Reliability-Based Analysis For Failure Modes Of Mansard Roof Truss According To Euro Code 3

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***Abstract:*-This paper shows the results of the safety assessment of Mansard steel roof truss using the First Order Reliability Methods. Four failure modes were considered in the studies: compression, bending, tension, as well as, combined compression and bending failure modes;**
From the results obtained from the First Order Reliability Method, the safety index values range between 0.076 to 11.90 for the members, while the values for the joints range from 0.677 to 1.89. A careful study of the results shows that the safety index values decreased as the load ratio increased and the safety index values at a particular load ratio are generally higher for the members than the joints. These results point to the fact that failure will be initialized at joint before progression to other members.

Keywords: Euro code, Failure mode, FORM, Reliability, Safety index, Stochastic model, Truss

1. INTRODUCTION

Steel Trusses are normally designed to support imposed, wind and dead loads. Various types of steel trusses such as Mansard, Storage, Scissors, Fink, Howe, and others are opted for in designs. These trusses are selected based on design criteria before being constructed. These criteria include amongst others their usage, cost, Span - to - depth and so on [1].
Engineering design and analysis are based often on deterministic thinking, characterized in design calculations by the use of specified minimum material properties, specified load intensities and by prescribed procedures for computing stresses and displacements. This has led to uncertainties, and conversely the application of prescribed safety factors in the analysis and design [2].
Because of these uncertainties, "risk", the probability of structural failure from all possible causes, both from violation of predefined limit state and from other causes is unavoidable. Though, uncertainties and their importance on structural safety and performance can be analysed thoroughly only through the technique of probability [3]. Reliability-based design involves incorporating these uncertainties in current methods of design, thereby using probability methods to design at pre-defined safety levels [4]. [5] looked at the reliability of structure by means of different target safety levels and probability distribution functions. The results showed that high target safety index makes it difficult to determine the safety factors such that the same safety level can be reached for different structures.
In this work, a reliability-based analysis for failure modes of steel mansard roof trusses to Euro code 3 [6] was carried out. The proposed design procedure considers the design of the mansard roof truss using First Order Reliability Method (FORM) at target safety levels which the current Codes of Practice do not provide.

2. LIMIT STATE EQUATIONS

The top chord is subjected to compression and bending. The bottom chord, tie member and strut member are subjected to tension and compression.

2.1 Limit State Equation due to Compression

The failure mode due to compression is thus:
G (x) = permissible compression load – applied compression load.
The design buckling resistance of a compression member should be taken as:

$$N_{b,Rd} = \frac{\chi A F_y}{Y_{m1}} \qquad (1)$$

Where $N_{b,Rd}$ is the permissible compression load, χ is the reduction factor, A, is the section area, Y_{m1} is partial factor and F_y is the yield strength.
For axial compression in members, the value χ of the appropriate non-dimensional slenderness λ should be determined from the relevant buckling curve according to (Euro code 3).

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \qquad (2)$$

But,

$$\phi = 0.5\,[1 + \alpha(\lambda - 0.2) + \lambda^{-2}]$$

[EC 3, 1993]

Where ϕ is value to determine the reduction factor, α is imperfection factor, λ is non-dimensional slenderness. Therefore,

$$\chi = \frac{1}{0.5[1 + \alpha(\lambda - 0.2) - \lambda^{-2}] + \sqrt{[0.5(1 + \alpha(\lambda - 0.2) + \lambda^{-2})^2 - \lambda^{-2}}} \qquad (3)$$

[EC 3, 1993]

$$\text{but,} \quad \lambda_1 = \pi\sqrt{E/F_y} = 93.9E \qquad (4)$$

$$E = \sqrt{235/F_y}$$

[EC3, 1993]

Where λ_1 is the slenderness value to determine the relative slenderness,
E is youngs modulus, F_y is the yield strength, ε is the Coefficient depending on yield strength
From

$$\lambda = \lambda_z/\lambda_1\beta_A^{0.5} \qquad (5)$$

where β_A is the correction factor, λ_z is the Slenderness ratio
therefore, equation (5) after substitution becomes

$$\chi = \frac{1/}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}–0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2} \left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}}\right)} \qquad (6)$$

From equation 1.

$$N_{b,Rd} = \frac{AF_y}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}–0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2} \left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}}\right)} \qquad (7)$$

[EC 3, 1993]

Therefore,

$$G(x)=N_{b,Rd}–N_{sd} \qquad (8)$$

$$N_{sd}=1.5xX(5)(1 + 0.9 \text{ x Alpha}). \qquad (9)$$

Where Alpha is the ratio of dead to live load , X(5) is Imposed Loadings.
Therefore,

$$G(x) = \frac{AF_y}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}–0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2} \left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}\,Y_{m1}}\right)}$$

$$- \; (1.5 \text{ x } X(5)x(1 + 0.9 \text{ x Alpha})) \qquad (10)$$

[EC 3, 1993]

The design value of the compression force must satisfy:

$$\frac{N_{sd}}{N_{b,Rd}} \leq 1.0 \qquad (11)$$

Where N_{sd} is the applied compression force, $N_{b,Rd}$ is permissible compression force.

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2.2 Limit State Equation for Bending

The failure mode due to bending is thus: G(x) permissible bending moment – applied bending.
The design resistance for bending about one principal axis of a cross section is determine as follows:

$$M_{c,Rd} = \frac{W_p F_y}{Y_{m1}} \qquad (12)$$

Where $M_{c,\,Rd}$ is the permissible bending moment, W_p is the Section Modulus, Fy is the yield strength, Y_{m1} is the partial factor.

But,

$$W_p = \frac{I_{xx}}{Y} \qquad (13)$$

Where I_{xx} is the moment of inertia, Y is the distance from the centroid axis to the top fibre.

But,

$$Y = d - C_x \qquad (14)$$

d is the depth of section, C_x is the centroid

Therefore,

$$I_{xx}=t_w(d-T_f)^3/12+3(d-T_f)t_w/2–(C_x+T_f)^2/12+BT_f^3/2+BT_f(C_x-T_f)^2 \qquad (15)$$

$$W_p=t_w(d-T_f)^3/12+3(d-T_f)t_w/2–(C_x+T_f)^2/12+BT_f^3/2+BT_f(C_x-T_f)^2/d-C_x \qquad (16)$$

$$M_{c,Rd}=(t_w(d-T_f)^3/12+3(d-T_f)t_w/2–(C_x+T_f)^2+BT_f^3/12+BT_f(C_x-T_f/2)^2)F_y)/(d-C_x)Y_{m1} \qquad (17)$$

Where t_w is the thickness of the web, T_f is the thickness of the flange, B is the width of the flange.
The design moment for bending moment must satisfy

$$\frac{M_{Ed}}{M_{c,\,Rs}} \leq 1.0 \qquad (18)$$

Where M_{Ed} is the design moment

$$M_{Ed} =1.5xX(5)xa(1+0.9Alpha)(L^2–a^2)/2L \qquad (19)$$

Where,
a is the Support distance, L is the Span length, Alpha is the ratio of live to dead load
The failure mode is thus,

$$G(x)=[tw(d-T_f)^3/12+3(d-T_f)tw/2–(C_x+T_f)^2+BT_f^3/12+BT_f(C_x-T_f/2)^2)F_y]/(d-Cx)Y_{m1}-[1.5xX(5)xa(1+0.9Alpha)(L^2–a^2)/2L)] \qquad (20)$$

2. 3 Limit State Equation for Top chord

The limit state equation due to compression and bending is thus:

$$G(x)=\frac{\text{Applied Compression}}{\text{Permissible compression}} + \frac{\text{Applied bending}}{\text{Permissible bending}} < 1.0 \qquad (21)$$

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$$\frac{Af(1.5xX(5)x\,(1+0.9Alpha)}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}-0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2-}\left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}}\;Y_{m1}\right)}$$

$$+$$

$(1.5\ x\ X(5)\ x\ a(1+0.9\ Alpha)\ (L^2-a^2)/2L)/\ tw(d-T_f)^3/12+3(d-T_f)tw/2\ -\ (C_x+T_f)^2+BT_f^3/12\ +\ BT_f(C_x\text{-}T_f/2)^2)F_y/(d-C_x)Y_{m1}$ (22)

2.4 Limit State Equation for Bottom Chord

The limit state equation for bottom Chord is thus:

G(x) = permissible tensile load – Applied tensile load

The design resistance for tension is given as follows:

$$N_{t,Rd}=\frac{Af_y}{Y_{m1}} \quad (23)$$

Where A is the area of the section, f_y is the Design Strength, Y_{m1} is the Partial factor. The design value for tension force N_{Ed} must satisfy:

$$\frac{N_{Ed}}{N_{E,Rd}} \leq 1.0 \quad (24)$$

Where N_{Ed} is the Applied tensile force.

$$N_{Ed} = 1.5xX(4)(1+0.9Alpha) \quad (25)$$

$$N_{E,Rd} = \frac{Af_y}{Y_{m1}} \quad (26)$$

Therefore,

$$\frac{1.5\ x\ X(4)x(1+0.9Alpha)}{Af_y/Y_{m1}} \leq 1.0 \quad (27)$$

$G(X) = N_{ERd} - N_{Ed}$

$$G(X) = AFy/\gamma_{m1} - 1.5\ x\ X(4)(1+0.9ALPHA) \quad (28)$$

2.5 Limit State Equation for Tie members

The limit state equation for tie member is thus:
G(x) = permissible tensile load – Applied tensile load
The design resistance for tension is given as follows:

$$N_{t,Rd}=Af_y/Y_{m1} \quad (29)$$

Where A is the area of the section, f_y is the Design Strength, Y_{m1} is the Partial factor
The design value for the tension force N_{Ed} must satisfy

$$\frac{N_{Ed}}{N_{E,Rd}} \leq 1.0$$

Where N_{Ed} is the Applied tensile force, N_{ERd} is the Permissible tensile load.

$$N_{Ed} = 1.5xX(4)(1+0.9Alpha) \quad (30)$$

$$N_{t,Rd} = Af_y/\ Y_{m1}$$

Therefore,

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$$\frac{1.5 \times X(4)x(1 + 0.9Alpha)}{Af_y/Y_{m1}} \leq 1.0 \quad (31)$$

$$G(X) = AFy/\gamma_{m1} - (1.5 \times X(4)(1+0.9ALPHA)) \quad (32)$$

2.6 Limit State Equation for Strut members

The failure mode due to Strut members is thus

G (x) = permissible compression load – applied compression load.

The design buckling resistance of a Strut member should be taken as:

$$N_{b,Rd} = \frac{\chi Afy}{Y_{m1}} \quad (33)$$

Where $N_{b,\,Rd}$ is the permissible compression load, χ is the reduction factor,
A is the section area, F_y is the yield strength, Y_{m1} is the partial factor.

For axial compression in members, the value of x the appropriate non-dimensional slenderness λ should be determined
from the relevant buckling curve according to

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \quad (34)$$

But,

$$\phi = 0.5[1+\alpha(\lambda - 0.2) + \lambda^{-2}] \quad (35)$$

where ϕ is the value to determine the reduction factor, α is the imperfection factor, λ is the non-dimensional slenderness.

Therefore,

$$\chi = \frac{1}{0.5[1 + \alpha(\lambda - 0.2) - \lambda^{-2}] + \sqrt{[0.5(1 + \alpha(\lambda - 0.2) + \lambda^{-2})^2 - \lambda^{-2}}} \quad (36)$$

but, $$\lambda_1 = \pi\sqrt{E/F_y} = 93.9E \quad (37)$$

$$E = \sqrt{235/F_y} \quad (38)$$

[EC 3, 1993]

Where λ_1 is the slenderness value to determine the relative slenderness, E is the young's modulus, F_y is the yield strength, ε is the Coefficient depending on yield strength.

from, $$\lambda = \lambda_z/\lambda_1\beta_A^{0.5} \quad (39)$$

Where β_A is the correction factor

$$\chi = 1/ \quad (40)$$

$$0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}-0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2 - \left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}}\right)}$$

$$N_{b,Rd} = \frac{AF_y}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}-0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2 - \left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}}\right)}} \quad (41)$$

therefore,

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$$G(x)=N_{b,Rd}-N_{sd} \quad (42)$$

$$N_{sd}=1.5xX(5)(1+0.9xAlpha) \quad (43)$$

Where Alpha is the ratio of dead to live load, X (5) is applied to load

Therefore:

$$G(x) = \frac{AF_y}{0.5[1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}-0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^2+\sqrt{(0.5(1+\alpha(\lambda_z/\lambda_1\beta_A^{0.5}0.2)+(\lambda_z/\lambda_1\beta_A^{0.5})^{-2})^2-}\left(\frac{\lambda_z}{\lambda_z\beta_A^{0.5}\,Y_{m1}}\right)}$$

$$-\quad (1.5xX(5)x(1+0.9xAlpha)) \quad (44)$$

The design value of the compression force satisfies,

$$\frac{N_{sd}}{N_{b,Rd}}\leq 1.0 \quad (45)$$

Where N_{sd} is the applied compression force, $N_{b,Rd}$ is the permissible compression force.

2.7. Limit state Equation for Connections

The limit state equation for the joints is thus:

$$G(X) = P_{weld} - F \quad (46)$$

$$P_{weld} = \frac{0.9\ x\ l\ x\ Ax\ fu\sqrt{2}}{\gamma_{mw}} \quad (47)$$

$$F = 1.5x\ X(5)a(1+0.9ALPHA)(L^2-a^2)/2L \quad (48)$$

Where l length of the weld (mm), A is the Throat thickness (mm), f_u is the Yield Strength (N/mm^2), γ_{mw} is the Partial Safety of factor.

Therefore, the limit state equation is given by:

$$G(X) = 0.9\frac{x\ l\ x\ A\ x\ f_u\ \ -\ \ 1.5\ x\ X(5)}{\gamma_{mw}}a(1+0.9ALPHA)(L^2-a^2)/2L \quad (49)$$

Where, l Length of the weld (mm), A is the Throat thickness (mm), f_u is the Yield Strength, ALPHA is the ratio of dead to live and; a is the Support distance; L is the Member length.

3.0 METHODOLOGY

3.1 First Order Reliability Procedure

The reliability function of a given surroundings is the probability that the system survives all the actions exerted upon it by the surroundings [7]. The reliability is defined as the systematic calculations and prediction of the probability of limit state violation [8]. Probabilistic design is concerned with the probability that a structure will realize the functions assigned to it. If R is the strength capacity and S the loading effect(s) of a structural system which are random variables, the key objective of the reliability index of any component is to confirm that R is at no time surpassed by S. in practice, R and S are typically functions of different basic variables [7]. To examine the effect of the variables on the performance of a structural system, there are five elementary steps required to arrive at a value for the reliability index β.

The limit state equation is referred to as the performance or state function and expressed as:

$$g(x_i) = g(x_1,x_2..., x_n) = R - S \quad (50)$$

where, xi for i=1, 2,…n, represent the basic design variables.

The limit state of the system can be expressed as:

$$g(x_i) = 0 \quad (51)$$

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Graphically, the line $g(x_i) = 0$ represents the failure surface while $g(x_i) > 0$ represents the safe region and $g(x_i) < 0$ corresponds to the failure region as depicted in Fig.1.
Adopting a reduced, normally distributed variable expression for each variate.

$$X_i = \frac{(x_i - \mu_{xi})}{\sigma_{xi}},\ i = 1,2,\ldots n \qquad (52)$$

and in terms of these reduced variates, the limit state equation becomes:

$$g(s_{xi}X`_1+\mu X_i, s_{x2}X`_2,+\mu_{x2},\ldots,s_{xn}X`_n+\mu_{xn})=0 \qquad (53)$$

where μ and S are the means and standard deviations of the design variables.

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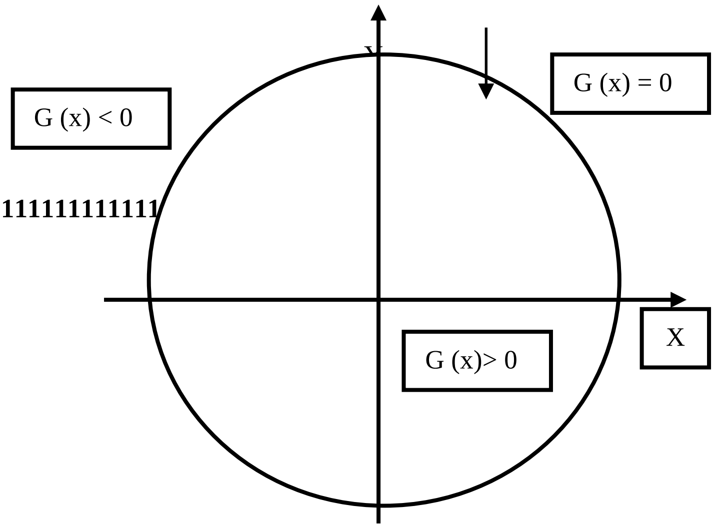

Fig.1: The most likely Failure Point [7]

Finding an expression of the distance from the failure surface to the origin of the reduced variate space: let the distance in question be D. the distance D, from a point $X_i = (X_1, X_2\ldots, X_n)$ on the failure surface $g(x_i) = 0$ to the origin x_i space is also given as:

$$D = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n)} \qquad (54)$$

in matrix form, $D = (X'_1 X'_2 \ldots X'_n) = \begin{vmatrix} X'_1 \\ X'_2 \\ X'_n \end{vmatrix})^{½}$ (55)

The point on the failure surface $(X'^*_1, X'^*_2\ldots, X^*_n)$, is minimized, having a distance to the origin may be determined by subjecting G(xi) = 0, by means of the Lagrange's multiplier and $g(x_i)$ is the limit state function. The minimum distance is obtained by introducing the gradient vector [7].

$$G^1 = \frac{\partial g}{\partial X'_1},\ \frac{\partial g}{\partial X'_2}, \ldots\ldots \frac{\partial g}{\partial X'_n} \qquad (56)$$

In which,

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X_i} \cdot \frac{\partial X'}{\partial X'_i} = \sigma x_i \frac{\partial g}{\partial X_i} \qquad (57)$$

Therefore, in vector form we have

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0 \qquad (58)$$

From which,

$$X' = \lambda DG \qquad (59)$$

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From equation (55)

$$D = [(\lambda D G^t) \ ((\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2} \qquad (60)$$

$$\lambda = (G^t G)^{1/2} \qquad (61)$$

Where G^t is the transpose of the gradient vector G. substituting equation (60) into equation (59) gives,

$$X' = \frac{-GD}{(G^t G)^{1/2}} \qquad (62)$$

Multiplying both sides of the (62) by G^t, the transpose of the gradient vector matrix, we have

$$G^t X' = \frac{-G^t G D}{(G^t G)^{1/2}} = -(G^t G)^{1/2} D \qquad (63)$$

which implies

$$D = \frac{-G^t X'}{(G^t G)^{1/2}} \qquad (64)$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals β and therefore equation (64) becomes

$$\beta = \frac{-G^{*t} X'^{*}}{(G^{*t} G^{*})^{1/2}} \qquad (65)$$

Where G* is the gradient vector at the most probable failure point ($X'_1{}^*$, $X_2'^*$,…$X_n'^*$). It is the value of β which tells us of the safety of any given design under uncertainties in the decision variables.
In equation (65), where basic variables are assumed to be normally distributed and uncorrelated, Taylor's expansion of G(x) about the mean yield quite precise approximations of β. Though, G(x) may contain non-normally distributed and frequently correlated variables, the accuracy of the estimates of β be subject to heavily on the choice of the point of linearization. FORM procedures resolve this problem by introducing suitable transformations altering all variables into uncorrelated normal variables, subsequently linearizing the emergent non-linear function G(x) about a suitable point X*, selected through an optimization procedure, and hence a probability of failure is evaluated using standard normal integral.

4.0 Program Data Preparation

The mansard truss shown in Fig. 2 was designed following EC3. The means, standard deviations as well as the statistical distributions of the basic design variables were prepared as shown in Tables 1 to 5 by [7]

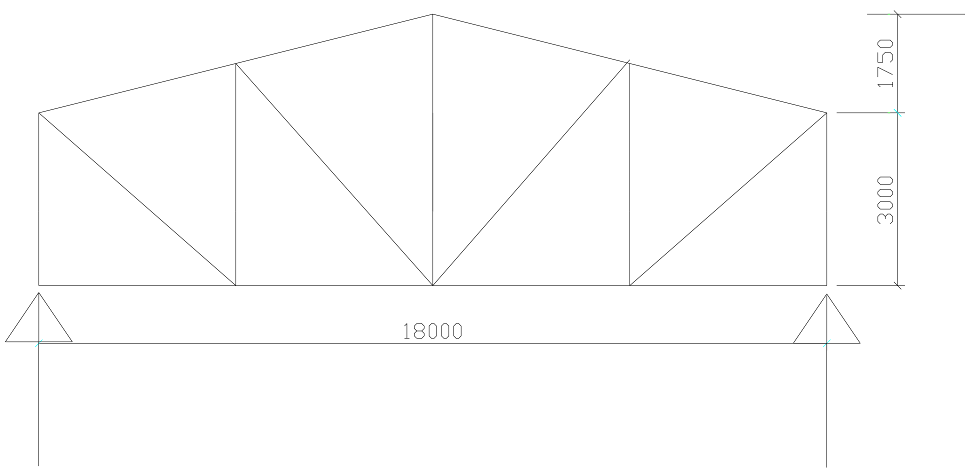

Fig. 2. Steel Mansard Truss

4.1 Parameters of Stochastic Model

The values of data used in the models are presented in Tables 1, 2,3,4 and 5.

Table 1: Parameters of Stochastic Model for Top Chord

S/N	BASIC VARIABLES	$E(X_i)$	$S(X_i)$	DISTRIBUTION	COV
1	Design strength F_y	$275N/mm^2$	$13.75N/mm^2$	Normal	0.05
2	Width of flange B	190.5mm	9.525mm	Normal	0.05
3	The thickness of flange T_f	14.5mm	0.725mm	Normal	0.05
4	The thickness of web t	9.1mm	0.455mm	Normal	0.05
5	Imposed load Q_k	1.5KN/m	0.225KN/m	Log-normal	0.15
6	Depth of section d	228.6mm	11.43mm	Normal	0.05
7	Centriode C_x	54.2mm	2.71mm	Normal	0.05
8	Root radius r	7.6mm	2.28mm	Log-normal	0.3

Table 2: Parameters of Stochastic Model for Bottom Chord

S/N	BASIC VARIABLES	$E(X_i)$	$S(X_i)$	DISTRIBUTION	COV
1	Design strength F_y	$275N/mm^2$	$13.75N/mm^2$	Normal	0.05
2	Width of flange B	190.5mm	9.525mm	Normal	0.05
3	The thickness of flange T_f	14.5mm	0.725mm	Normal	0.05
4	The thickness of web t_w	9.1mm	0.455mm	Normal	0.05
5	Live load Q_k	1.5KN/m	0.45KN/m	Log-normal	0.15
6	Root radius r	7.6mm	0.456mm	Log-normal	0.3
7	Depth of section d	228.6mm	11.43mm	Normal	0.05

Table 3: Parameters of Stochastic Model for Tie Member

S/N	BASIC VARIABLES	$E(X_i)$	$S(X_i)$	DISTRIBUTION	COV
1	Design strength F_y	$275N/mm^2$	$13.75N/mm^2$	Normal	0.05
2	Member length l	120mm	6.0mm	Normal	0.05
3	Member thickness t	8mm	0.4mm	Normal	0.05
4	Live load Q_k	1.5KN/m	0.225KN/m	Log-normal	0.15

Table 4: Parameters of Stochastic Model for Strut Members

S/N	BASIC VARIABLES	$E(X_i)$	$S(X_i)$	DISTRIBUTION	COV
1	Design strength F_y	$275N/mm^2$	$13.75N/mm^2$	Normal	0.05
2	Member length h	120mm	6mm	Normal	0.05
3	Member thickness t	10mm	0.5mm	Normal	0.05
4	Heel root radius r_1	10mm	0.5mm	Log-Normal	0.05
5	Toe radius r_2	5mm	1.5mm	Log-normal	0.3
6	Applied loading Q_k	1.5KN/m	0.225KN/m	Log-normal	0.15

Table 5: Parameters of Stochastic Model for Connections

S/N	BASIC VARIABLES	$E(X_i)$	$S(X_i)$	DISTRIBUTION	COV
1	Design strength F_y	$275N/mm^2$	$13.75N/mm^2$	Normal	0.05
2	Length of weld L	639mm	31.95mm	Normal	0.05
3	The thickness of weld t	56.6mm	2.83mm	Normal	0.05
4	Imposed Load Q_k	1.5KN/m	0.225KN/m	Log-normal	0.15

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Table 6: Safety Index values, β, and their corresponding probabilities of Failure, P_f, for Members

Top Chord Members

Member	Alpha 0.2		Alpha 0.4		Alpha 0.6		Alpha 0.8		Alpha 0.9		Alpha 1.0	
	β	P_f	β	P_f	β	P_f	β	P_f	β	P_f	β	P_f
a-2	8.67	$0.222x10^{-17}$	7.30	$0.143x10^{-12}$	6.50	$0.402x10^{-10}$	5.93	$0.150x10^{-8}$	5.70	$0.606x10^{-8}$	5.49	$0.201x10^{-7}$
b-4	8.52	$0.796x10^{-17}$	7.16	$0.425x10^{-12}$	6.35	$0.106x10^{-9}$	5.78	$0.368x10^{-8}$	5.55	$0.143x10^{-7}$	5.35	$0.462x10^{-7}$
BOTTOM CHORD MEMBERS												
m-1	10.0	$0.713x10^{-23}$	5.96	$0.130x10^{-8}$	3.56	$0.185x10^{-3}$	1.85	$0.320x10^{-1}$	1.15	0.125	0.524	0.300
m-3	9.63	$0.293x10^{-21}$	5.58	$0.124x10^{-7}$	3.18	$0.744x10^{-3}$	1.47	$0.711x10^{-1}$	0.766	0.222	0.138	0.445
TIE MEMBERS												
1-2	10.70	$0.497x10^{-26}$	6.84	$0.395x10^{-11}$	4.53	$0.294x10^{-5}$	2.88	$0.199x10^{-2}$	2.19	$0.142x10^{-1}$	1.52	$0.554x10^{-1}$
3-4	10.07	$0.350x10^{-23}$	6.17	$0.345x10^{-9}$	3.85	$0.585x10^{-4}$	2.19	$0.140x10^{-1}$	1.52	$0.644x10^{-1}$	0.908	0.181
STRUT MEMBERS												
1-a	9.42	$0.237x10^{-20}$	5.49	$0.197x10^{-7}$	3.11	$0.945x10^{-3}$	1.41	$0.793x10^{-1}$	0.764	0.222	0.076	0.470
2-3	10.70	$0.497x10^{-26}$	6.89	$0.282x10^{-11}$	4.55	$0.274x10^{-5}$	2.87	$0.204x10^{-2}$	2.25	$0.142x10^{-1}$	1.56	$0.597x10^{-1}$
4-5	11.90	$0.639x10^{-32}$	8.35	$0.342x10^{-16}$	6.06	$0.701x10^{-9}$	4.41	$0.529x10^{-5}$	3.73	$0.956x10^{-4}$	3.11	$0.945x10^{-3}$

Table 7: Safety Index values, β, and their corresponding probabilities of Failure, P_f, for Joints

Joint	Member	Alpha=0.2		Alpha=0.4		Alpha=0.6		Alpha=0.8		Alpha=0.9		Alpha=1.0	
		β	Pf	β	Pf	β	Pf	β	Pf	β	Pf	β	Pf
1	NM	1.89	$0.296x10^{-1}$	1.77	$0.383x10^{-1}$	1.65	$0.492x10^{-1}$	1.53	$0.626x10^{-1}$	1.47	$0.704x10^{-1}$	1.41	$0.788x10^{-1}$
	NP	1.86	$0.316x10^{-1}$	1.71	$0.435x10^{-1}$	1.53	$0.626x10^{-1}$	1.41	$0.788x10^{-1}$	1.34	$0.905x10^{-1}$	1.26	0.103
2	MN	1.83	$0.337x10^{-1}$	1.65	$0.492x10^{-1}$	1.47	$0.704x10^{-1}$	1.29	$0.981x10^{-1}$	1.20	0.115	1.11	0.133
3	OP	1.80	$0.359x10^{-1}$	1.59	$0.556x10^{-1}$	1.38	$0.834x10^{-1}$	1.17	0.121	1.07	0.143	0.964	0.167
	OR	1.83	$0.337x10^{-1}$	1.65	$0.492x10^{-1}$	1.47	$0.704x10^{-1}$	1.29	$0.981x10^{-1}$	1.20	0.115	1.11	0.133
4	PO	1.89	$0.296x10^{-1}$	1.77	$0.383x10^{-1}$	1.65	$0.492x10^{-1}$	1.53	$0.626x10^{-1}$	1.47	$0.704x10^{-1}$	1.41	$0.788x10^{-1}$
	PN	1.86	$0.316x10^{-1}$	1.71	$0.435x10^{-1}$	1.53	$0.626x10^{-1}$	1.41	$0.788x10^{-1}$	1.34	$0.905x10^{-1}$	1.26	0.103
5	QR	1.83	$0.337x10^{-1}$	1.65	$0.492x10^{-1}$	1.47	$0.704x10^{-1}$	1.29	$0.981x10^{-1}$	1.20	0.115	1.11	0.133
6	RQ	1.86	$0.316x10^{-1}$	1.71	$0.435x10^{-1}$	1.53	$0.626x10^{-1}$	1.41	$0.788x10^{-1}$	1.34	$0.905x10^{-1}$	1.26	0.103
	RO	1.89	$0.296x10^{-1}$	1.77	$0.383x10^{-1}$	1.65	$0.492x10^{-1}$	1.53	$0.626x10^{-1}$	1.47	$0.704x10^{-1}$	1.41	$0.788x10^{-1}$

4.2. Data from componential reliability analysis

Since the member forces are expressed in terms of the dead to live load ratio, the ratio was varied from 0.2 to 1.0 and necessary computation using FORM was carried out to determine the probability of failure "P_f'' and the corresponding safety index, β. This was done for the Top Chord members, Bottom Chord members, Web members, as well as the Joints. The results are tabulated in Tables 6 and 7. From Table 6 the beta values range 0.076 to 11.90 for the members, while the values for the joints in Table 7 range from 0.677 to 1.89. A careful study of the results shows that the β values decreases as the load ratio increases and the β values at a particular load ratio are generally higher for the members than the joints. These results point to the fact that failure will be initialized at joint before progression to other members.

A closer look at the results in Table 6 show that the safety levels (β values) at all α-levels, exhibit the following trend: The Strut members are the safest having the highest safety level, Tie members come next in succession; The Bottom Chord members are next with the Top chord following. Failure in the members will be in progression, starting with the members having the least safety level. It should be borne in mind that since it is statically determinate structures, failure of any member or joint denotes truss failure [8].

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5.0 CONCLUSION AND RECOMMENDATIONS

The reliability-based design of Euro code 3 procedures for a Mansard roof truss system has been carried out with the aid of computer programs in the FORTRAN language. The FORM of analysis for the elements was carried using FORM5 [7].

From the results obtained for the First Order Reliability Method, the safety indices range between 0.076 to 11.90 for the members, while the values for the joints range from 0.677 to 1.89. Joint failure of a mansard roof truss designed by Euro code 3, will therefore be initiated before progressing to members under increased loading.

It is recommended that:

- Mansard roof trusses should be designed at a lower value of ratio dead- to – live loads.
- the design of the joints in Mansard Steel roof trusses should be considered more seriously.
- The reliability-based design needs to be carried out considering other wind speed in different Nations.

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