented to streamline and systematize the extraction of data of interest on the content established in the first phase. The use of AI is done through the following instruction: You are a researcher conducting a systematic review. Identify the following variables of interest: mathematical object addressed in the paper, theoretical framework underlying the research, study subjects addressed in the research. The results obtained are verified by the researchers to guarantee the veracity of the information.

In the extraction of the information, that is, the fourth phase, Excel is used to record the information found in the previous step. They are discriminated according to the database, obtaining a total of 16 research by Scholar Google, 6 by ScienceDirect, 40 by Scopus, 3 in Taylor and Francis, and finally, 3 in ERIC, for a total of 68 researches. Due to the rigorousness of the previous phase, no repeated documents were found, which corresponds to the first filter of information collection (Manchado Garabito et al., 2009). The second filter proposed by the authors deals with the exclusion of documents under the exclusion criteria. Finally, those excluded by emphasis. Figure 1 summarizes the fourth phase of the systematic review.

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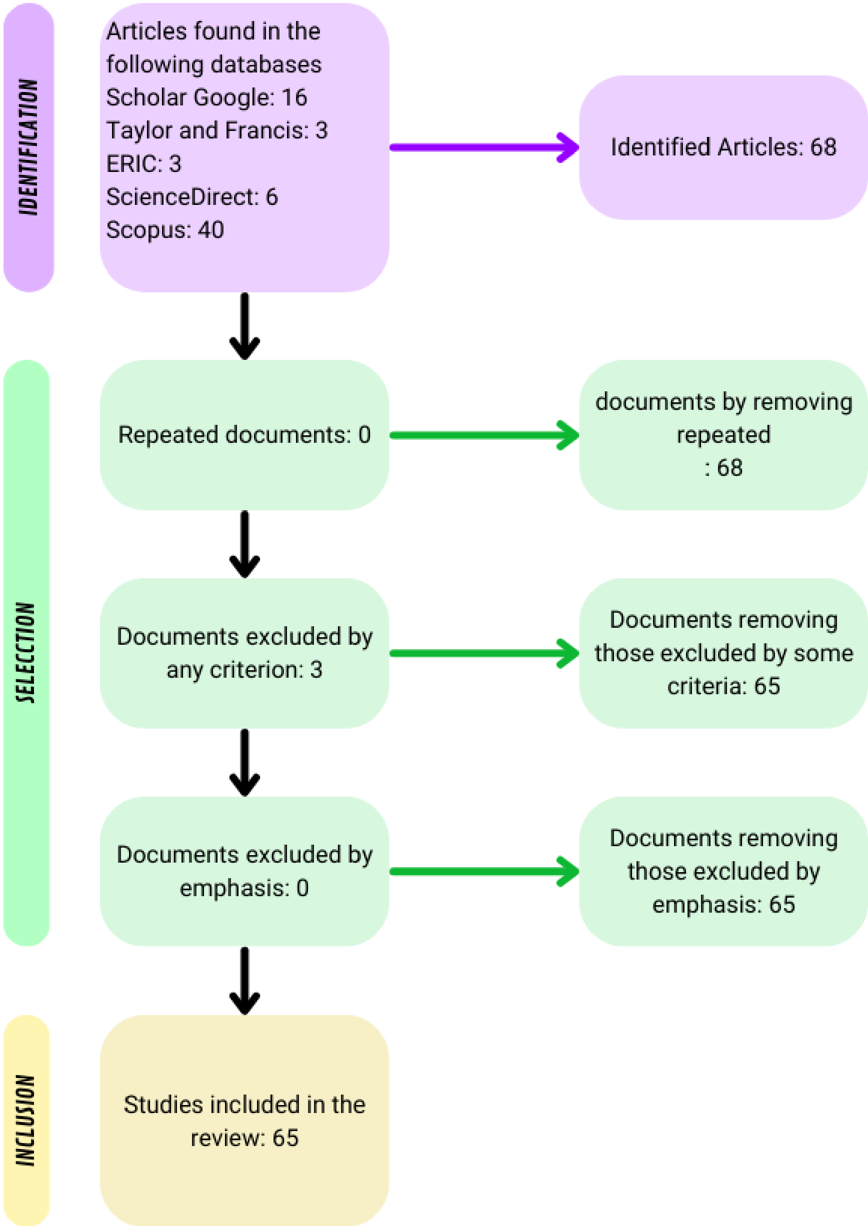

Figure 1 Synthesis of data extraction (Source: Author´s own elaboration).

Finally, the last phase compiles the various institutional meanings found in the literature that are easily categorized within the primary objects (concepts, language, situations, procedures, propositions and arguments) conceived by the EOS (Godino et al., 2007). From the concept, rational numbers are defined, according to Vamvakoussi &; Vosniadou (2010).

$$\{a \div b \ / \ a, b \in \mathbb{Z}, b \neq 0\}$$

That is, they are all the numbers that can be expressed as the division of two whole numbers and the divisor other than zero. This definition explains the relationship between two magnitudes, where b is the number of parts into which the whole or unit is divided, and a is the number of parts taken from the whole (Gómez &;

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Perez, 2016, (Baiduri, 2020; Hurst &; Cordes, 2018; Mazzocco &; Devlin, 2008; Norton &; Wilkins, 2010; Schiller, Abreu-Mendoza, Siegler, et al., 2024; Schiller, Abreu-Mendoza, Thompson, et al., 2024; Schiller et al., 2022). Rationals, on the other hand, function as operators, i.e., transformers or change functions of a given initial state (Gómez &; Perez, 2016; Ben-Chaim et al., 1998; Lortie-Forgues &; Siegler, 2017). For example, percentage situations or fractions of a quantity. Finally, from Euclid's elements, rationals are endowed with one more meaning, as a measure; due to, when measuring quantities of magnitudes that being commensurable do not correspond to a multiple in integer quantities of the unit (Gómez &; Perez, 2016).

From the language, rational numbers, by admitting both fractional and decimal representations, offer a diversity of linguistic perspectives for their conceptualization. The multiple representations, including symbols, graphs and definitions (Yopp, 2018), facilitate access to these concepts and allow establishing relationships between quantities, even in the absence of a precise numerical value (Moss &; Case, 1999; Hackenberg &; Sevinc, 2022; Moskal &; Magone, 2000; Nelson et al., 2018; Newton, 2012; Weller et al., 2009).
That is, the use of expressions such as: “a piece of ...”, “a part of ...”, “a half of ...”, which even without the use of exact quantities is understood to refer to part-whole relationships.
The equivalence between different representations (Xu et al., 2024) underlines the flexibility of the numerical, graphical representation system and its role in the construction of mathematical meanings from problem situations (Pantoja et al., 2016). The traditional association of rationals with fractions (Ni &; Zhou, 2005) is enriched by this perspective of multiple representations.

It is part of the teacher's job to propitiate situations and anticipate in advance the approaches and errors that can be found in the development of activities around the mathematical object (Nunokawa, 2005; Arieli-Attali &; Cayton-Hodges, 2014; Avcu, 2024; Flores et al., 2015; Heo & Choi, 2014; Ketterlin-Geller et al., 2015; Powell &; Nelson, 2021; Yi et al., 2024). As well as, to present a mastery in the diverse topics that have connection with the mathematical object (among others, direct and inverse proportionality, trigonometric ratios, probability and similarity of figures) (Rojas, et al., 2015).

Khashan (2014), states that procedures refer to the mathematical skills to solve a problem; these skills are the set of rules, laws, algorithms, etc. that the individual learns and uses to respond to problem situations around mathematical objects. In the case of rational numbers, the algorithms of addition, subtraction, multiplication and division; the algorithms of direct and inverse proportion (rule of three) (DeWolf et al., 2015; Ervin, 2017; Hackenberg et al., 2017; Jarrah et al., 2022; McMullen et al., 2017; Rakes &; Ronau, 2019; Rojo et al., 2024); calculation of percentages and probabilities, among others.

Rational numbers, similarly, to natural numbers and integers build their properties from two operations: addition (+) and multiplication (×); and their order relations: greater (>), lesser (<) and equal (=) (Yetim &;

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Alkan, 2013; Yilmaz &; Ay, 2018). Marjanovic and Kadelburg (2019) list the properties that rational numbers adopt in Figure 2.

$$\text{(i)}\ (\forall a)(\forall b)\, a+b=b+a \qquad \text{(v)}\ (\forall a)(\forall b)\, ab=ba$$
$$\text{(ii)}\ (\forall a)(\forall b)(\forall c)\, (a+b)+c=a+(b+c) \qquad \text{(vi)}\ (\forall a)(\forall b)(\forall c)\, (ab)c=a(bc)$$
$$\text{(iii)}\ (\exists 0)(\forall a)\, a+0=a \qquad \text{(vii)}\ (\exists 1)(0<1\ \mathit{and}\ (\forall a)\, a\cdot 1=a)$$
$$\text{(iv)}\ (\forall a)(\exists b)\, a+b=0 \qquad \text{(viii)}\ (\forall a\neq 0)(\exists b)\, ab=1$$
$$\text{(ix)}\ (\forall a)(\forall b)(\forall c)\, a(b+c)=ab+ac$$
$$\text{(x)}\ (\forall a)(\forall b)\, (a<b \iff (\exists c>0)\, a+c=b)$$
$$\text{(xi)}\ (\forall a)(\forall b)\, (a<b\ \mathit{or}\ a=b\ \mathit{or}\ b<a)$$
$$\text{(xii)}\ (\forall a)(\forall b)(\forall c)\ (a<b \iff a+c<b+c) \qquad \text{(xiii)}\ (\forall a)(\forall b)(\forall c>0)\ (a<b \iff ac<bc)$$

figure 2: properties of rational numbers. Taken from (Marjanovic &; Kadelburg, 2019)

Finally, arguments, according to Godino et al. (2007), are all those used to validate, explain and demonstrate the properties, propositions and procedures that compose the mathematical object (Smith, 1995; Whitenack &; Ellington, 2013; Wright, 2014; Yetim &; Alkan, 2013).

Figure 3 presents in synthesized form the institutional meanings of rational numbers found in the literature review. This in relation to the primary objects that shape it under the theoretical lens of the Ontosemiotic Approach.

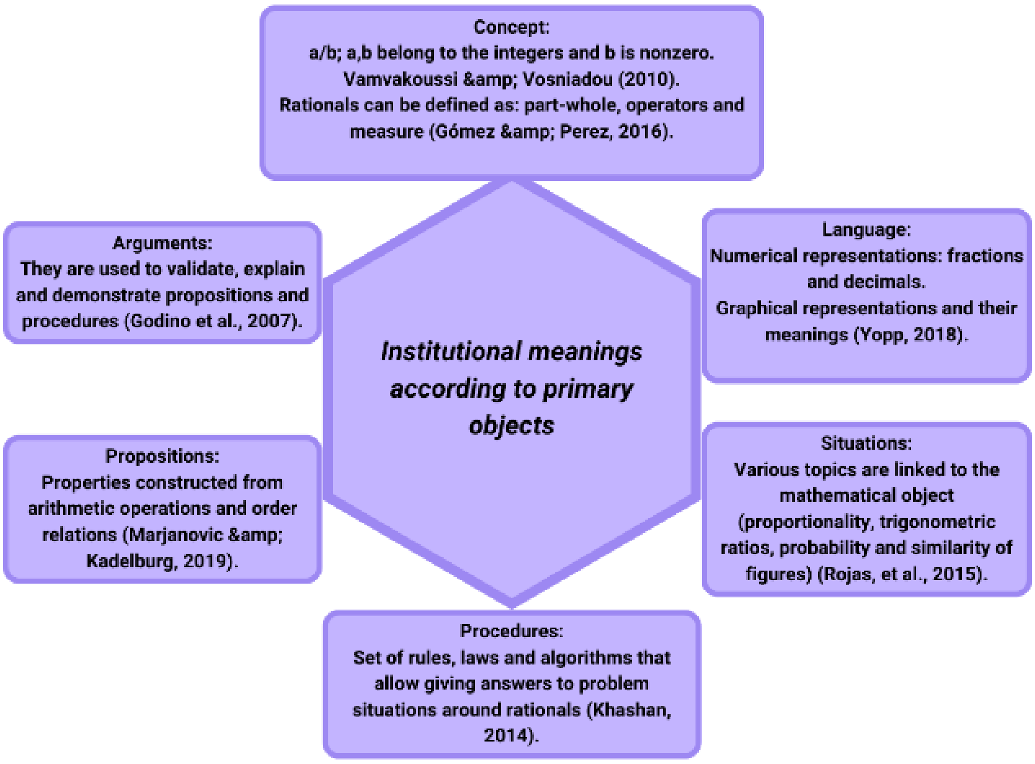

Figure 3: Institutional meanings found in the systematic review that are categorized within the six primary objects (Source: Author´s own elaboration).

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5. DISCUSSION

From the results obtained in the literature review of the analysis of the indexed articles, it is observed that the institutional meaning that predominates in rational numbers is the part - whole relationship. This, in turn, allows highlighting an existing absence in the meaning of rational numbers seen as a measure (Gómez &; Pérez, 2016). It is, therefore, that the exploration of rational numbers not only reveals their fundamental importance in the numerical, graphic system, but also, in the system of measurements and in the various forms that are manifested in everyday contexts.

Through the diverse institutional meanings found, it is essential for the resolution of problems in diverse areas, the connection between them. Allowing the understanding of their properties, their application and their relationship with other mathematical concepts.

Due to the importance of identifying these meanings found in the literature, a comparison is made with VOSviewer software in its version 1.6.20 to analyze the correlation of the key terms of these institutional meanings selected in the SCOPUS database. This with the purpose of showing the nodes that have a direct relationship with the main node (Rational Numbers). These nodes, which we will denote secondary, are related to the meaning of percentages, decimals and fractions. Secondary nodes represent fundamental ways of expressing and working with rational numbers. Their close relationship to the main node underscores their role as concrete representations and operational tools of the mathematical object. Understanding how rational numbers manifest themselves through percents to express relative proportions, decimals to facilitate calculations and comparisons, and fractions to represent parts of a whole or relationships between quantities is key to a deep understanding of the numerical set. This connection allows students and professionals to move fluidly between the different representations, apply the concept in a variety of contexts, and solve problems effectively in areas ranging from finance to science to everyday life. Exploring these secondary relationships in the literature, as facilitated by analysis with VOSviewer, helps to identify research patterns and possible areas where conceptual understanding and practical application of rational numbers across their various representations could be strengthened. (see Figure 4).

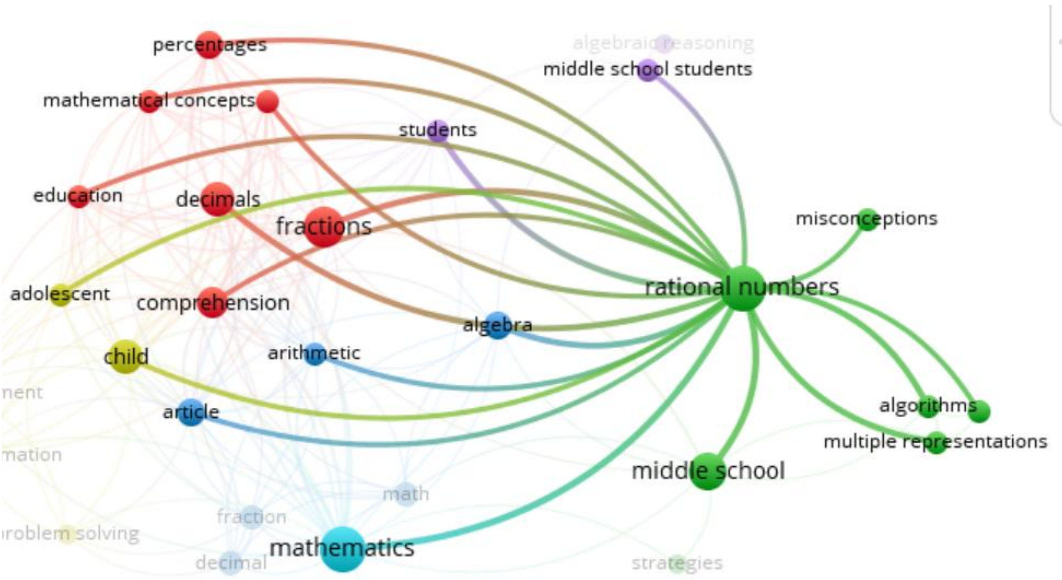

Figure 4: VOSviewer analysis of the institutional meanings found in the literature review (Source: Author´s own elaboration).

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6. CONCLUSIONS

In the field of teaching and learning rational numbers, it is essential to address the complexity of this mathematical concept in a comprehensive manner. The pedagogical challenge lies in achieving cognitive transversality, creating meaningful connections between the various meanings of rational numbers so that students can build a deep and multifaceted understanding. Identifying these meanings highlights the need for an appropriate didactic transposition that is adapted to the particularities of each student and allows them to move easily between these different forms of expression. Ultimately, teaching that balances the different meanings and strengthens the links between their representations will not only enrich the conceptual understanding of rational numbers but also empower students to apply them confidently and effectively in a wide range of everyday and academic situations.

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