Reliability of Time Dependent Stress-Strength System for Half Logistic Distribution

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***Abstract* - Failure of a system may occur due to certain type of stresses acting on them. If these stresses do not exceed a certain threshold value the system may work for a long period. On the other hand, if the stresses exceed the threshold they may fail within no time. There is uncertainty about stress and strength random variables at any instant of time and also about the behavior of the variables with respect to time and cycles. Time dependent stress- strength models are considered with repeated application of stress and also the change of the strength with time. Reliability of time dependent stress-strength system is carried out by considering each of stress variables are random- fixed and strength variables are random – independent and vice versa and deterministic stress and random independent strength and vice-versa for stress –strength follow Half Logistic Distribution. It is observed from the computations that the reliability of the system is depending on the stress parameter and the strength parameter and another constant parameters**

Key words: Half Logistic distribution, stress strength model, deterministic , random fixed.

INTRODUCTION

Time dependent stress strength models by considering with the repeated application of stress considered as the change of the distribution of strength with time. “Stress” is used to indicate any agency that tends to induce “failure”, while “Strength” indicates any agency resisting “failure”. “Failure is defined to have occurred when the actual stress exceed the actual strength.

There is uncertainty about the stress and strength random variables at any instant of time and also about the behavior of the random variables with respect to time and cycles .The two terms ‘deterministic’ and ‘random fixed’ are used to describe these two uncertainties. In deterministic, the variables assume values that are exactly known a priori. Random fixed refers to the behavior of the variable with respect to time is fixed or the variable varies in time in a known manner .The failure of components under repeated stresses had been investigated primarily. Repeated stresses are characterized by the time, each load applied and the behavior of time intervals between the applications of loads.

The reliability after n cycles R_n to R(t), the reliability at time t , where t is continuous. Simply when cycle times are deterministically known $R(t) = R_n$, $t_n < t \le t_{n+1}$,where t_i is instant in time at which the i^{th} cycle occurs .The time dependent load was discussed by several researchers. Some of them are Bilikam etal [1] ,Kechengshen[2] , M.N.Gopalan[3] and Dongshang chang[4].

In the present paper, we have discussed deterministic stress and random fixed strength and vice versa, we had take Half logistic distribution. Reliability computations were done for different cycle lengths .The result is that the system reliability rapidly changes in Rayleigh distribution than the Exponential distribution.

STATISTICAL METHOD

X and Y denote the stress and strength of the system. f (X) and g(Y) are probability density functions of X and Y . Then the reliability of the system is

$$R = \int_{-\infty}^{\infty} f(X)\left[\int_{X}^{\infty} g(Y)dY\right]dX$$

Or

$$R = \int_{-\infty}^{\infty} g(Y)\left[\int_{-\infty}^{Y} f(X)d\,X\right]dY$$

The reliability computations for deterministic cycle times can take two cases

Case 1: Deterministic stress and random fixed strength

Let the stress be x_0, a constant and the strength on the i^{th} cycle Y_i given by

$$Y_i = Y_0 - a_i\ ,\ \ i = 1\ 2\ 3 \text{-------}$$

Where $a_i \le 0$ are known constants. Further, the a_i's are assumed non- decreasing in time. The p.d.f of y_0 , $g_0(y_0)$ is assumed known. Then

$$p[E_n] = p(x_n \le y_n)$$

$$= \int_{x_0+a_n}^{\infty} g_0(y_0)\,dy_0$$

But $\quad R_n = p[E_1, E_2, ---E_n]$

$$R_n = p[E_1/E_2, ---E_n] * p[E_2/E_3\,, ---E_n]$$

$$--- * p[E_{n-1}/E_n] * p[E_n]$$

All but the last term in the R.H.S of above equations are 1's because of restrictions on the a_i's which cause the strength y_i to decrease in time. Hence

$$R_n = p[E_n] = \int_{x_0+a_n}^{\infty} g_0(y_0)dy_0$$

Case 2: Random fixed stress and deterministic strength

Let $X_i = X_0 + b_i$, i = 1,2,3,--- denote the stress in cycle i , where b_i's are known non negative constants , non decreasing in time . Further let the strength be held constant at y_0. The p.d.f. of $X_0, f_0(x_0)$ is assumed known.

The restrictions on $b_i's$ guarantee non-decreasing. Stress, which in turn ensure that

$$R_n = p[E_n]$$
$$= p(X_n \leq Y_n)$$
$$= p(X_0 + b_n \leq y_0)$$
$$R_n = \int_0^{y_0-b_n} f_0(x_0)dx_0$$

A random variable X is said to have Half logistic distribution if its p.d.f. is given by

$$f(x) = \frac{2e^{-x}}{(1+e^{-x})^2}, x \geq 0$$ and

Similarly a random variable Y is said to have Half logistic distribution if its p.d.f. is given by

$$g(y) = \frac{2e^{-y}}{(1+e^{-y})^2}, y \geq 0$$

For deterministic stress and random fixed strength, the system reliability

$$\text{R(t)} = R_n = p[E_n]$$
$$= p(x_n \leq y_n)$$
$$= \int_{x_0+a_n}^{\infty} g_0(y_0)\,dy_0$$
$$= \int_{x_0+a_n}^{\infty} \frac{2e^{y_0}}{(1+e^{y_0})^2}\,dy_0$$

Take $1 + e^{y_0} = t$

Then

$$R(t) = \int_{1+e^{-x_0+a_n}}^{1} \frac{-2}{t^2}dt$$
$$= 2\left[1 - \frac{1}{1+e^{-x_0+a_n}}\right]$$
$$= \frac{2e^{-x_0+a_n}}{1+e^{-x_0+a_n}}$$

For random fixed stress and deterministic strength, the reliability of the system is

$$\text{R(t)} = R_n = p[E_n]$$
$$= p(x_n \leq y_n)$$
$$= \int_0^{y_0-b_n} f_0(x_0)dx_0$$
$$= \int_0^{y_0-b_n} \frac{2e^{x_0}}{(1+e^{x_0})^2}\,dx_0$$

Take $1 + e^{x_0} = t$

Then

$$R(t) = \int_2^{1+e^{-y_0+a_n}} \frac{-2}{t^2}dt$$
$$= \frac{1-e^{-y_0+b_n}}{1+e^{-y_0+b_n}}$$

Reliability Computations:

Table 1

x_0	a_n	R
0.1	0.01	0.95503
0.2	0.01	0.905285
0.3	0.01	0.856008
0.4	0.01	0.807435
0.5	0.01	0.759787
0.6	0.01	0.71327
0.7	0.01	0.668066
0.8	0.01	0.624337
0.9	0.01	0.58222
1	0.01	0.541824
1.1	0.01	0.503237

Figure 1

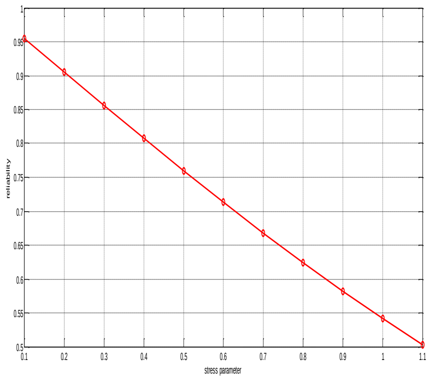


Table 2

x_0	a_n	R
0.2	0.01	0.905285
0.2	0.02	0.910242
0.2	0.03	0.915204
0.2	0.04	0.92017
0.2	0.05	0.92514
0.2	0.06	0.930114
0.2	0.07	0.935091
0.2	0.08	0.940072
0.2	0.09	0.945055
0.2	0.1	0.950042
0.2	0.11	0.95503

Figure 2

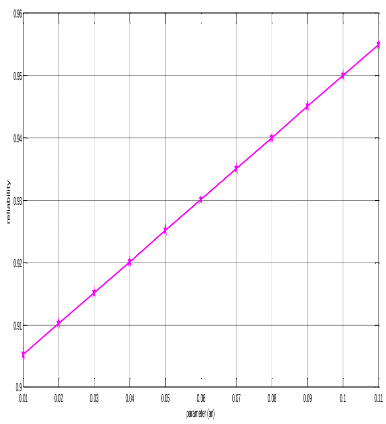


Table 3

y_0	b_n	R
1	2	0.268941
2	2	0.5
3	2	0.731059
4	2	0.880797
5	2	0.952574
6	2	0.982014
7	2	0.993307
8	2	0.997527
9	2	0.999089

Figure 3

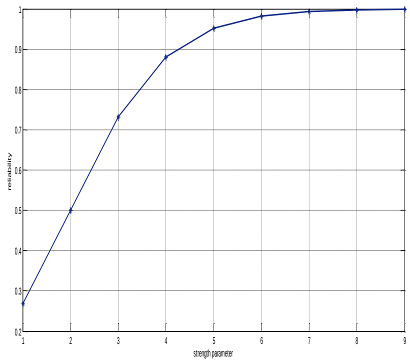
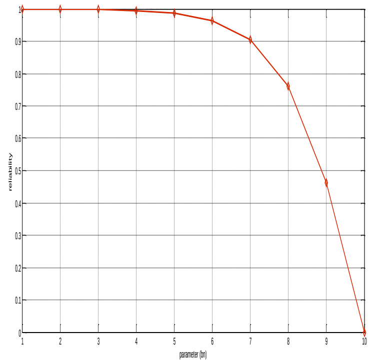


Table 4

y_0	b_n	R
10	1	0.999753
10	2	0.999329
10	3	0.998178
10	4	0.995055
10	5	0.986614
10	6	0.964028
10	7	0.905148
10	8	0.761594
10	9	0.462117
10	10	0

Figure 4



CONCLUSION

The stress and strength follow Half Logistic distribution for deterministic stress and random- fixed strength and vice versa . In computations we observed that if the Stress parameter value (a_n), increases the reliability also increases and constant value(x_0) increases, the reliability decreases. The strength parameter value (b_n) is increases, then reliability value decreases and constant value (y_0) increases then reliability also increases.

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