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# Review Paper on Crack Modeling and Crack Detection Methods

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*Abstract*— **In this review paper various crack modeling and crack detection methods have been discussed which could be utilized for crack identification in beam. After studying various techniques it is found that roational massless spring for crack modeling and genetic algorithm for crack detection are most suitable. The differential equations for the free bending vibrations are established and then solved individually for each segment with the corresponding boundary conditions and the appropriated compatibility conditions at the cracked section. To solve the inverse problem, genetic algorithm is used to search optimal solution for detection of multiple cracks locations and their intensities in Timoshenko beam.**

***Keywords— Timoshenko Beam, Multiple Cracks, Characteristic equation, Mathematical model, Inverse Problem, Crack identification, Genetic Algorithm.***

## 1. INTRODUCTION

The presence and propagation of crack damage in mechanical and civil structures can cause catastrophic failure. So crack damage identification is of important concern to the structural engineering community. Traditional methods also called as conventional NDT methods like ultrasonic methods, magnetic field methods, x-ray methods and radiography need the vicinity of the damage, and the portion of the structure being inspected should be readily accessible. These methods basically have local nature over damage that is it requires particular area of damage to determine crack. Though these methods are accurate, reliable and effective they require full accessibility and thorough scanning of entire component. Also it becomes more time consuming, laborious and expensive for large slender beams, pipes, rails etc, which are widely encountered in power plants, chemical plants, off shore oil installations. This has motivated development of alternate methods.

Many researchers have presented various methodologies for model a cracks having different degree of accuracies. Several approaches are also adopted for detection of single or multiple cracks in thick or thin beams, rotors, pipes etc. here few crack modeling methods and crack detection strategies adopted by various researchers are reviewed and presented their summaries

## 2. CRACK MODELING METHOD

Large amount of work have been done in crack modelling because it plays a vital role in crack detection. The modelling of crack is complex due to many reasons:

Physical discontinuity at the crack location

Large stress concentration near the crack tip

The crack leads to significant localization of deformation which leads to increase in flexibility, decrease in stiffness and change in damping. Damping is quite sensitive to material compositions, manufacturing processes and environmental conditions; hence it is difficult to propose a model to reflect the change in damping. The crack may be considered either open or breathing. In case of open crack, crack remains open during the entire cycle of vibration. While in breathing crack, crack remains open only during part of the cycle of vibration. Most cracks, that either develops or propagates in a component in-service due to fatigue, creep, manufacturing defects, corrosion, etc., behave more like breathing (closing) crack. However, the breathing crack model includes non-linearity in the formulation

### *2.1 Reduced section method*

Bovsunovsky and Matveev [2000] have odelin crack by considering short sections with a reduced cross-sectional moment of inertia. The reduction of the cross-sectional moment of inertia causes the change in strain energy. This is made basis to determine crack parameters. The section depth is reduced over a span near the crack location. At the same time, the cross-section area of the beam was supposed to be constant.

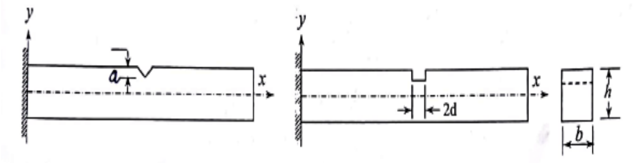


Figure 2.1: Three segment beam representation[Bovsunovsky and Matveev(2000)]

As shown in above fig the section depth is reduced near the crack. The length of segment is calculated considering strain energy. Addition of segment leads to conversion into three segment beam. The span size, d for this reduced section is given by

$$d = \frac{0.3675h(1-a/h)\left[(1-a/h)^6 - 3(1-a/h)^2 + 2\right]}{1-(1-a/h)^3} \quad (1)$$

Where a is crack depth, h is height of beam, b is width of beam.

The limitation of the above-mentioned approach is that the stress distribution is assumed independent of the crack size. It means shallow and deep crack does not make any variation in the stress field. This may lead to larger inaccuracies in the detection of crack. Murigendrappa [2004] improved the approach by odeling the crack into five segments. It improves the results up to a certain extend compared to three segments. To improve this the beam wih crack is modeled into five segments due to which better results are obtained compared to three segments.

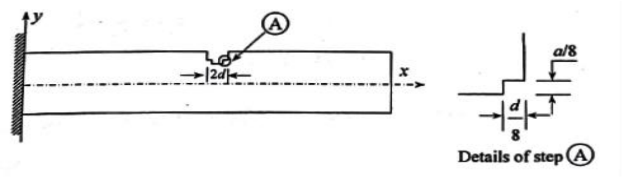


Figure 2.2: Five segment beam representation [Murigendrappa (2004)]

## *2.2 One dimensional method*

The presence of the crack affects stress and strain distributions in the vicinity of cracked section. There are large stress concentrations around the crack tip and also stress distribution, at the cracked section, is not linear. This stress field around the crack tip decreases with the distance, away from the crack region. The modification of the stress field induced by crack was incorporated using the local empirical function by Christides and Barr.

This local empirical function assumed exponential decay of stress field with the distance from the crack and also included one dimensionless parameter which could be evaluated experimentally. They have developed a cracked Euler-Bernoulli beam theory by deriving the differential equation for uniform beam containing one or more pairs of symmetric cracks. By choosing stress, strain, displacement and momentum fields, this derivation has been reduced to one spatial dimension using integrations over the cross-section. They have solved the governing equation using two-term Rayleigh-Ritz solution to evaluate the fundamental natural frequency of cracked beam. They have treated the crack as a notch.

It has been found that the two-term solution proposed by Christides and Barr‘s is not fully convergence, and the convergence of the Galerkin‘s procedure is also very slow. To improve fast convergence, they have modified Galerkin‘s approach which combines Fourier series expansion with an additional function satisfying the continuity characteristics of exact solutions. This leads to increased computational work.

## *2.3 One dimensional method*

Various finite element models were employed to model cracks in a beam. Papadopoulos and Dimarogonas [1987a, b] have used flexibility matrix method using two parts. First part only considers original flexibility matrix of crack-free beam. The second part incorporates an additional matrix due to the crack. This additional matrix accounts for additional deformation and energy release due to crack in the structure. Gounaris et al; Gounaris and Dimarogonas have generated cracked beam element and used this element in conjunction with stiffness formulation.

Special elements have also been developed to model crack and incorporate additional flexibility. Ostachowicz and Krawczuk have introduced point finite elements (PFE) to incorporate additional local flexibility due to crack. Krawczuk and Ostachowicz [1993] proposed an algorithm of a linear and a geometrical stiffness matrix calculation for an element modelled instead of crack in a cantilever beam. They have also studied transverse vibrations of a beam with crack subjected to a constant axial force.

For small cracks the crack flexibilities are very small and the elements of the stiffness matrix [K] are correspondingly larger. This might lead to numerical problems during solution. To overcome this, this stiffness matrix for a cracked beam element should be developed.

## *2.4 Rotational spring model method*

In case of transverse vibration of beams, it is generally assumed that there is an extra angular rotation at the crack location proportional to the bending moment at the section. Hence it can be modeled as a massless rotational spring of infinitesimal length inserted at the location. This method separates the beam into two segments having different deflection pattern. The infinite stiffness value of the spring represents no crack whereas zero stiffness shows complete separation of the member. The magnitude between zero to infinite define presence of crack with certain severity, inverse function of spring stiffness. Hence decreasing the stiffness value indicates increasing the crack severity. To deal with the effects of the crack on the vibration characteristics of the Timoshenko beam, based on fracture mechanics, the crack of a beam is modeled as a simplified rotational spring model, and it is assumed that the crack is open and has a uniform depth in width.

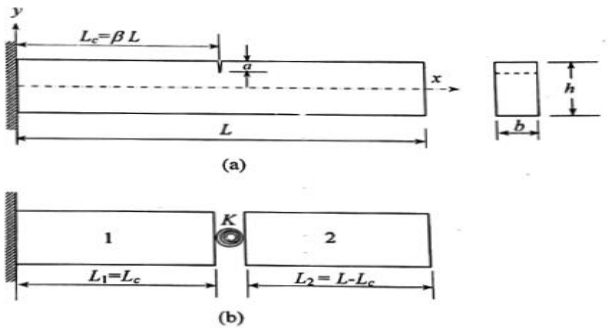


Figure 2.3: (a) Cantilever beam with crack
(b) Representation of crack by Rotational spring
[Ostachowicz and Krawczuk(1993)]

The crack of the beam is assumed to be open, and with a uniform depth and width. The length, height and width of the beam are L, h and b, respectively. And there is a crack at a distance Lc from the left end of the beam, as shown in Fig.1.In the analysis, a steel beam is considered, thus, the influence of material damping on the resonant frequencies is ignored, and only bending vibrations are considered.

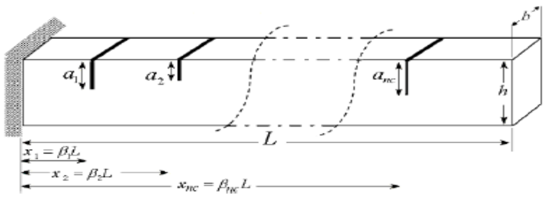

Figure 2.4: Cantilever beam having multiple cracks [Dimarogonas and Paipetis (1983)]

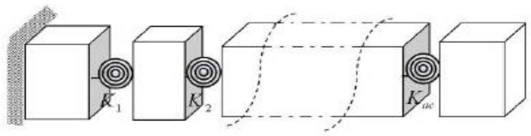

Figure 2.5: Modelling multiple cracked cantilever beam with rotational spring [Dimarogonas and Paipetis (1983)]

From the crack strain energy function, rotational/torsional spring constant near the vicinity of the transverse cracked section of the beam have been correlated [Dimarogonas, 1976; Dimarogonas and Paipetis, 1983]. The correlations are given as follows

$$Kr = \frac{EI}{5.346hf\left(\frac{a}{h}\right)} \quad (2)$$

$f\left(\frac{a}{h}\right)$ is local compliance function, which is computed from strain energy density function. It is defined as

$$f\left(\frac{a}{h}\right) = 1.8624\left(\frac{a}{h}\right)^2 - 3.95\left(\frac{a}{h}\right)^3 + 16.375\left(\frac{a}{h}\right)^4 - 37.226\left(\frac{a}{h}\right)^5 + 76.81\left(\frac{a}{h}\right)^6 - 126.9\left(\frac{a}{h}\right)^7 + 172\left(\frac{a}{h}\right)^8 - 143.97\left(\frac{a}{h}\right)^9 + 66.56\left(\frac{a}{h}\right)^{10}$$

The Rotational spring stiffness corresponding to a given crack can be determined experimentally through deflection method or inverse vibration analysis. It can also be determined by finite element method.

## 3. CRACK DETECTION METHOD

Development of crack in a beam or any component leads to change its vibration parameters such as stiffness, mass, natural frequency, mode shapes etc. These methods make use of one of these parameters

### *3.1 Natural frequency method*

The natural frequency of component decreases because of crack. In these method crack can be detected by measuring the natural frequencies. These include forward problem of determination of natural frequencies knowing the crack details and inverse problem of determination of crack details from natural frequencies. For a beam having uniform cross section they have obtained a relationship between the crack location, stiffness and natural frequency. For the first three modes of vibration they have plotted the variation of right hand side of equation with crack position. The common intersection of these curves is taken as crack location.

$$\frac{EA}{K} = \frac{\cot\lambda\beta + \cot\lambda(1-\beta)}{\lambda} \quad (3)$$

Where E is Young's Modulus, A is cross sectional area, K is spring stiffness, $\lambda$ is frequency parameter and $\beta$ is crack position.

The natural frequency is not suitable for locating the defects of structures because of their inherent global property, especially symmetrical structures, and the mode shape data are often difficult to measure with sufficient accuracy for damage detection in practice. These disadvantages of crack detection methods based on modal parameters seriously restrict their development in the field of structural engineering.

In the case of small measured frequency errors, the predicted crack parameters are in good agreement with the measured crack parameters. However, in the case of large measured frequency errors, the predicted crack parameters only give roughly estimated results. However this method cannot predict crack size.

### *3.2 Impedence Analysis Method*

This method shows that the location of a single moderate crack of beam can be identified by monitoring the change in the first anti-resonance as a function of the measuring location along the beam length. The mechanical impedances of the cracked Timoshenko beam are calculated in this part. The driving-point impedance at an arbitrary position x along the beam length can be derived according to the expression

$$Z(x) = \frac{Fo}{j2\pi f w(x)} \quad (4)$$

Where j is the unit of an imaginary number, w(x) is the corresponding displacement and f is the frequency.

In this method cracked beam is excited by a harmonic force with wide scanning frequencies, and the driving force is moved from the left end to the right end of the beam. At the same time, the mechanical impedance at each driving point location is measured, which can be computed numerically by using impedance equation Subsequently, the first anti-resonant frequencies are extracted from the graphs, and a figure correlating the first anti-resonant frequency with the driving point location along the beam is plotted

It is quite difficult to predict and read the impedance versus anti resonance frequency graph for crack detection. Also the numerical results show that when a crack occurs in

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the beam, the curve of the first anti-resonant frequency versus driving-point location will not be smooth but show discontinuity at the crack position, and the degree of the discontinuity depends on the depth of the crack

### *3.3 Mode Shape Method*

This method is useful for detection of both location and size and is demonstrated for cantilever beam with normal edge crack. Curvature at a point v is given in Structural and Health Monitoring 5 (2014) which is presented as

$$v = \frac{M}{EI} \quad (5)$$

Where M is bending moment, E is modulus of elasticity and I is second moment of inertia of cross section. Since the presence of crack reduces EI locally, the curvature will increase. This local increase in curvature can be used to detect crack.

Curvature mode shape can be useful for detection of crack location, it suffers from drawback that large volume of data is to be obtained experimentally. Mode shape method could have problems due to inaccuracy of the model, environmental and other non stationary effects on measurements, and lack of data in suitable frequency ranges. In fact, it is difficult to construct models of most existing structures with high accuracy. MSs are not sensitive to damage of small extent, curvatures of MS, or curvature mode shapes (CMSs), are used to locate damage. These disadvantages of crack detection method based on modal parameters seriously restrict their development in the field of structural engineering

### *3.4 Wavelet Based Method*

The wavelet transform is useful mathematical tool to detect changes in the mode shapes of a structure and therefore to detect damage. This technique is based on the wavelet analysis of the difference of mode shapes corresponding to a reference state and a potentially damaged state. The wavelet coefficients of each mode shape difference are added up to obtain an overall graphical result along the structure.

The limitation of this method is that the mathematical analysis is not always clear a priori and in most applications these parameters are chosen depending on previous results or on trial and error.

### *3.5 Artificial Neural Network*

In this method, study intends to train a neural network using antiresonant frequencies and determine its feasibility to assess experimental damage. Hence, it was decided to work with the simplest neural network that has been able to detect, locate and quantify structural damage. This is a multilayer perceptron (MLP) with three layers (input, hidden and output).

Disadvantage of ANN is the need of large training sets. It is extremely difficult and time-consuming to produce large enough training data sets from experiments. An alternative to generate training samples is to use a numerical model of the structure. Castelli and Revel showed that it is possible to produce correct damage predictions in an experimental structure using a neural network that was trained with samples generated by a finite element model. Nevertheless, this approach is highly dependent on the accuracy of the numerical model.

### *3.6 Genetic Algorithm*

Genetic algorithms are based on the theory of natural selection and work on generating a set of random solutions and making them compete in an arena where only the fittest survive. Each solution in the set is equivalent to a chromosome. A set of such solutions forms a population. The algorithm then uses three basic genetic operators: (i) Reproduction (ii) cross over (iii) Mutation together with a fitness function to evolve a new population or the next generation.

Starting from a random set of solutions the algorithm uses these operators and the fitness function to guide its search for the optimal solution. It is thus based on a guided random search mechanism. The fitness function gauges how good the solution in question is and provides a measure to its adaptability or survivability. The genetic operators copy the mechanisms based on the principles of human evolution. The best and quickest way of explaining the working of algorithm is to describe how they can be used to solve problems

The solution of inverse problems, such as crack identification in beams, may be basically considered as an optimization problem. The GA is an optimization method that simulates the natural evolution phenomena based on Darwin’s theory. The GA operates on an initial population of randomly generated candidate solutions, encoded as chromosomes. Applying the principle of survival of the fittest to hopefully produce better and better approximations, the GA may gradually find the best individual, achieved through the evolution, as the solution of the inverse problem.

## 4. DETECTION OF MULTIPLE CRACKS

For a cantilever beam with two cracks, size of characteristic determinant becomes 12 x 12. For every single additional crack the size of determinant increases by 4. The crack is represented by massless rotational spring. This method leads to a characteristic equation of size (n+2) in case of n cracks.

Khiem and Lien have developed dynamic stiffness matrix(DSM) method based on equivalent rotational spring and transfer matrix for forced vibration of multiple cracked beam. They have successfully employed DSM model to detect numerous cracks in beams by using data on natural frequencies. Patil and Maiti have proposed a method for detecting multiple cracks in a slender Euler Bernoulli beam based on frequency measurement. In this method, the transverse vibration has been modeled through Tansfer Matrix Method (TMM) and the crack is represented by a rotational spring. “Liang et al” has virtually divided beam into number of segments and each segment is considered to be associated with a damage parameter. This procedure gives a linear relationship explicitly between the changes in natural frequencies of the beam and damage parameters.

## 5. TRANSFER MATRIX METHOD (TMM)

A transfer matrix relates state vectors at two points on a component/structure. The transfer matrix method (TMM) is applicable to beams with a large number of geometric segments, intermediate supports etc. This method offers definite advantage over the approaches based on modal analysis. Ostachowitcz and Krawczuk have shown that for n cracks, size of characteristic determinant becomes 4n+4. So it becomes mathematically expensive for greater number of cracks, while TMM gives only 2 x 2 determinant for natural frequency calculation for any number of cracks. Thus, size of characteristic determinant reduces considerably. For intermediate rigid supports the size can be reduced to (n+2), where n is the number of supports, irrespective of the number of beam segments.

To illustrate TMM, consider a segment j of a beam of which state vectors include displacement w, slope $\theta$ bending moment M and shear force V, in case of bending.


Figure 2.6: Segment of a beam

$$\begin{bmatrix} w \\ \theta \\ M \\ V \end{bmatrix}_i = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \begin{bmatrix} w \\ \theta \\ M \\ V \end{bmatrix}_{i-1}$$

$$\text{i.e. } S_i = T_{ij} S_{i-1}$$

$$T_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/K & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence Tij is a transfer matrix and it relates the state vectors of the two ends of the segment. At crack location, there is continuity in displacement, bending moment, shear force and only a jump in slope $\theta$.

By using above two equations, transfer matrix for whole structure containing cracks can be easily written. After inserting the boundary conditions, size of this matrix reduces to 2 x 2, whose determinant gives natural frequency of the structure.

## 6. SUMMARY

The review of different crack detection methodologies given by various researchers clearly shows that several approaches have been developed to locate crack in slender beams. Different geometries, crack locations etc have also been addressed.

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

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