International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

# Removal of Impulsive Noise from MRI Images using Quadratic Filter

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***Abstract*— In the past few decades there had been significant progress in the theory of linear time invariant (LTI) systems which successfully can model many real time systems and phenomena. But most of the natural systems and processes are inherently nonlinear, and there arise the need for nonlinear systems. The power series proposed by Vito Volterra can model mild polynomial nonlinearities. This paper focuses on quadratic volterra filter for removing impulsive noise from MRI images, the acquisition of which happens at extremely noisy conditions. By conventional filtering methods, it is very difficult to remove impulsive noise added with the raw MRI data due to the rapidly changing magnetic field. Comparisons of performance parameters with conventional systems establish the superiority of quadratic systems in removing impulsive noise.**

***Keywords*— MRI, Impulsive Noise, Volterra Series, Quadratic Filter, Singular Value Decomposition**.

## 1. INTRODUCTION

Magnetic resonance imaging (MRI), is a medical imaging technique used in radiology to investigate the anatomy and function of the body. This imaging technique has a wide range of applications in medical diagnosis such as to find problems such as tumors, bleeding, injury, blood vessel diseases, or infection. Unlike images generated by digital cameras, images generated by medical imaging systems such as MRI machine are largely corrupted by impulsive noise as the acquisition of image happens under strong and rapidly varying magnetic fields, typically 1.5 Tesla to 2 Tesla. So noise removal is essential for improving the interpretability of information in images and for generating perceptually more pleasing images from the given input images.

Both linear and nonlinear filters can be used for noise removal. Linear filters are useful in many image processing applications. The obvious advantage of a linear filter is its simplicity in design and implementation. But linear systems fail when it comes to removing impulsive noise added with the raw MRI data due to the rapidly changing magnetic field. Besides, medical images are formed by complex nonlinear processes and the inherent nonlinearities in them cannot easily be modeled by conventional linear systems. Mild polynomial nonlinearities can be modeled by Volterra series. It is a power series that add quadratic, cubic and higher order components in parallel with the linear term. So this series has the added advantage that existing LTI systems can be augmented by adding parallel polynomial filters to yield improved performance. It is widely observed that much of the nonlinear behavior can be modeled with the quadratic term alone. So the present work was to design and implement a quadratic filter that can take into account the inherent nonlinearities in image formation while removing the impulsive noise.

## 2. DISCRETE VOLTERRA SERIES

Although the theory of linear systems is very advanced and useful, most of the real life and practical systems are nonlinear, making them difficult for mathematical modeling. Mild polynomial nonlinearities can be modeled by Volterra power series. An Nth order Volterra filter with input vector x[n] and output vector y[n] is realized by [1]

$$y[n] = h_0 + \sum_{r=1}^{\infty}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\dots\sum_{n_r=1}^{N} h_r[n_1, n_2, \dots n_r]\, x[n-n_1]\, x[n-n_2]\dots x[n-n_r] \quad (1)$$

where r indicates the order of nonlinearity, with r=1 implying a linear system, r=2 implying a quadratic system and so forth. The term $h_0$ denotes the output offset when no input is present and the term $h_r[n_1,n_2,\dots n_r]$ denotes the $r_{th}$ order Volterra kernel. Identification of this kernel for a nonlinear system is the chief issue in designing polynomial systems [2].

In practical systems, the polynomial nonlinearities are often comprised of the quadratic term alone. So it is proposed that a two dimensional quadratic filter can model and process inherent nonlinearities in medical images, resulting in better noise removal and sharper edges.

## 3. TWO DIMENSIONAL QUADRATIC VOLTERRA FILTER

The two dimensional quadratic filter is governed by the equation Eq. 2

$$y[n_1,n_2] = \sum_{m_{11}=0}^{N_1-1}\sum_{m_{12}=0}^{N_2-1}\sum_{m_{21}=0}^{N_1-1}\sum_{m_{22}=0}^{N_2-1} h_2[m_{11},m_{12},m_{21},m_{22}] \times x[n_1-m_{11},n_2-m_{12}]\,x[n_1-m_{21},n_2-m_{22}] \quad (2)$$

Out of the four indices $m_{11}$ , $m_{12}$ , $m_{21}$ , $m_{22}$ of the kernel $H_2$ , two stem from the quadratic nature of the kernel and the remaining two denote the two dimensions of the signal processed. Eq. 2 is represented in the matrix form as in Eq. 3.

$$y[n_1,n_2] = X^T[n_1,n_2]H_2X[n_1,n_2] \quad (3)$$

where

$$H_2 = \begin{bmatrix} H[0,0] & \cdots & H[0,N_2-1] \\ \vdots & \ddots & \vdots \\ H[N_2-1,0] & \cdots & H[N_2-1,N_2-1] \end{bmatrix} \quad (4)$$

where each sub-matrix H[i, j] is given by

$$H[i,j] = \begin{bmatrix} h[0,i,0,j] & \cdots & h[0,i,N_1-1,j] \\ \vdots & \ddots & \vdots \\ h[N_1-1,i,0,j] & \cdots & 0,i,N_1-1,j \end{bmatrix} \quad (5)$$

The principal issues in Volterra systems are the identification of the kernel and its computationally efficient implementation. Unlike in linear filtering, there are no general design rules for finding $H_2$. Design of two dimensional kernels for specific applications can be done using methods like optimization, bi-impulse response method etc [3].

The current work uses the optimization of mean square error using Powell method [4] as it yields faster convergence. The second issue is in realizing the kernel with minimum computational complexity [5], [6]. A feasible implementation can be done with an appropriate decomposition of $H_2$ like LU or singular value decomposition (SVD). In this paper quadratic filter is implemented using SVD.

## 4. METHODOLOGY

The methodology of noise removal is as outlined in Fig. 1. The first phase is the design of the quadratic kernel for noise removal $H_{2noise}$. There is no general methodology for the design of quadratic systems unlike in the case with LTI systems. Mostly, the methods are heuristic and application dependent but bestows the designer with the power to customize the filter to suit the application. It can often be expressed as one which could be solved by a suitable method of optimization. Here Powell method of optimization is used as described in Sec. V. This step of optimization minimizes the mean square error between a known image and the output of a quadratic filter that receives the noisy version of the image at its input. The optimization is done repeatedly until minimum mean square error yields.

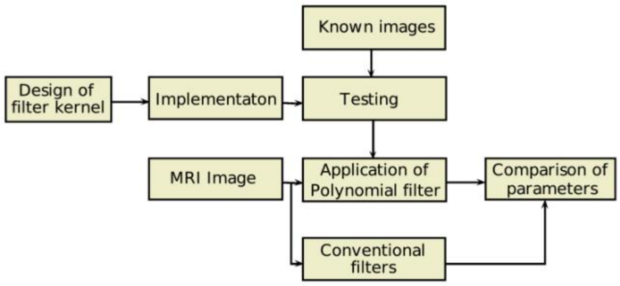


Fig.1. Flow of work

The solution for this minimum error is selected as the filter kernel. As the direct implementation of this kernel is computationally complex, singular value decomposition is done on $H_{2noise}$ to yield an approximate realization $\widetilde{H}2_{noise}$. In the third phase, the kernel is tested with known images for ascertaining the improvement in signal to noise ratio and peak signal to noise ratio before applying to noisy MRI images. The noisy MRI image is taken and is subjected to filtering by $\widetilde{H}2_{noise}$ as implemented for removing impulsive noise.

## 5. DESIGN AND IMPLEMENTATION OF QUADRATIC FILTER

The design of quadratic filter includes the discovery of its kernel coefficients. The main criteria that followed for the design purpose of filter is the minimization of the difference in desired output and observed output. The important design method based on this criterion is optimization technique. Here we used the Powell's conjugate direction method for obtaining $H_{2noise}$ [4], since this algorithm has a fast rate of convergence.

A synthetic image, $X[n_1,n_2]$ of 64 × 64 dimension, corrupted by impulsive noise of known variance $\sigma_N^2$ is simulated. Its noiseless version $Y_d[n_1,n_2]$ of identical dimension is also simulated. The output of the quadratic filter is assumed as:

$$y[n_1,n_2] = X^T[n_1,n_2]H_{2_{noise}}X[n_1,n_2] \quad (6)$$

Let the MSE between $y[n_1,n_2]$ and $y_d[n_1,n_2]$ is

$$\varepsilon = E[|y_d[n_1,n_2] - y[n_1,n_2]|^2] \quad (7)$$

$$\varepsilon = E[|y_d[n_1,n_2] - X^T[n_1,n_2]H_{2_{noise}}X[n_1,n_2]|^2] \quad (8)$$

The MSE ξ is minimized to yield an optimum kernel $H_{2noise}$. The kernel is plotted in Fig. 2.

ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

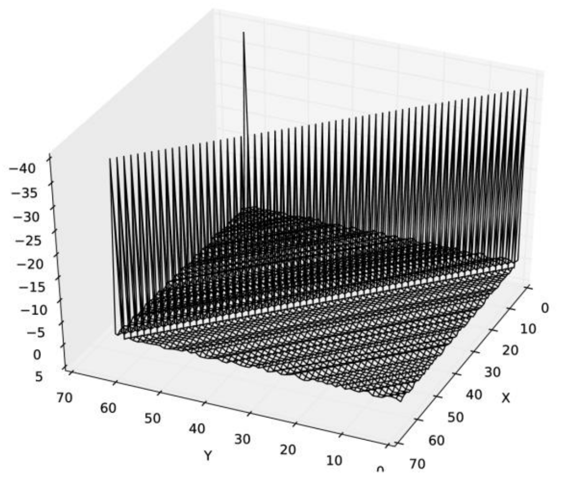


Fig.2. Quadratic filter kernel

A direct implementation of this kernel as in Eq. 3 is computationally complex. Instead SVD decomposition is performed on $H2_{noise}$ to yield an approximation as:

$$\widetilde{H2_{noise}} = \sum_{i=1}^{r} \lambda_i S_i S_i^T \quad (9)$$

where λi are the singular values arranged in a decreasing order and Si are the orthonormal singular vectors. Each 64 × 1 vector can be resized as a 3 × 3 FIR image filter that is equivalent to H(i, j) in Eq. 5.

The outputs of FIR filters are squared and a weighted sum with λi values yields the filter output.

## 6. EXPERIMENT

The filter kernel $\widetilde{H}2_{noise}$ designed in Sec. V is simulated in Python with the help of scipy and pylab modules. The noisy images are imported into Python using the image processing toolbox and subjected to filtering by $\widetilde{H}2_{noise}$. The filtered images are compared with those processed by spatial filters like median filter in terms of visual quality. For testing the noise invulnerability of $\widetilde{H}2_{noise}$, the experimental set up in Fig. 3 is used. In this, impulsive noise of known variance is added with the images and subjected to filtering. Quantitative measures like signal to noise ratio and peak signal to noise ratio are calculated with reference to the noisy image. Experiment is repeated for conventional filter like median and the results are compared with those of quadratic filter. Conventional filters blur the image while removing noise. Quadratic filter gives a better result.

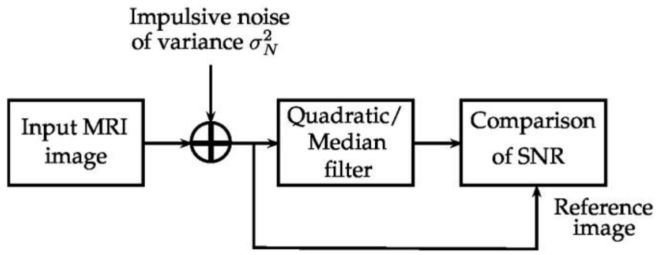


Fig.3. Experimental setup for ascertaining the performance parameters

## 7. RESULTS

Quadratic filter is designed and implemented with the objective of removing impulsive noise from raw MRI data which is acquired under strong and rapidly changing magnetic fields. The MRI images corrupted by impulsive noise of different noise variances are applied to the quadratic filter implemented by SVD method. The resulting output is compared with those of spatial filter like median filter. It is observed that quadratic filter removes the impulsive noise better than the median filter.

Fig. 4 show the outputs of various filters for impulsive noise variance $\sigma_N^2$=200. The noisy MRI image is as in Fig. 4(b). The output of median filter is shown in Fig. 4(c) and quadratic filter output is in Fig. 4(d). As claimed the output of the quadratic filter is the least noisy. Median filter fails to clean much of the impulsive noise in the input image.



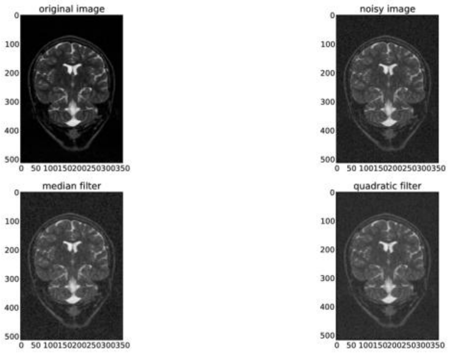


Fig.4. Output of different filters

### *7.1 Signal to Noise Ratio*

The improvement in signal to noise ratio is computed as per the experimental setup in Fig. 3. The SNR is expressed as:

$$SNR = 10\log_{10}\left[\frac{\sum_{n_1}\sum_{n_2} r^2[n_1,n_2]}{\sum_{n_1}\sum_{n_2}[r^2[n_1,n_2] - t^2[n_1,n_2]]}\right] \quad (10)$$

where $r[n_1,n_2]$ denotes the reference image and $t[n_1,n_2]$denotes the test image. N1 × N2 is the size of the images. There is 12 dB improvement in using quadratic filter.

Besides the improvement in SNR, quadratic filter has the advantage that the edges are not blurred on filtering, ensuring that the periphery of a possible pathological disorder like a tumour remains unambiguous.

## 8. CONCLUSION

Magnetic resonance imaging (MRI), is a medical imaging technique used to find problems such as tumors, bleeding, injury, blood vessel diseases, or infection. By conventional filtering methods, it is very difficult to remove impulsive noise added with the raw MRI data due to the rapidly changing magnetic field.

Conventional nonlinear filters employed for noise removal from images are median filter, mean filter, Gaussian filter etc. Such filters often suffer from poor edge resolution, blurring and poor signal to noise ratio. The paper summarizes the design and implementation of a quadratic noise removal filter based on Volterra series. The design method used for the quadratic kernel is method of optimization. Then it subjected to singular value based decomposition to yield an approximate but computationally simple implementation. The quadratic filtering operation is observed to remove the impulsive noise present in raw MRI data much better than the conventional filtering methods.

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