

Review on Comparative Analysis of Load Flow Calculation Methods

Mohammad Nail Alzyod

MCS student at Electrical Engineering Department at King Fahd University of Petroleum & Minerals (KFUPM).
Dahran, Saudi Arabia

I.O. Habiballah

Associate Professor of Electrical Engineering Department at King Fahd University of Petroleum & Minerals (KFUPM). Dahran, Saudi Arabia

Abstract:- The active and reactive power flow between the generating plant and the load is analyzed in a three phase power system throughout several networks, buses, and branches. Power flow studies provide a systematically organized approach for the computation under steady state condition of different bus voltages, their phase angle, as well as active and reactive power flows among different generators, loads and branches. An analysis of power flow consists of determining the magnitude and angle at each busbar until power mismatches are insignificant at an equilibrium in active and reactive power. There are several commonly used methods for solving transmission power flow problems, including Gauss-Seidel (G-S), Newton power flow (N-R), and Fast Decoupled Load Flow (FDLF).

List of symbols

P	Real power
Q	Reactive Power
V	Voltage Magnitude
Δ	Voltage Angle
Y_{bus}	bus admittance matrix
G-S	Gauss-Seidel
N-R	Newton power flow
FDLF	Fast Decoupled Load Flow
PV	Generator bus
PQ	Load bus

I. INTRODUCTION

Among the most essential tools for grid operators is a power flow calculation that determines the steady state behaviour of the network. If a power flow calculation is determined to be correct, we can figure out if the power system can handle the given generation and consumption. As a consequence, power flow computations are carried out to control, plan and operate the power system[1]. However, Load flow studies are indispensable in studying transient stability [2]. Hence, they are of great importance when studying power systems. Therefore, they also comprise the determination of voltages as well as both real and reactive power flows for a specific bus of the system depending on its conditions.

By performing a flow calculation, you can determine the effects of changes and modifications in the network layout in terms of the operating units and the load on system power quality and safe economic operation. Considering that the grid scheduling simulation is based on the system power flow calculation model, this directly impacts the quality of the simulator, as the grid scheduling simulation is conducted under a variety of simulated operating modes, this requires the flow calculation model to have good convergence, and the results should be comparable to the actual grid operation, and the convergence rate should be fast enough to support real-time requirements[3].

The power flow or load flow problem is the problem of computing the voltage magnitude $|V_i|$ and angle δ_i in each bus of a power system where the power generation and consumption are specified. The transmission networks have evolved various power flow solutions techniques. There are several commonly used methods for solving transmission power flow problems, including Gauss-Seidel (G-S), Newton power flow (N-R), and Fast Decoupled Load Flow (FDLF). In this paper, a detailed comparison will be made between these methods.

II. LOAD FLOW STUDIES

An analysis of power flow is a fundamental part of contingency assessment and real-time monitoring systems. The active and reactive power flow between the generating plant and the load is analysed in a three phase power system throughout several networks, buses, and branches. Power flow studies provide a systematically organized approach for the computation under steady state condition of different bus voltages, their phase angle, as well as active and reactive power flows among different generators,

loads and branches[4]. During the planning and operation of power distribution systems, power flow analysis is widely used by professionals. Under load flow analysis, steady state operation of a power system is analysed [2] .

Voltages at different buses, line flows in the network, and system losses are the outputs of the power flow model. This is accomplished by solving nodal power balance equations. Due to the nonlinearity of these equations, iterative techniques are typically applied to solve this problem [2]. An analysis of power flow consists of determining the magnitude and angle at each busbar until power mismatches are insignificant at an equilibrium in active and reactive power. Especially for heavily loaded and complex systems, it may not always be easy to calculate the final values of the busbars voltage. Identifying the system's knowns and unknowns is the first step in resolving power flow problems. According to these variables, buses can be classified into three main types: slack, generation, and load buses as shown in table 1:

Table 1 Slack, generation, and load buses

Types of Bus	Known Variables	Unknown Variables
Slack	$ V , \delta$	P, Q
PV	P, $ V $	Q, δ
PQ	P, Q	$ V , \delta$

As a condition of the load flow analysis being acceptable, it would have to satisfy the conditions of low space usage, fast performance, high reliability, as well as simplicity and versatility to some extent[5]. The studies of load flow, however, have the many objectives [6]. This analysis helps determine the preferable location for a forthcoming generating station, substation, and new lines, as well as their optimal capacity. However, by utilizing the load flow solution, any nodal voltages or phase angles are determined, and thus the injection of power at all the buses, and power flows through interconnecting power channels. In addition to that, it determines the voltage of the buses. A voltage level must be maintained within a closed tolerance on certain buses.

Many load flow analysis methods did not take into account the limits of the reactive power for PV buses nodes. However, in order for the incorporation of the reactive power limits of the PV buses nodes to be automated, the rough complementary constraints of the voltage magnitude as well as the reactive power for PV buses nodes are considered [6], [7]. In some cases, the rough constraint equations at some points are not differentiable, so convergence is somewhat low. In order to address this problem, a scalar was incorporated into the integral constraints in order to smooth the equations, thereby improving convergence [9], [10].

Studies of power systems must essentially incorporate load flow into their analysis. However, the study of load flow enables the analysis of the consequences of an instantaneous loss of generating stations or transmission lines on power flow. It enables analysis of line losses in different conditions of power flow. Additionally, studies of load flows are also useful in determining both the optimal size and the most suitable location to place power capacitors, for the sake of enhancing power factor as well as raising network voltage [11]. Consequently, it contributes to the selection of best locations, along with the optimal capacity of the proposed power plants, new lines and substations.

III. CONVENTIONAL METHODS OF POWER STUDY

Newton-Raphson Power Flow Method

The Newton Raphson method can be used to solve non-linear equations numerically. The method is often referred to as an iterative root finding scheme. This method is called root finding because it is geared toward solving equations such as $f(x)=0$. In such an equation, the solution will be called x^* (or x^*), and it will be a root of the function $f(x)$ (or $f(x)$). Power flow calculations are typically performed by the first order Newton-Raphson (NR) method.

Since successive approximations are required to reach a solution, it is iterative[12]. Below is a general overview of the process. To start, guess a solution. The guess will, of course, be wrong, unless we are very fortunate. In other words, we decide to update the "old" solution with a "new" solution with the intention that the "new" solution is closer to the correct solution than the "old" solution.

The update is obtained in this procedure as a key aspect of this type of method. In order to guarantee that the "new" solution is always closer to the correct solution than "old", it must be ensured that the update is improving the solution. Then such a procedure can be guaranteed to work if only computing takes enough updates, i.e., if only iteration takes enough times. The following is a brief overview of some of the positives and negatives of this approach to problem solving

Using power and current-mismatch functions in polar, Cartesian, and complex forms, the Newton-Raphson method is incorporated into a methodology for solving power flow problems. A power flow problem can be solved using the Newton-Raphson method in six different ways based on these two mismatch functions and three coordinates[1].

In this iterative procedure, the choice of the estimated starting points for the unknowns heavily affects the convergence to the desired solution. Once the result of the iterations is close enough to the solution, under moderate regularity conditions and under the assumption of the Jacobian being non-singular, the algorithm moves toward the solution superlinearly. However, Newton Raphson method requires no more than few iterations regardless of number of buses considered, therefore power flow equations can be solved quickly[13].

However, ill-conditioned behaviour may result from systems with high R/X ratios, unreliable interconnections, or overloaded buses. Those mentioned factors contributed to the instability of the Newton Raphson method. Unfortunately, the N R method, tended to be slow to converge when applied under ill-conditions. Particularly, when a system loading reaches critical load, the sparsity of the Jacobian matrix decreases, which results in the Jacobian matrix becoming singular [14]. Occasionally, power mismatches could be observed between two successive iterations, particularly when the load flow model is ill-conditioned. It is most common for these fluctuations to appear at the beginning when the power mismatches are high [15].

Most often, it is difficult or not practical to obtain a close enough initial guess to ensure that you obtain an asymptotic convergence result within a few iterations[16]. In fact, NR's algorithm may not converge at all to the intended solution if the initial guess is too far off[3]. There is well-known asymptotic convergence of NR's algorithm when the initial guess is sufficiently close to the solution.

Additionally, several convergence enhancing approaches such as the Levenberg-Marquardt method (LMM) and the Acceleration Factor Method (AFM) have been included in the proposed methods to speed up convergence [17]. However, for power flow analysis, analysis, Newton-Raphson power flow algorithms utilize these methods to enhance the convergence speed by changing the method of obtaining the system variables' mismatch vector.

It is generally accepted that power flow approaches which employ the NR method provide sufficient accuracy [13]. Unfortunately, this approach is rather slow and computationally burdensome. The reason for this is the complexity associated with inverse Jacobian matrix calculations throughout each iteration.

There are several positive aspects of the Newton Raphson method: Convergence occurs quickly if the initial guess is close to the correct solution [19]. Also, it can be converted to multiple dimensions, and it can be used to polish roots found by other methods. Additionally, it has a large convergence region. However, despite the longer time required per iteration for Newton Raphson method, the overall time for iterative process is less compared to Gauss Seidel method because there are fewer iterations for convergence. Otherwise, there are several negative aspects of the Newton Raphson method: It takes much longer for each iteration. In addition to that, it is also more difficult to code, specifically when using sparse matrix algorithms.

Brief steps of applying N-R method [20]:

- Construct Ybus in Per Unit
- Establish an initial guess for unknown magnitude and angle of voltages for a flat start
- For iteration k, find the mismatch vector
- For iteration k, obtain the Jacobian matrix J
- Identify the error vector
- Using an iteration number of (k + 1), verify that the power mismatches are acceptable, if so, you can continue, otherwise, go back to the second step.
- Calculate active and reactive power of the Slack Bus
- Calculate Line Flows

Gauss-Seidel Power Flow Method

It appears that the computations in the Gauss Seidel method are serial. In addition, every component of each iteration is dependent on all the previous ones. Hence, updates cannot occur simultaneously. Additionally, new iterations depend on the order in which equations are evaluated. By changing this ordering, the components (rather than their ordering) of each new iteration also change. In view of these limitations, engineers and researchers turn to Newton Raphson method. Further, the performance of this method is highly dependent on the initial estimation of the system variables solution [21], which means that the rate of convergence would rely on the nearness of the solution values to the actual values. However, as the number of iterations increases with the size of the network, the longer it takes for the Gauss-Seidel method to reach a solution [21].

There are several positive aspects of the Gauss-Seidel method: It's easy to program it. Also, each iteration has a relatively short computation time (the order of computation depends on the number of branches and buses in the system). In addition, it uses less memory space than the NR method. Otherwise, there are several negative aspects of the Gauss-Seidel method: It has a tendency to converge relatively slowly, but that can be improved by acceleration. Also, it is prone to missing solutions, particularly when dealing with large systems. Moreover, it generally diverges when there's a negative branch reactance (often seen with compensated lines). Furthermore, complex numbers are needed in programming.

Brief steps of applying gauss seidel method[11], [20]:

- Formulate and Assemble Ybus in Per Unit.
- Specify Initial Guesses to Unknown Voltage Magnitudes and Angles, it is referred as the flat start solution
- For load buses, calculate the new iterative bus voltage
- For voltage-controlled buses, calculate the new iterative bus reactive power, then calculate the new iterative bus voltage.
- Correct the bus voltage magnitude to the specified magnitude
- For Faster Convergence, apply acceleration factor to load buses
- Check Convergence
- Find slack bus power
- Find All Line Flows
- Compute the complex power loss in the line by summing the losses over all the lines.

Fast Decoupled Power Flow Method

For high-voltage transmission systems, there is a relatively small voltage angle between adjacent buses. Furthermore, the ratio of X to R is high. There is a strong correlation between real power and voltage angle, as well as between reactive power and voltage magnitude because of these two properties. On the other hand, real power is weakly related to power magnitude and voltage angle, and reactive power is weakly related to voltage angle[12]. Real power flows from the bus with a higher voltage angle to the bus with a lower voltage angle when considering adjacent buses. Similar to that, reactive power flows from a higher voltage magnitude bus to a lower voltage magnitude bus

Fast Decoupled is superior to Newton-Raphson in terms of computation time. In order to accomplish this, the Jacobian matrix is not taken into account, but the powerful couplings between active powers and phase angles, as well as reactive powers and voltages, are utilized[12], [22]. This attribute enables the skipping of some of the calculations in the iterative processes, which effectively reduces the complexity of the computation. However, when FDLF is used, the matrix does not contain elements relevant to Slack bus and Jacobian. In other words, the matrix does not contain elements relevant to Slack bus as well as PV buses. However, when FDLF is used, the matrix does not contain elements relevant to Slack bus and Jacobian. In other words, the matrix does not contain elements relevant to Slack bus as well as PV buses[4]. Unfortunately, the FD method, tended to be slow to converge when applied under ill-conditions. However, when a system loading reaches critical load.[14]

This technique consists of two steps: Separating (decoupling) the calculation of real and reactive power, in addition to the direct extraction of Jacobian matrix elements from the Y-bus. For large bus systems, the matrix size is very large, and so for faster performance and lower memory consumption, it is preferable to decoupled load flow where P is taken independent of V and Q is taken independent of δ , so those Jacobian elements are considered zeros.

There are several positive aspects of the Fast Decoupled method: The convergence occurs faster than using other methods. Also, there is a relatively small memory requirement as compared to other methods [4], such as the Newton Raphson method, the Gauss-Seidel method, etc. However, calculating the power flow is easier with this method since it's less complicated than other methods. In addition to that, the number of iterations is lower, and the equation size is smaller. Otherwise, there are several negative aspects of the Fast Decoupled method: More iterations are needed, even if the time for each iteration is less than with the NR method. However, three factors essentially determine how accurate a fast-decoupled load flow can be: the size and structure of the system Tolerances for convergence, the loading level of the system, and particularly in large and heavily loaded systems, even relatively small errors in state variables can result in larger errors in real and reactive power, however compared to line ratings, these errors are small. Moreover, the accuracy of the solution which is a controllable parameter can be improved by using a smaller convergence tolerance, which is a controllable parameter.

FDLF and Newton-Raphson methods have proven that they are effective and can be applied to larger and more complicated network structures, but FDLF enhances computational performance and can result in a cost reduction [21]. Accordingly, the method to be selected is dependent upon the overall costs involved in solving load flow issues as well as response time and accuracy.

In the Fast Decoupled Power Flow method, lines are assumed to be extremely inductive, i.e., the R/X ratio is quite low. Since these assumptions are taken into account, the Fast Decoupled Power Flow method creates constant matrices that are used during all iterations and, as a result, they only required to be factorized once. In cases where this assumption is flawed (e.g., in distribution networks and certain subtransmission networks), the convergence for this method may be slow or may not take place at all [23]. To handle such a problem, it has been proposed to use an axis rotation method by rescaling both the bus admittances and the complex power injections using a unit-magnitude complex scalar factor. Due to the fact that this complex factor modifies the ratio of lines R/X , Fast Decoupled Power Flow method can be improved with an optimal selection of this factor [22]. However, it is proposed to evaluate an axis rotation method that involves various complex factors for each bus[23].

With the fast decoupled power flow algorithm, it is possible that the Newton-Raphson algorithm can be simplified by exploiting the strong coupling between real power and bus voltage phase angles, and reactive power and bus voltage magnitudes that is found in power systems[12]. By approximating the partial derivatives of the real power equations as zeros with respect to the bus voltage magnitudes, the Jacobian matrix can be simplified. In a similar way, the partial derivatives of the reactive power equations are approximated as zeros with respect to the phase angles of the bus voltage. Furthermore, the remaining partial derivatives are often approximated based only on the imaginary portion of the bus admittance matrix [18].

IV. COMPARISONS

In this study, Gauss-Seidel, Newton-Raphson and Fast-Decoupled load flow methods will be compared in terms of several parameters for enhancing accuracy, stability, and reliability of the system as well as establishing a framework for further research. However, it might be possible to develop innovative techniques that could make the system more futuristic.

Comparative parameters are as follows[6]:

- A method's ability to solve a problem is judged by how long it takes to solve it, one of the most important parameters of comparison. A comparison of methods based on how long each method takes will help to identify the most appropriate approach to solving a problem with less time.
- There are some methods that can solve a problem quickly, however, they are less effective than others, or the results are not as accurate, so power system engineers do not favor them. It is about accuracy
- Complexity is taken into consideration since it increases computation time and decreases accuracy of results. The comparison will illustrate an idea for approaching a normal power system problem with a less complicated computation. Computer systems become increasingly complex as they are upgraded.
- As technology capabilities have only recently been developed to enable more widespread and cheaper implementation, convergence is considered a new trend. Convergence provides the ability to perform multiple tasks on a single device, and conserving information storage and power use. Algorithms are compared by how fast they converge. Basically, convergence rate refers to the speed at which a convergent sequence approaches its limit.

This is only a summary about the most important parameters that are taken into account when comparing load flow analysis methods, there will be many other additional parameters in the upcoming comparisons within this paper.

Comparing Gauss-Seidel method with Newton-Raphson method[6], [11], [25]:

- While the Gauss-Seidel method is widely known and established, Newton Raphson is the most recent and sophisticated method of studying power flows.
- For N-R, polar coordinates are preferred, and for Gauss Seidel, rectangular coordinates.
- In Gauss Seidel method, one iteration of computation takes less time than in N-R method, but the number of iterations required is more than that in N-R method.
- While G-S method has a slow rate of convergence and linear convergence characteristics, N-R method has a quadrature convergence characteristic.
- In comparison to N-R method, G-S method requires more computer time and costs more.
- In small power systems, G-S methods are used for solving problems, while N-R methods are most useful for large power systems.
- Newton-Raphson method has a quadratic convergence, thus it is mathematically superior to Gauss- Seidel method.
- Compared to Newton-Raphson's quadratic convergence, the Fast decouple method provides geometric convergence, resulting in a high number of iterations.

Comparing Gauss-Seidel method, Newton-Raphson method and Fast-Decoupled Method [26]:

- Gauss-Seidel method is simply and easily executed, but as the number of buses increases, it becomes more time-consuming (more iterations) in comparison with all other methods.
- Newton Raphson is more accurate and provides better results in fewer iterations in comparison with all other methods.
- Fast Decoupled method is the fastest in comparison with all other methods, but means less accuracy because assumptions are made to calculate faster.
- Gauss-Seidel requires a greater number of iterations for the same values of voltage magnitude, angle, active and reactive power.
- Newton-Raphson offers significantly better results than GS with a smaller number of iterations.
- In just the same number of iterations, Fast Decoupled method gives similar results almost identical to NR method.

Comparing Gauss-Seidel method with Newton Raphson Method shown in Table 2.

Table 2 Comparing Gauss-Seidel method with Newton Raphson Method [2], [7], [12], [14], [15], [21], [22], [25], [27]

	Gauss Seidel	Newton Raphson
Co-ordinates	Has good performance with rectangular coordinates.	It is preferable to use polar coordinates, since rectangular coordinates occupy more memory.
Arithmetical operations	A minimum number for completing one iteration.	During each iteration, jacobian elements will be calculated.
Time	Time for each iteration is reduced, but increases as the number of buses increases.	Iterations take 7 times as long as GS and increase with the number of buses.
Speed for iterations	Fastest	About one sixth of what GS is
Convergence	Linearly	Quadratic
No. of iterations	A large number, increasing as buses increase.	It is very little (about 3 to 5) for large system and is almost constant.
Slack bus selection	Convergence is adversely affected by the choice of slack bus	Minimal sensitivity to this
Accuracy	Generally less accurate	More accurate
Memory	Due to the sparsity of the matrix, there is less memory usage.	Despite compact storage scheme, there is large memory usage.
Usage/application	Small size system	A large system, ill-conditioned problems, optimal load flow studies.
Programming Logic	Easy	Very difficult
Reliability	Reliable only for a small system.	Also, Reliable for large system
Maximum Power Mismatch	Highest	Lowest
Computational burden	Low	High

Comparing Gauss-Seidel method with Fast-Decoupled Method shown in Table 3.

Table 3 Comparing Gauss-Seidel method with Fast-Decoupled Method [2], [12], [14], [21], [22], [25], [28], [29]

	Gauss Seidel	Fast Decoupled
Co-ordinates	Has good performance with rectangular coordinates.	Polar coordinates
Arithmetical operations	A minimum number for completing one iteration.	Less than Newton Raphson.
Time	Time for each iteration is reduced, but increases as the number of buses increases.	Less time
Speed for iterations	Fastest	It is about 2/3 of what the GS is
Convergence	Linearly	Geometric
No. of iterations	A large number, increasing as buses increase.	For practical accuracies, only 2 to 5 iterations
Slack bus selection	Convergence is adversely affected by the choice of slack bus	Moderate
Accuracy	Generally less accurate	Moderate
Memory	Due to the sparsity of the matrix, there is less memory usage.	Only 60% of NR memory
Usage/application	Small size system	Optimization studies, multi-load flow studies, contingency evaluation for security assessment and enhancement.
Programming Logic	Easy	Moderate
Reliability	Reliable only for a small system.	More reliable than NR method.
Maximum Power Mismatch	Highest	Half of power mismatch value as compared to GS
Computational burden	Low	Higher than Gauss-Seidel

Comparing Newton-Raphson method with Fast-Decoupled Method shown in Table 4.

Table 4 Comparing Newton-Raphson method with Fast-Decoupled Method [2], [7], [12], [14], [15], [21], [22], [25], [28], [29]

	Fast Decoupled	Newton Raphson
Co-ordinates	Polar coordinates	It is preferable to use polar coordinates, since rectangular coordinates occupy more memory.
Arithmetical operations	Less than Newton Raphson.	During each iteration, jacobian elements will be calculated.
Time	Less time	Iterations take 7 times as long as GS and increase with the number of buses.
Speed for iterations	It is higher than NR by factor of 5	About one-fifth of that for FD
Convergence	Geometric	Quadratic
No. of iterations	For practical accuracies, only 2 to 5 iterations	It is very little (about 3 to 5) for large system and is almost constant.
Slack bus selection	Moderate	Minimal sensitivity to this
Accuracy	Moderate	More accurate
Memory	Only 60% of NR memory	Despite compact storage scheme, there is large memory usage.
Usage/application	Optimization studies, multi-load flow studies, contingency evaluation for security assessment and enhancement.	A large system, ill-conditioned problems, optimal load flow studies.
Programming Logic	Moderate	Very difficult
Reliability	More reliable than NR method.	Also, Reliable for large system
Maximum Power Mismatch	medium	The least
Computational burden	Lower than Newton Raphson	High

V. CONCLUSION

This paper presented a comprehensive and comparative study between load flow methods. Each one of the three methods has been presented and discussed. Then, depending on several factors, a detailed comparison was made between these methods

VI. ACKNOWLEDGEMENT

Acknowledgement is due to King Fahd University of Petroleum and Minerals for its support for providing a research environment. I wish to express my sincere appreciation to this university.

VII. REFERENCES

- [1] B. Sereeter, C. Vuik, and C. Witteveen, "On a comparison of Newton–Raphson solvers for power flow problems," *Journal of Computational and Applied Mathematics*, vol. 360, pp. 157–169, Nov. 2019, doi: 10.1016/j.cam.2019.04.007.
- [2] D. Y. Shende, S. R. Gaikwad, A. v Kale, A. S. Uike, S. A. Palekar, and V. B. Hardas, "LOAD FLOW ANALYSIS OF IEEE 14 BUS SYSTEMS IN MATLAB BY USING FAST DECOUPLED METHOD," *International Research Journal of Engineering and Technology*, 2020, [Online]. Available: www.irjet.net
- [3] W. Zheng, F. Yang, and Z. D. Liu, "Research on Fast Decoupled Load Flow Method of Power System," *Applied Mechanics and Materials*, vol. 740, pp. 438–441, Mar. 2015, doi: 10.4028/www.scientific.net/amm.740.438.
- [4] P. Sharma, N. Batish, S. Khan, and S. Arya, "Power Flow Analysis for IEEE 30 Bus Distribution System," *WSEAS TRANSACTIONS on POWER SYSTEMS*, 2018.
- [5] K. Nagaraju, S. Sivanagaraju, T. Ramana, and P. v. Prasad, "A novel load flow method for radial distribution systems for realistic loads," *Electric Power Components and Systems*, vol. 39, no. 2, pp. 128–141, Jan. 2011, doi: 10.1080/15325008.2010.526984.
- [6] A. Shrivastava, P. Badarayani, and D. Arora, "Comparison of Different Load Flow Techniques & Implementation using Newton Raphson Method," vol. 2, no. 8, pp. 621–624, [Online]. Available: <http://www.krishisanskriti.org/jbaer.html>
- [7] L. Sundaresh and P. S. Nagendra Rao, "A modified Newton-Raphson load flow scheme for directly including generator reactive power limits using complementarity framework," *Electric Power Systems Research*, vol. 109, pp. 45–53, Apr. 2014, doi: 10.1016/j.epsr.2013.12.005.
- [8] P. S. Nagendra Rao and Lakshmi Sundaresh, "Nonlinear complementarity formulation for including generator Q limits directly into the Newton Raphson load flow method," *2014 Eighteenth National Power Systems Conference (NPSC)*, 2014.
- [9] T. Tinoco, D. Rubira, and A. Wigington, "Extending Complementarity-Based Approach for Handling Voltage Band Regulation in Power Flow," *2016 Power Systems Computation Conference (PSCC)*, 2016.
- [10] W. Murray, T. Tinoco De Rubira, and A. Wigington, "A robust and informative method for solving large-scale power flow problems," *Computational Optimization and Applications*, vol. 62, no. 2, pp. 431–475, Nov. 2015, doi: 10.1007/s10589-015-9745-5.
- [11] A. Dubey, "LOAD FLOW ANALYSIS OF POWER SYSTEMS," *International Journal of Scientific & Engineering Research*, vol. 7, no. 5, 2016, [Online]. Available: <http://www.ijser.org>
- [12] K. Singhal, "Comparison between Load Flow Analysis Methods in Power System using MATLAB," *International Journal of Scientific & Engineering Research*, vol. 5, no. 5, 2014, [Online]. Available: <http://www.ijser.org>
- [13] Wei-Tzer Huang and Wen-Chih Yang, "Power Flow Analysis of a Grid-Connected High-Voltage Microgrid with Various Distributed Resources," 2011.
- [14] G. Kaur, "Comparative Analysis of Load Flow Computational Methods Using MATLAB." [Online]. Available: www.ijert.org
- [15] S. Janković and B. Ivanović, "Application of combined Newton-Raphson method to large load flow models," *Electric Power Systems Research*, vol. 127, pp. 134–140, Jun. 2015, doi: 10.1016/j.epsr.2015.05.024.
- [16] F. Casella and B. Bachmann, "On the choice of initial guesses for the Newton-Raphson algorithm," *Applied Mathematics and Computation*, vol. 398, Jun. 2021, doi: 10.1016/j.amc.2021.125991.
- [17] A. A. Nazari, R. Keypour, M. H. Beiranvand, and N. Amjadi, "A decoupled extended power flow analysis based on Newton-Raphson method for islanded microgrids," *International Journal of Electrical Power and Energy Systems*, vol. 117, May 2020, doi: 10.1016/j.ijepes.2019.105705.
- [18] P. J. Lagace, M. H. Vuong, and I. Kamwa, "Improving Power Flow Convergence by Newton Raphson with a Levenberg-Marquardt Method."
- [19] U. Thongkrajay, N. Poolsawat, T. Ratnayomchai, and T. Kulworawanichpong, "Alternative Newton-Raphson Power Flow Calculation in Unbalanced Three-phase Power Distribution Systems," *WSEAS International Conference on Applications of Electrical Engineering, Prague*, 2006.
- [20] H. Wayne. Beaty, *Handbook of electric power calculations*. McGraw-Hill, 2001.
- [21] Vishnu Sidaarth Suresh, "Comparison of Solvers Performance for Load Flow Analysis," *Transactions on Environment and Electrical Engineering*, vol. Vol. 3, No. 1, pp. 26–32, 2019, doi: 10.22149/tee.v3i1.131.
- [22] Takamichi Ochi, Kaoru Koyanagi, and Ryuichi Yokoyama, "The development and the application of fast decoupled load flow method for distribution systems with high R/X ratios lines," *2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT)*, 2013.
- [23] P. Buason and D. K. Molzahn, "Analysis of Fast Decoupled Power Flow via Multiple Axis Rotations," Apr. 2021. doi: 10.1109/NAPS50074.2021.9449709.
- [24] Leonard L. Grigsby, Ed., *The Electric Power Engineering Handbook*. 2012.
- [25] A. Parwaiz et al., "Comparative Analysis of Load Flow Methods on Standard Bus System," *International Research Journal of Engineering and Technology*, vol. 775, 2008, [Online]. Available: www.irjet.net
- [26] A. Vijayvargia, S. Jain, S. Meena, V. Gupta, and M. Lalwani, "Comparison between Different Load Flow Methodologies by Analyzing Various Bus Systems," 2016. [Online]. Available: <http://www.irphouse.com>
- [27] A. Shahriari, H. Mokhlis, and A. H. A. Bakar, "Critical reviews of load flow methods for well, ill and unsolvable condition," *Journal of Electrical Engineering*, vol. 63, no. 3, pp. 144–152, 2012. doi: 10.2478/v10187-012-0022-x.
- [28] A. P. de Moura and A. A. F. de Moura, "Newton-Raphson power flow with constant matrices: A comparison with decoupled power flow methods," *International Journal of Electrical Power and Energy Systems*, vol. 46, no. 1, pp. 108–114, Mar. 2013, doi: 10.1016/j.ijepes.2012.10.038.
- [29] R. K. Portelinha, C. C. Durce, O. L. Tortelli, and E. M. Lourenço, "Fast-decoupled power flow method for integrated analysis of transmission and distribution systems," *Electric Power Systems Research*, vol. 196, Jul. 2021, doi: 10.1016/j.epsr.2021.107215.