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Reliability Assessment of Solar Power System by Employing Algebra of Logics

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***Abstract*--The author has been used algebra of logics for the formulation and solution of mathematical model of solar power system. Reliability of the whole system has been obtained. Reliability function for the system as a whole has been computed in two different cases e.g., when failures follow Weibull and exponential time distribution. An important reliability parameter, mean time to system failure (M.T.T.F.), has also been obtained to improve practical utility of the model. A numerical computation together with its graphical illustration has been mentioned in the end to highlight important results of the study.**

Keyword- Reliability Assessment, Solar Power System, Algebra of Logics, Boolean Algebra etc.

I. Introduction

In this model, the author has considered a solar power system for its reliability analysis by using algebra of logics. System configuration of considered system has been shown in fig-1. In this system, there are two solar panels in standby redundancy. On failure of solar panel x_1 we can follow online solar panel x_2 with the help of perfect switching device x_3. These solar panels produce the solar energy and it can be saved in four batteries B_1, B_2, B_3 and B_4. There are two automatic switching devices ASD_1 and ASD_2 to change the battery links. Batteries supply DC power to the sine-wave inverter x_{10} ,that converts this DC power into AC power. Finally, this AC power feed the connected electric equipments. The cables used to connect any two equipments are hundred percent reliable. Note that any one battery is sufficient to feed all the consumer requirements.

II. Assumptions

The following assumptions have been associated with this model:

(1) Initially, all the components are good and operable.
(2) Reliability of every component of the system is known in advance.
(3) Each component will remain either good or bad.
(4) Transaction of power from one component to any other is 100% reliable.
(5) Switching device used to online standby solar panel is perfect.
(6) Failures are statistically-independent.

SOLAR SYSTEM

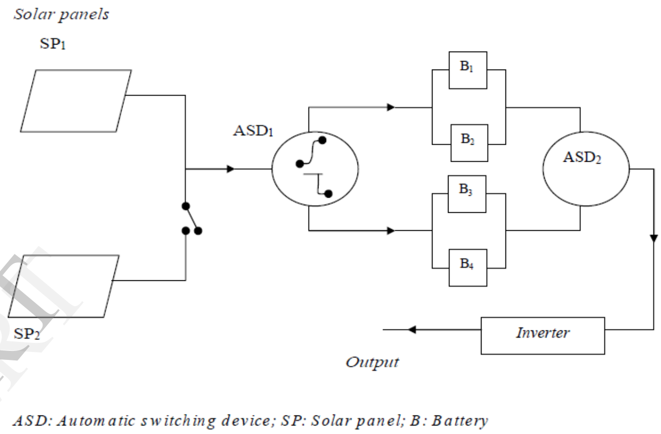

Fig-1 (System Configuration)

III. Notations Used

Following notations have been used throughout this study:

x_1, x_2	States of solar panels.
x_3	State of switching device.
x_5, x_6, x_7, x_8	States of batteries.
x_4, x_9	States of automatic switching devices.
x_{10}	State of sine-wave inverter.
x_i	0 in bad state, 1 in good state.
$\wedge / \vee$	Conjunction/disjunction.
R_S	Reliability of whole system.
R_i	Reliability corresponding to system state x_i.
$R_{SW}(t) / R_{SE}(t)$	Reliability functions for whole system, in case failures follow weibull/exponential time distribution.
$M.T.T.F.$	Mean time to failure.

IV. Formulation of Mathematical Model

The possible minimal paths for successful operation of the system, in terms of logical matrix, can be expressed as:

$$F(x_1, x_2 --, x_{10}) = \begin{bmatrix} x_1 & x_4 & x_5 & x_9 & x_{10} & \\ x_1 & x_4 & x_6 & x_9 & x_{10} & \\ x_1 & x_4 & x_7 & x_9 & x_{10} & \\ x_1 & x_4 & x_8 & x_9 & x_{10} & \\ x_2 & x_3 & x_4 & x_5 & x_9 & x_{10} \\ x_2 & x_3 & x_4 & x_6 & x_9 & x_{10} \\ x_2 & x_3 & x_4 & x_7 & x_9 & x_{10} \\ x_2 & x_3 & x_4 & x_8 & x_9 & x_{10} \end{bmatrix} \quad \ldots(1)$$

V. Solution Of The Model

We can write the equation (1) again as :

$$F(x_1, x_2 --, x_{10}) = \left| x_4 \;\; x_9 \;\; x_{10} \wedge f(x_1, x_2 --, x_{10}) \right| \quad \ldots(2)$$

where,

$$f(x_1, x_2 --, x_{10}) = \begin{bmatrix} x_1 & x_5 & \\ x_1 & x_6 & \\ x_1 & x_7 & \\ x_1 & x_8 & \\ x_2 & x_3 & x_5 \\ x_2 & x_3 & x_6 \\ x_2 & x_3 & x_7 \\ x_2 & x_3 & x_8 \end{bmatrix} \quad \ldots(3)$$

Substituting

$$B_1 = [x_1 \quad x_5] \quad \ldots(4)$$

$$B_2 = [x_1 \quad x_6] \quad \ldots(5)$$

$$B_3 = [x_1 \quad x_7] \quad \ldots(6)$$

$$B_4 = [x_1 \quad x_8] \quad \ldots(7)$$

$$B_5 = [x_2 \quad x_3 \quad x_5] \quad \ldots(8)$$

$$B_6 = [x_2 \quad x_3 \quad x_6] \quad \ldots(9)$$

$$B_7 = [x_2 \quad x_3 \quad x_7] \quad \ldots(10)$$

$$B_8 = [x_2 \quad x_3 \quad x_8] \quad \ldots(11)$$

in equation (3) and using orthogonalisation algorithm, we obtain:

$$f(x_1, x_2 --, x_{10}) = \begin{bmatrix} B_1 & & & & & & & \\ B_1' & B_2 & & & & & & \\ B_1' & B_2' & B_3 & & & & & \\ B_1' & B_2' & B_3' & B_4 & & & & \\ B_1' & B_2' & B_3' & B_4' & B_5 & & & \\ B_1' & B_2' & B_3' & B_4' & B_5' & B_6 & & \\ B_1' & B_2' & B_3' & B_4' & B_5' & B_6' & B_7 & \\ B_1' & B_2' & B_3' & B_4' & B_5' & B_6' & B_7' & B_8 \end{bmatrix} \quad \ldots(12)$$

Now we can obtain by using algebra of logics

$$B_1' = \begin{bmatrix} x_1' & \\ x_1 & x_5' \end{bmatrix}$$

$$\therefore B_1' B_2 = \begin{bmatrix} x_1' & \\ x_1 & x_5' \end{bmatrix} \wedge [x_1 \quad x_6]$$

$$= [x_1 \quad x_5' \quad x_6] \quad \ldots(13)$$

Similarly, we compute the following:

$$B_1' B_2' B_3 = [x_1 \quad x_5' \quad x_6' \quad x_7] \quad \ldots(14)$$

$$B_1' B_2' B_3' B_4 = [x_1 \quad x_5' \quad x_6' \quad x_7' \quad x_8] \quad \ldots(15)$$

$$B_1' B_2' B_3' B_4' B_5 = [x_1' \quad x_2 \quad x_3 \quad x_5] \quad \ldots(16)$$

$$B_1' B_2' B_3' B_4' B_5' B_6 = [x_1' \quad x_2 \quad x_3 \quad x_5' \quad x_6] \quad \ldots(17)$$

$$B_1' B_2' B_3' B_4' B_5' B_6' B_7 = [x_1' \quad x_2 \quad x_3 \quad x_5' \quad x_6' \quad x_7] \quad \ldots(18)$$

and

$$B_1' B_2' B_3' B_4' B_5' B_6' B_7' B_8 = [x_1' \quad x_2 \quad x_3 \quad x_5' \quad x_6' \quad x_7' \quad x_8] \quad \ldots(19)$$

Making use of equations (4) and (13) through (19), equation (12) becomes:

$$f(x_1, x_2 --, x_{10}) = \begin{bmatrix} x_1 & x_5 & & & & & \\ x_1 & x_5' & x_6 & & & & \\ x_1 & x_5' & x_6' & x_7 & & & \\ x_1 & x_5' & x_6' & x_7' & x_8 & & \\ x_1' & x_2 & x_3 & x_5 & & & \\ x_1' & x_2 & x_3 & x_5' & x_6 & & \\ x_1' & x_2 & x_3 & x_5' & x_6' & x_7 & \\ x_1' & x_2 & x_3 & x_5' & x_6' & x_7' & x_8 \end{bmatrix}$$

...(20)

Using (20), equation (2) gives:

$$F(x_1, x_2 --, x_{10}) = \begin{bmatrix} x_1 & x_4 & x_5 & x_9 & x_{10} & & & & \\ x_1 & x_4 & x_5' & x_6 & x_9 & x_{10} & & & \\ x_1 & x_4 & x_5' & x_6' & x_7 & x_9 & x_{10} & & \\ x_1 & x_4 & x_5' & x_6' & x_7' & x_8 & x_9 & x_{10} & \\ x_1' & x_2 & x_3 & x_4 & x_5 & x_9 & x_{10} & & \\ x_1' & x_2 & x_3 & x_4 & x_5' & x_6 & x_9 & x_{10} & \\ x_1' & x_2 & x_3 & x_4 & x_5' & x_6' & x_7 & x_9 & x_{10} \\ x_1' & x_2 & x_3 & x_4 & x_5' & x_6' & x_7' & x_8 & x_9 & x_{10} \end{bmatrix}$$

...(21)

Since, R.H.S. of equation (21) is disjunction of pair-wise disjoint conjunctions, therefore, the reliability of whole solar system is given by:

$$R_S = P_r\{F(x_1, x_2 --, x_{10}) = 1\} =$$

$$R_4R_9R_{10}[R_1R_5 + Q_5R_1R_6 + Q_5Q_6R_1R_7 + Q_5Q_6Q_7R_1R_8 + Q_1R_2R_3R_5$$
$$+ Q_1Q_5R_2R_3R_6 + Q_1Q_5Q_6R_2R_3R_7 + Q_1Q_5Q_6Q_7R_2R_3R_8]$$

where

R_i = reliability of the component corresponding to system state x_i

and $Q_i = 1 - R_i$

Thus, we may write

$$R_S = R_4R_9R_{10}[R_1R_5 + R_1R_6 + R_1R_7 + R_1R_8 + R_2R_3R_5 + R_2R_3R_6 + R_2R_3R_7 + R_2R_3R_8$$
$$+ R_1R_5R_6R_7 + R_1R_5R_6R_8 + R_1R_5R_7R_8 + R_1R_6R_7R_8 + R_1R_2R_3R_5R_6$$
$$+ R_1R_2R_3R_5R_7 + R_1R_2R_3R_6R_7 + R_2R_3R_5R_6R_7 + R_1R_2R_3R_5R_8$$
$$+ R_1R_2R_3R_6R_8 + R_1R_2R_3R_7R_8 + R_2R_3R_5R_6R_8 + R_2R_3R_5R_7R_8$$
$$+ R_2R_3R_6R_7R_8 + R_1R_2R_3R_5R_6R_7R_8 - R_1R_5R_6 - R_1R_5R_7$$
$$- R_1R_6R_7 - R_1R_5R_8 - R_1R_6R_8 - R_1R_7R_8 - R_1R_2R_3R_5 - R_1R_2R_3R_6$$
$$- R_2R_3R_5R_6 - R_1R_2R_3R_7 - R_2R_3R_5R_7 - R_2R_3R_6R_7 - R_1R_2R_3R_8$$
$$- R_2R_3R_5R_8 - R_2R_3R_6R_8 - R_2R_3R_7R_8 - R_1R_5R_6R_7R_8 - R_1R_2R_3R_5R_6R_7$$
$$- R_1R_2R_3R_5R_6R_8 - R_1R_2R_3R_5R_7R_8 - R_1R_2R_3R_6R_7R_8 - R_2R_3R_5R_6R_7R_8]$$

...(22)

VI. Some Particular Cases

CASE I: When each component has the reliability R:

In this case, the reliability of the whole system can be obtained from equation (22),

$$R_S = 4R^5 - 2R^6 - 6R^7 + 9R^8 - 5R^9 + R^{10}$$

...(23)

CASE II: When failure rates follow weibull time distribution:

In this case, the reliability of the whole system is given by:

$$\mathrm{R_{SW}}(t) = \sum_{i=1}^{23} \exp.\{-a_i t^\alpha\} - \sum_{j=1}^{22} \exp.\{-b_j t^\alpha\}$$

...(24)

where α is a real positive parameter and

$a_1 = c + \lambda_1 + \lambda_5$

$a_2 = c + \lambda_1 + \lambda_6$

$a_3 = c + \lambda_1 + \lambda_7$

$a_4 = c + \lambda_1 + \lambda_8$

$a_5 = c + \lambda_2 + \lambda_3 + \lambda_5$

$a_6 = c + \lambda_2 + \lambda_3 + \lambda_6$

$a_7 = c + \lambda_2 + \lambda_3 + \lambda_7$

$a_8 = c + \lambda_2 + \lambda_3 + \lambda_8$

$a_9 = c + \lambda_1 + \lambda_5 + \lambda_6 + \lambda_7$

$a_{10} = c + \lambda_1 + \lambda_5 + \lambda_6 + \lambda_8$

$a_{11} = c + \lambda_1 + \lambda_5 + \lambda_7 + \lambda_8$

$a_{12} = c + \lambda_1 + \lambda_6 + \lambda_7 + \lambda_8$

$$a_{13} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6$$
$$a_{14} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7$$
$$a_{15} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7$$
$$a_{16} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$$
$$a_{17} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_8$$
$$a_{18} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_8$$
$$a_{19} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_7 + \lambda_8$$
$$a_{20} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8$$
$$a_{21} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8$$
$$a_{22} = c + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 + \lambda_8$$
$$a_{23} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$$
$$b_1 = c + \lambda_1 + \lambda_5 + \lambda_6$$
$$b_2 = c + \lambda_1 + \lambda_5 + \lambda_7$$
$$b_3 = c + \lambda_1 + \lambda_6 + \lambda_7$$
$$b_4 = c + \lambda_1 + \lambda_5 + \lambda_8$$
$$b_5 = c + \lambda_1 + \lambda_6 + \lambda_8$$
$$b_6 = c + \lambda_1 + \lambda_7 + \lambda_8$$
$$b_7 = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5$$
$$b_8 = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6$$
$$b_9 = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6$$
$$b_{10} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_7$$
$$b_{11} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7$$
$$b_{12} = c + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7$$
$$b_{13} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_8$$
$$b_{14} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_8$$
$$b_{15} = c + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_8$$
$$b_{16} = c + \lambda_2 + \lambda_3 + \lambda_7 + \lambda_8$$
$$b_{17} = c + \lambda_1 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$$
$$b_{18} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$$
$$b_{19} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8$$
$$b_{20} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8$$
$$b_{21} = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 + \lambda_8$$
$$b_{22} = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$$

and $c = \lambda_4 + \lambda_9 + \lambda_{10}$

where λ_i is the failure rate of system state $x_i, \forall i = 1,2,--10$

CASE III: When failures follow exponential time distribution:

Exponential time distribution is the particular case of weibull time distribution for $\alpha = 1$ and is very useful in numerous practical problems. Therefore the reliability function for the whole system at any time‘t’, in this case, is given by:

$$\mathrm{R}_{\mathrm{SE}}(t) = \sum_{i=1}^{23} \exp.\{-a_i t\} - \sum_{j=1}^{22} \exp.\{-b_j t\} \quad \ldots(25)$$

where $a_i's$ and $b_j's$ have been mentioned earlier.

Also, in this case, an important reliability parameter M.T.T.F , is given by

$$M.T.T.F. = \int_0^\infty R_{SE}(t)dt$$

$$= \sum_{i=1}^{23} \frac{1}{a_i} - \sum_{j=1}^{22} \frac{1}{b_j} \quad \ldots(26)$$

VII. Results and Conclusion

For a numerical computation, let us consider the values:

(i) $\lambda_i(i = 1,2,--10) = \lambda = 0.001,\ \mathrm{t} = 0,1,2---$ and $\alpha = 2$. Using these values in equation (24), we compute the table -1. The corresponding graph has been shown in fig-2.

(ii) $\lambda_i(i = 1,2,3--10) = \lambda = 0.001$, and $\mathrm{t} = 0,1,2---$. Using these values in equation (25), we compute the table-1 and the corresponding graph has been shown in fig-2.

(iii) Putting $\lambda_i(i = 1,2,3--10) = \lambda = 0.001, 0.002\ ---0.01$ in equation (26), we compute table-2 and its graphical reorientation has been shown in fig-3.

Analysis of table -1 and fig-2 reveals that values of reliability function decreases catastrophically, in case, failures follow weibull time distribution but it decreases approximately in constant manner for exponential time distribution. Therefore, reliability function remains better when failures follow exponential time distribution.

A critical examination of table -2 and fig-3 concludes that M.T.T.F. decreases rapidly for the lower values of failure rate λ but it decreases smoothly for higher values of λ.

TABLE-1

t	$R_{SW}(t)$	$R_{SE}(t)$
0	1	1
1	0.997003	0.997003
2	0.988040	0.994010
3	0.973206	0.991023
4	0.952657	0.988040
5	0.926626	0.985063
6	0.895421	0.982091
7	0.859436	0.979124
8	0.819147	0.976162
9	0.775117	0.973206
10	0.727979	0.970254

Fig-2

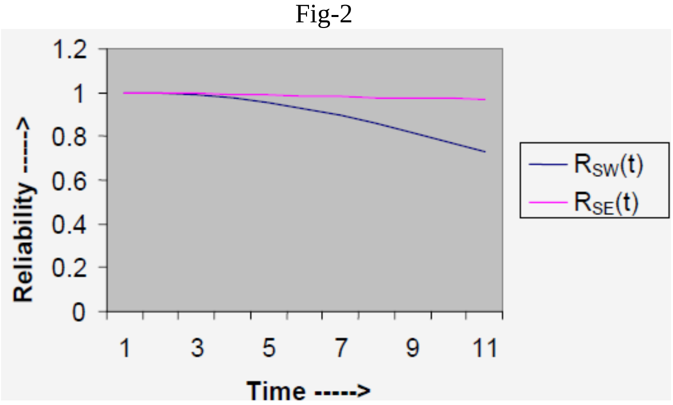

TABLE-2

λ	M.T.T.F.
0	∞
0.001	278.9683
0.002	139.4841
0.003	92.9894
0.004	69.7421
0.005	55.7937
0.006	46.4947
0.007	39.8526
0.008	34.8710
0.009	30.9965
0.010	27.8968

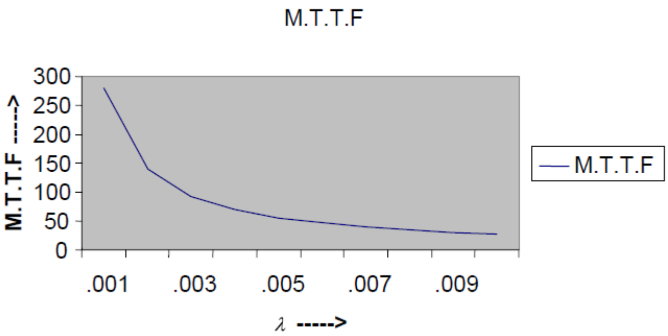

Fig-3

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