

Reliability Enhancement using Different Redundancy Configurations Applied in a Network with two failure modes – Simulation Approach

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Abstract -Reliability is one of the most important concepts in the electronics systems conclude the optimal design. Reliability can be defined as the probability that component parts of system will perform its intended functions under given circumstances, without failure at stated period of time. Redundancy represents the most common technique used to enhance the reliability. It measures to extend a component or system's life by reducing its potential failure modes. In this paper, two basic challenges are examined. The first of them is the selection of the best formula for reliability measure. The second challenge is to maintain the optimal number of redundancy in the selected configurations. In the presented simulation, a comparison is performed using series - parallel and parallel - series configurations. Typically, the failure of system is subject to be only by two modes of failures, which exemplifies the network system.

Finally, the redundancy may increase or decrease the reliability of the system. For example, assume a network consisting of (n) relays in series has two different failure modes an open mode or closed mode, in addition to variant number of subsystems (m). We try to change number of subsystems (m) to find the optimum value (m*) at different relays of fixed (n) and the result was that the optimum value of m* shouldn't be the maximum and any increment of n will decrease the reliability. This analysis will be inspected for different configurations of two types of failure modes.

Keywords -reliability, series-parallel systems, parallel-series systems redundancy

I. INTRODUCTION

Due to rapid development of technologies in electronics, components and systems increasingly complex and reliability has been given more considerable attention by both researchers and practitioners. Generally, reliability is the study of how, why and when failures occur [1]. From the viewpoint of engineering, reliability engineering deals with measures to extend a component or system's life by reducing its potential failure modes. Many systems use redundancy to achieve their intended reliability [2]. The reliability can be defined as the probability that a system, subsystem, component or part will perform its intended functions under defined conditions at a designated time for a specified operating period [3]. Reliability, in the quantitative sense as used here, is

defined above as a probability. For the present, it seems that quantitative treatment of reliability will involve probability and statistical inference. Reliability predictions may be performed for any of the following reasons: Potential technical contribution, financial implications, and Compulsory. Each of these could apply to the user of a system as well as to the supplier [4]. For example, he may decide to search for areas needing reliability improvement. However, the other reasons do occur. Financial implications arise in a fixed price or incentive contract which also has an associated reliability requirement and method of measurement. The compulsory reason may typically apply to a government agency because of policy and to a supplier because of contract requirements. The numerical reliability prediction number and its attendant measure of uncertainty are usually necessary in order to respond to any of the reasons performing a prediction which are noted above [5]. That is, response to such questions as "Can the mission be achieved?" or "What are the possibilities of making a profit?" or simply here is what the customer asked for. Searching for reliability improvements and probing around for weaknesses in the design and the operational procedure is the most technically appealing use. It is this use that often results in a reliability prediction going into more detail than it otherwise might. That is, comparative detailed values are sought rather than absolute gross values. Hopefully new alternatives will be opened up and the really bad choices can be eliminated. Linear optimization techniques, such as dynamic programming algorithms, offer the potential of improved allocation of overall reliability among the items comprising the system [6]. Of these uses, obtaining the prediction number and searching for improvements have seen more application than the other two. With the extensive experience accumulated with reliability prediction, it is now possible to make some intelligent judgments on accuracy even if only qualitative. When there is a fair amount of historical data and the equipment is not excessively complex or new, a crude rule-of-thumb for electronic equipment would be to expect the actual meantime between failures (MTBF) to be within the range of 50 to 200 percent of the predicted MTBF.

A component is subject to failure in either open or closed modes. Networks of relays, fuse systems for warheads, diode circuits, fluid flow valves, etc. are a few examples of such components. Generally, redundancy can be used to enhance the reliability of a system without any change in the reliability of the individual components that form the system. However, in a two-failure mode problem, redundancy may either increase or decrease the system's reliability [7]. For example, a network consisting of n relays in series has the property that an open-circuit failure of any one of the relays would cause an open-mode failure of the system and a closed-mode failure of the system. (The designations "closed mode" and "short mode" both appear in this chapter and we will use the two terms interchangeably.) On the other hand, if the n relays were arranged in parallel, a closed-mode failure of any one relay would cause a system closed-mode failure, and an open-mode failure of all n relays would cause an open-mode failure of the system. Therefore, adding components in the system may decrease the system reliability [8]. Diodes and transistors also exhibit open-mode and short-mode failure behavior. For instance, in an electrical system having components connected in series, if a short circuit occurs in one of the components, then the short circuited component will not operate but will permit flow of current through the remaining components so that they continue to operate. However, an open-circuit failure of any of the components will cause an open-circuit failure of the system.

System reliability where components have various failure modes is covered in reference [9]. Series-parallel and parallel-series systems have been studied in many references where the size of each subsystem was fixed, but the number of subsystems was varied to maximize reliability. Determining a value of k that maximizes the reliability of k -out-of- n systems is analyzed in reference. Reliability optimization of series, parallel, parallel-series, series-parallel, and k -out-of- n systems subject to two types of failure will be discussed next. In general, the formula for computing the reliability of a system subject to two kinds of failure is [10]:

$$\begin{aligned} \text{System reliability} &= \Pr\{\text{system works in both modes}\} \\ &= \Pr\{\text{system works in open mode}\} \\ &\quad - \Pr\{\text{system fails in closed mode}\} \\ &\quad + \Pr\{\text{system fails in both modes}\} \end{aligned} \quad (1.1)$$

When the open- and closed-mode failure structures are dual of one another, i.e. $\Pr\{\text{system fails in both modes}\} = 0$, then the system reliability given by Equation 1.1 becomes:

$$\begin{aligned} \text{System reliability} &= 1 - \Pr\{\text{system fails in open mode}\} \\ &\quad - \Pr\{\text{system fails in closed mode}\} \end{aligned} \quad (1.2)$$

The next sections will give a brief introduction for several reliability configurations, Series, Parallel, Parallel-Series, Series-Parallel.

II. RELIABILITY EVALUATION

The reliability of the system is defined as the probability of obtaining the correctly processed message at the output. To derive a general expression for the reliability of the system, we use an adapted form of the total probability theorem as translated into the language of reliability.

Let A denote the event that a system performs as desired, let X_i and X_j be the event that a component X (e.g. converter, monitor, or switch) is good or failed respectively. Then

$$\begin{aligned} \Pr\{\text{System works}\} &= \Pr\{\text{system works when unit } X \text{ is good}\} * \Pr\{\text{unit } X \text{ is good}\} \\ &\quad + \Pr\{\text{system works when unit } X \text{ is failed}\} * \Pr\{\text{unit } X \text{ is failed}\}. \end{aligned} \quad (2.1)$$

The above equation provides a convenient way of calculating the reliability of complex systems. In order for the system to operate when the first converter works and the first monitor fails, the first switch must work and the remaining system of size $n - 1$ must work. The reliability of the system consisting of n non-identical converters can be easily obtained.

III. REDUNDANCY OPTIMIZATION

Assume the following notations

q_{o1} : Probability of component failure in open mode due to the 1st reason.

q_{o2} : Probability of component failure in open mode due to the 2nd reason.

q_{s1} : Probability of component failure in short mode due to the 1st reason

q_{s2} : Probability of component failure in short mode due to the 2nd reason.

* implies an optimal value.

m : number of subsystems in a system (or subsystem size).

n : number of components in each subsystem.

$h_{o1}(m)$: probability of system failure in open mode due to the 1st reason.

$h_{o2}(m)$: probability of system failure in open mode due to the 2nd reason.

$h_{s1}(m)$: probability of system failure in short mode due to the 1st reason.

$h_{s2}(m)$: probability of system failure in short mode due to the 2nd reason.

$P(m)$: Average system profit.

B : conditional probability (given system failure) that the system is in open mode

$1 - \beta$: conditional probability (given system failure) that the system is in short mode

c_1, c_3 : gain from system success in open, short mode

c_2, c_4 : gain from system failure in open, short mode; $c_1 > c_2$,

$c_3 > c_4$.

Assume the following assumptions

- The system consists of (m) subsystems, each subsystem contains (n) statistically independent and identical distribution (i.i.d.) components (in other words, the failure of one component is no way affects the probability of failure of the other components).
- A component is either good, failed open, or failed short.
- The system has two failure modes:
 - An open failure of at least one component in each subsystem causes the system to have an open failure (1st or 2nd reason).
 - A short failure of all components in any subsystem causes the system to have a short failure (1st or 2nd reason).
- The unconditional probabilities of component failure in open and short modes are known, and constrained (i.e. $q_{o1}, q_{o2}, q_{s1},$ and $q_{s2} > 0; q_{o1} + q_{o2} + q_{s1} + q_{s2} < 1$).
- Costs of system failure in open and short modes are known and can be different.
- The system can be failed open when all components in any subsystem fail open.
 - The probability of system failure in 1st open mode
$$h_{o1}(m) = 1 - (1 - q_{o1})^m \quad (2.2)$$
 - The probability of system failure in 2nd open mode
$$h_{o2}(m) = 1 - (1 - q_{o2})^m \quad (2.3)$$
- The system can be failed short if at least one components in each subsystem fails short.
 - The probability of system failure in 1st short mode
$$h_{s1}(m) = [1 - (1 - q_{s1})^n]^m \quad (2.4)$$
 - The probability of system failure in 2nd short mode
$$h_{s2}(m) = [1 - (1 - q_{s2})^n]^m \quad (2.5)$$

A. Series - Parallel Redundant Systems Analysis

In this section, we analyze the series-parallel system in which the components are arranged so that there are (m) subsystems operating in series. Each subsystem consists of (n) identical components in parallel. Two reasons of failure are assumed.

In order to maximizing the average system profit, we settle the following equations.

The average system profit is given by:

$$P(m) = \beta [c_1(1 - \sum_{i=1}^2 h_{oi}(m)) + c_2(1 - \sum_{i=1}^2 h_{si}(m))] + (1 - \beta) [c_3(1 - \sum_{i=1}^2 h_{si}(m)) + c_4(1 - \sum_{i=1}^2 h_{oi}(m))] \quad (2.6)$$

$$P(m) = \beta [c_1 - (c_1 - c_2)(h_{o1}(m) + h_{o2}(m))] + (1 - \beta) [c_3 - (c_3 - c_4)(h_{s1}(m) + h_{s2}(m))] \quad (2.7)$$

$$P(m) = -(1 - \beta)(c_3 - c_4) [(h_{s1}(m) + h_{s2}(m)) + a(h_{o1}(m) + h_{o2}(m))] + b \quad (2.8)$$

Where

$$a = \beta(c_1 - c_2) / [(1 - \beta)(c_3 - c_4)]$$

$$b = \beta c_1 + (1 - \beta)c_3$$

$$\Delta P = P(m+1) - P(m),$$

It will conclude to be:

$$h_{s1}(m+1) - h_{s1}(m) = -(1 - q_{s1})^n [1 - (1 - q_{s1})^n]^m \quad (2.9)$$

Similarly:

$$h_{s2}(m+1) - h_{s2}(m) = -(1 - q_{s2})^n [1 - (1 - q_{s2})^n]^m \quad (2.10)$$

Also,

$$h_{o1}(m+1) - h_{o1}(m) = (1 - (q_{o1})^n)^m [1 - 1 + (q_{o1})^n] \quad (2.11)$$

$$h_{o2}(m+1) - h_{o2}(m) = (q_{o2})^n [1 - (q_{o2})^n]^m \quad (2.12)$$

$$\Delta P = -(1 - \beta)(c_3 - c_4) \{ -(1 - (1 - q_{s1})^n)^m (1 - q_{s1})^n - (1 - (1 - q_{s2})^n)^m (1 - q_{s2})^n + a[(q_{o1})^n (1 - (q_{o1})^n)^m + (q_{o2})^n (1 - (q_{o2})^n)^m] \} \quad (2.13)$$

Let

$$\Delta P = 0$$

$$a[(q_{o1})^n (1 - (q_{o1})^n)^m + (q_{o2})^n (1 - (q_{o2})^n)^m] = (1 - q_{s1})^n (1 - (1 - q_{s1})^n)^m + (1 - q_{s2})^n (1 - (1 - q_{s2})^n)^m \quad (2.14)$$

1) First Case:

For $a < 1$:

Let $\beta = 0.4 \quad c_1 = 500 \quad c_2 = 50 \quad c_3 = 400$
 $c_4 = 80 \quad q_{s2} = 0.01 \quad q_{o2} = 0.01$
 $a = 15/16$

The next figures illustrate the variation of m^* versus q_{o1} with different q_{s1} .

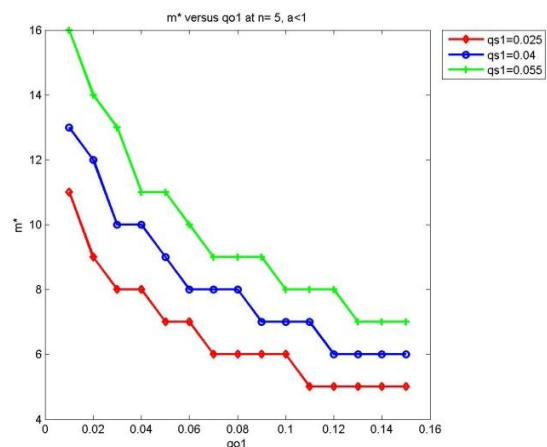


Figure 1: m^* versus q_{o1} at $n=5$ for $q_{s1}=0.025, 0.04,$ and 0.055 (Series Parallel Configuration for $a < 1$)

Figure 1 shows that m^* increases with larger q_{s1} at $n=5$ for $q_{s1}=0.025, 0.04,$ and 0.055 .

In other words optimality can be reached with lower number of subsystems (m^*), also m^* decreases with q_{o1} increase. Reaching better results can be attained by increasing n value but to a certain value then again the optimality is reached with large value of m^* .

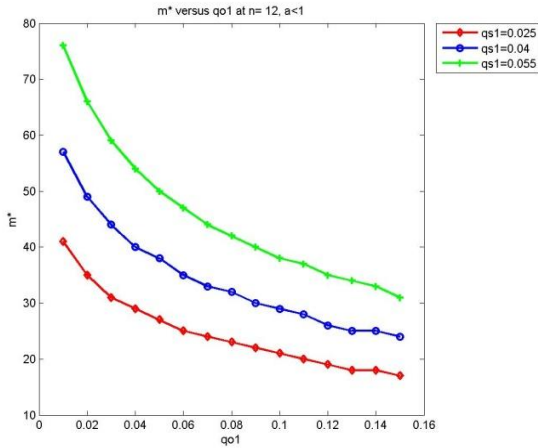


Figure 2: m^* versus q_{o1} at $n=12$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Series Parallel Configuration for $a < 1$)

Figure 2 illustrates the variation of m^* with q_{o1} at $n=12$ for $q_{s1}=0.025, 0.04,$ and 0.055 . Optimality can be reached with lower number of subsystems (m^*), also m^* decreases as q_{o1} increases. Reaching better results can be attained by increasing n value but to a certain value then again the optimality is reached with large value of m^* .

2) Second Case:

For $a > 1$:

Let $\beta=0.6$ $c_1=500$ $c_2=50$ $c_3=400$ $c_4=80$
 $q_{s2}=0.01$ $q_{o2}=0.01$ $a=135/64$

The next figures show the variation of optimal number of subsystem m^* for different q_{s1} and q_{o1} at specific value of n .

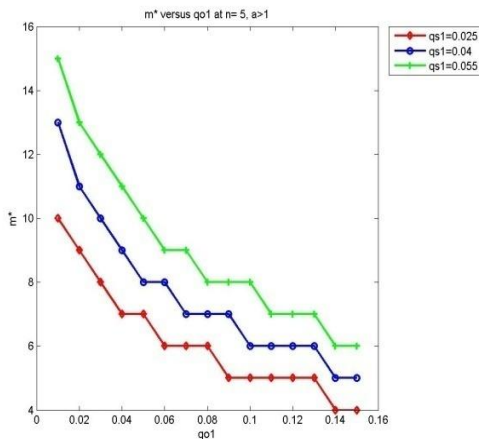


Figure 3: m^* versus q_{o1} at $n=5$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Series Parallel Configuration for $a > 1$)

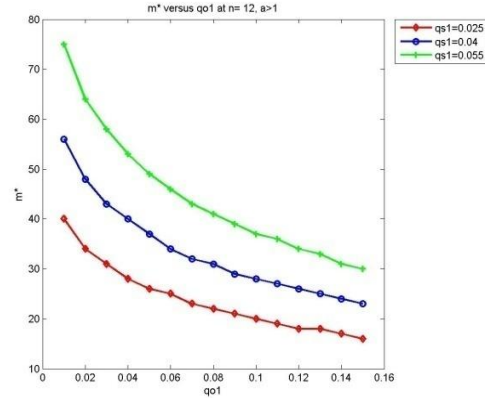


Figure 4: m^* versus q_{o1} at $n=12$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Series Parallel Configuration for $a > 1$)

We can summarize the results as:

- 1) m^* is increasing as q_{s1} increases.
- 2) m^* is decreasing as q_{o1} increases.
- 3) Case two ($a > 1$) required higher m^* .
- 4) m^* decreases with β .
- 5) It can be realized that m^* increases with increasing the value of the subsystem size (n).

B. Parallel – Series Redundant Systems Analysis:

This system is considered to have components arranged so that there are m subsystems operating in parallel, each subsystem consists of (n) identical components in series. The system consists of components that can fail in two mutually exclusive ways. In this case, the following assumptions are considered:

1. The system can be failed open when one component in any subsystem failed open.

- The probability of system failure in 1st open mode $h_{o1}(m) = (1 - (1 - q_{o1})^n)^m$ (2.15)
- The probability of system failure in 2nd open mode $h_{o2}(m) = (1 - (1 - q_{o2})^n)^m$ (2.16)

2. The system can be failed short when all components in any subsystem are failed short.

- The probability of system failure in 1st short mode $h_{s1}(m) = 1 - (1 - (q_{s1})^n)^m$ (2.17)
- The probability of system failure in 2nd short mode $h_{s2}(m) = 1 - ((1 - (q_{s2})^n)^m)$ (2.18)

Similarly, from above sections, to maximizing the average system profit, it can be proved that the equation which calculates the optimal number of subsystems that maximizes the average system profit:

$$a[(1 - q_{o1})^n (1 - (1 - q_{o1})^n)^m + (1 - q_{o2})^n (1 - (1 - q_{o2})^n)^m] = (q_{s1})^n (1 - (q_{s1})^n)^m + (q_{s2})^n (1 - (q_{s1})^n)^m \quad (2.19)$$

1) First Case:

For $a < 1$:

Let $\beta=0.4$ $c_1=500$ $c_2=50$
 $c_3=400$ $c_4=80$ $q_{s2}=0.01$ $q_{o2}=0.01$
 $a=15/16$

The next figures show the variation of optimal number of subsystem m^* for different q_{s1} and q_{o1} at specific value of n

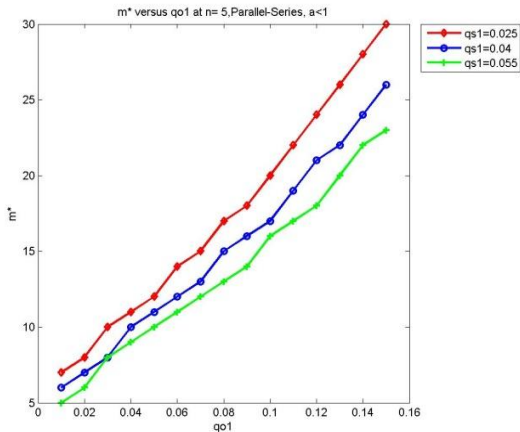


Figure 5: m^* versus q_{o1} at $n=5$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Parallel Series Configuration for $a < 1$)

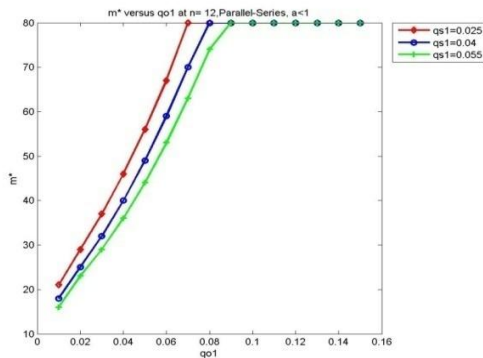


Figure 6: m^* versus q_{o1} at $n=12$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Parallel Series Configuration for $a < 1$)

From these figures, it can be concluded that for $a < 1$, m^* is decreasing as q_{s1} increases, also m^* is increasing with higher values of q_{o1} .

2) Second Case:

For $a > 1$:

$$\begin{aligned} \text{Let } \beta &= 0.6 & c_1 &= 500 & c_2 &= 50 \\ c_3 &= 400 & c_4 &= 80 & q_{s2} &= 0.01 & q_{o2} &= 0.01 \\ a &= 135/64 \end{aligned}$$

The next figures show the variation of optimal number of subsystem m^* for different q_{s1} and q_{o1} at specific value of n .

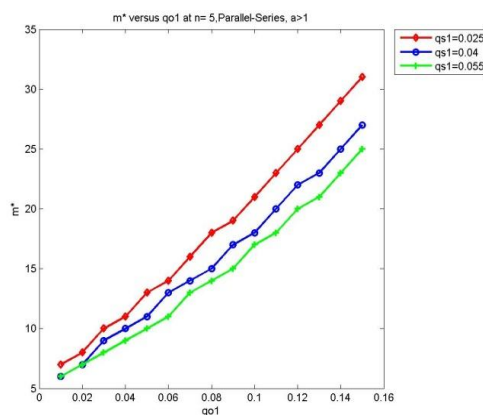


Figure 7: m^* versus q_{o1} at $n=5$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Parallel Series Configuration for $a > 1$)

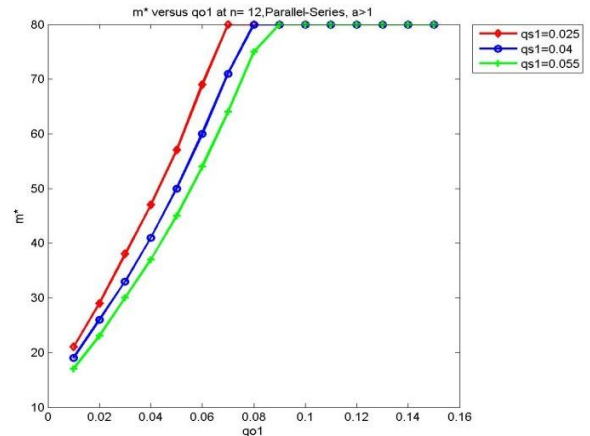


Figure 8: m^* versus q_{o1} at $n=12$ for $q_{s1}=0.025, 0.4,$ and 0.055 (Parallel Series Configuration for $a > 1$)

We can summarize the results as:

- 1) It can be concluded from the figures that for $a < 1$, m^* is decreasing as q_{s1} increases.
- 2) For $a > 1$, m^* is increasing as q_{o1} increases.
- 3) Case two ($a > 1$) required higher m^* .
- 4) m^* increases with β .

IV. CONCLUSIONS

There are many definitions to the Reliability of the system, still trying to quantify this expression. Redundancy doesn't increase the reliability in linear relation, i.e. increasing the number of components (n) or the subsystem (m), not always reach the maximum reliability. The optimal value, we try to achieve is the maximum reliability with minimum number of components. Redundancy has many shapes or arrangement (series, parallel, Series-Parallel, Parallel-Series, and k -out-of- n). Every arrangement has different relation or variation for the optimal value m^* . In Parallel-Series for $a < 1$, m^* is decreasing with q_{s1} whereas, Series-Parallel for $a < 1$, m^* is increasing with q_{s1} . For $a > 1$, m^* is increasing with q_{o1} in Parallel-Series, and decreasing in Series-Parallel.

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