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Reliability and Availability for Non-Repairable & Repairable Systems using Markov Modelling

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***Abstract*—Markov model provides great flexibility in modelling the timing of events. Markov analysis is a method of analysis that can be applied to both repairable and non-repairable types of systems. In this paper, Markov modelling technique is used to compute the reliability for non-repairable system and defined the mean time to failure of non-repairable systems with different failure rates. This technique is also used to compute the steady-state availability for repairable systems and to derived the mean time between failure of repairable systems with different failure rates and repair rates.**

Keywords*— Markov model; Repairable and Non-repairable systems; MTTF; MTBF; Failure rate; Repair rate; Reliability;

INTRODUCTION

Markov analysis is the mathematical abstractions to model simple or complex concepts in quite easily computable form. The Markov analysis is also considered powerful modelling and analysis tool in solving reliability tribulations. Markov analysis is a tool for modelling complex system designs involving timings, sequence, repair, redundancy and fault-tolerance.
Mean Time To Failure (MTTF) describes the expected time to failure for a non-repairable system. MTTF is commonly refer to as the life time of any product or a device. MTTF can be mathematically calculated by

$$MTTF = \int_0^\infty R(t)\,dt$$

Mean Time Between Failure (MTBF) is the predicted time that passes between one previous failure of a mechanical/electrical system to the next failure during normal operation or, the time between one system breakdown and the next.
James Li [1] & [2] derived the reliability for a parallel redundant system with different failure rate & repair rate using Markov modelling and reliability comparative evaluation of active redundancy vs. Standby redundancy respectively. M.A. El-Damcese and N.S. Temraz [3] studied analysis for a parallel repairable system with different failure modes. Jacob Cherian et al [4] has described reliability of a standby system with repair. Garima Chopra [5] studied reliability measures of two dissimilar units parallel system using Gumbel-Hougaard family copula. M.A. El-Damcese et al [6] estimated availability and reliability analysis of three elements parallel system with fuzzy failure and repair rate.

MATHEMATICAL MODEL:

(i) 3-component non-repairable system with different failure rates

A non-repairable system has finite life time. The probability of a system failure will increase over time during system operation. A non-repairable system remains failed, after it failed once.

3-component system can have $2^3 = 8$ distinct states. The failure rates are λ_1, λ_2 and λ_3. The transition rate diagram is given below.

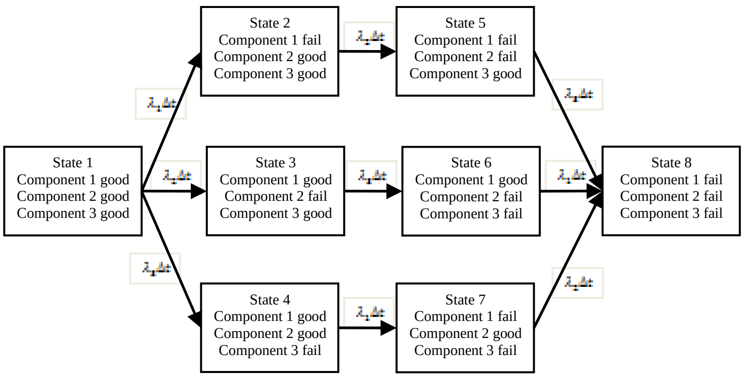

Figure 1: Non-repairable system state diagram

The probability of the system state one i.e., the probability of 3-component good at time is

$$P_1(t+\Delta t) = (1-(\lambda_1+\lambda_2+\lambda_3)\Delta t)P_1(t)$$

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = -(\lambda_1+\lambda_2+\lambda_3)P_1(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_1(t)}{dt} = -(\lambda_1+\lambda_2+\lambda_3)P_1(t)$$

Solving the above equation by using the Laplace transformations, and taking assumption that the system good at initial time $t = 0$, hence $P_1(0) = 1$

$$P_2(0) = 0, P_3(0) = 0, P_4(0) = 0, P_5(0) = 0, P_6(0) = 0, \& P_7(0) = 0$$

then the following equation is obtained

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$$P_1(t) = e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of the system in state two at $t+\Delta t$ time is

$$P_2(t+\Delta t) = (\lambda_1 \Delta t)P_1(t) + (1-\lambda_2 \Delta t)P_2(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t)$$

Taking Laplace transform, the equation is

$$P_2(s) = \frac{\frac{\lambda_1}{(\lambda_1+\lambda_3)}}{(s+\lambda_2)} - \frac{\frac{\lambda_1}{(\lambda_1+\lambda_3)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform

$$P_2(t) = \frac{\lambda_1}{(\lambda_1+\lambda_3)}e^{-(\lambda_2)t} - \frac{\lambda_1}{(\lambda_1+\lambda_3)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of the system in state three at $t+\Delta t$ time is

$$P_3(t+\Delta t) = (\lambda_2 \Delta t)P_1(t) + (1-\lambda_3 \Delta t)P_3(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_3 P_3(t)$$

Taking Laplace transform, the equation is

$$P_3(s) = \frac{\frac{\lambda_2}{(\lambda_1+\lambda_2)}}{(s+\lambda_3)} - \frac{\frac{\lambda_2}{(\lambda_1+\lambda_2)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform, we get

$$P_3(t) = \frac{\lambda_2}{(\lambda_1+\lambda_2)}e^{-(\lambda_3)t} - \frac{\lambda_2}{(\lambda_1+\lambda_2)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of the system in state four at $t+\Delta t$ time is

$$P_4(t+\Delta t) = (\lambda_3 \Delta t)P_1(t) + (1-\lambda_1 \Delta t)P_4(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_4(t)}{dt} = \lambda_3 P_1(t) - \lambda_1 P_4(t)$$

Taking Laplace transform, the equation is

$$P_4(s) = \frac{\frac{\lambda_3}{(\lambda_2+\lambda_3)}}{(s+\lambda_1)} - \frac{\frac{\lambda_3}{(\lambda_2+\lambda_3)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform, we get

$$P_4(t) = \frac{\lambda_3}{(\lambda_2+\lambda_3)}e^{-(\lambda_1)t} - \frac{\lambda_3}{(\lambda_2+\lambda_3)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of being in state five at $t+\Delta t$ time

$$P_5(t+\Delta t) = (\lambda_2 \Delta t)P_2(t) + (1-\lambda_3 \Delta t)P_5(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_5(t)}{dt} = \lambda_2 P_2(t) - \lambda_3 P_5(t)$$

Taking Laplace transform, the equation is

$$P_5(s) = \frac{\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_3)(-\lambda_2+\lambda_3)}}{(s+\lambda_2)} + \frac{\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)(-\lambda_3+\lambda_2)}}{(s+\lambda_3)} + \frac{\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_3)(\lambda_1+\lambda_2)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform, we get

$$P_5(t) = \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_3)(-\lambda_2+\lambda_3)}e^{-(\lambda_2)t} + \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)(-\lambda_3+\lambda_2)}e^{-(\lambda_3)t} + \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_3)(\lambda_1+\lambda_2)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of the system in state six at $t+\Delta t$ time

$$P_6(t+\Delta t) = (\lambda_3 \Delta t)P_3(t) + (1-\lambda_1 \Delta t)P_6(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_6(t)}{dt} = \lambda_3 P_3(t) - \lambda_1 P_6(t)$$

Taking Laplace transform, the equation is

$$P_6(s) = \frac{\frac{\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(-\lambda_1+\lambda_3)}}{(s+\lambda_1)} + \frac{\frac{\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(-\lambda_3+\lambda_1)}}{(s+\lambda_3)} + \frac{\frac{\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_2)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform, we get

$$P_6(t) = \frac{\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(-\lambda_1+\lambda_3)}e^{-(\lambda_1)t} + \frac{\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(-\lambda_3+\lambda_1)}e^{-(\lambda_3)t} + \frac{\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_2)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The probability of the system in state seven at $t+\Delta t$ time is

$$P_7(t+\Delta t) = (\lambda_1 \Delta t)P_4(t) + (1-\lambda_2 \Delta t)P_7(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_7(t)}{dt} = \lambda_1 P_4(t) - \lambda_2 P_7(t)$$

Taking Laplace transform, the equation is

$$P_7(s) = \frac{\frac{\lambda_1\lambda_3}{(\lambda_2+\lambda_3)(-\lambda_1+\lambda_2)}}{(s+\lambda_1)} + \frac{\frac{\lambda_1\lambda_3}{(\lambda_1+\lambda_3)(-\lambda_2+\lambda_1)}}{(s+\lambda_2)} + \frac{\frac{\lambda_1\lambda_3}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_3)}}{(s+(\lambda_1+\lambda_2+\lambda_3))}$$

Using inverse Laplace transform, we get

$$P_7(t) = \frac{\lambda_1\lambda_3}{(\lambda_2+\lambda_3)(-\lambda_1+\lambda_2)}e^{-(\lambda_1)t} + \frac{\lambda_1\lambda_3}{(\lambda_1+\lambda_3)(-\lambda_2+\lambda_1)}e^{-(\lambda_2)t} + \frac{\lambda_1\lambda_3}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_3)}e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The reliability of the 3-component system, if at least one component should operate

$$R(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) + P_6(t) + P_7(t)$$

$$R(t) = \left[\frac{\lambda_2\lambda_3(\lambda_2+\lambda_3) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(\lambda_3-\lambda_1)(\lambda_2-\lambda_1)}\right]e^{-\lambda_1 t} + \left[\frac{\lambda_1\lambda_3(\lambda_1+\lambda_3) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_3)(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)}\right]e^{-\lambda_2 t}$$

$$+\left[\frac{\lambda_1\lambda_2(\lambda_1+\lambda_2) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_2-\lambda_3)(\lambda_1-\lambda_3)}\right]e^{-\lambda_3 t} + \left[\frac{2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_3+\lambda_1)}\right]e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

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Mean Time To Failure (MTTF) of the system

$$MTTF = \int_0^\infty R(t)\,dt$$

$$MTTF = \left[\frac{\lambda_2\lambda_3(\lambda_2+\lambda_3) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_2+\lambda_3)(\lambda_3-\lambda_1)(\lambda_2-\lambda_1)}\right]\frac{1}{\lambda_1} +$$

$$\left[\frac{\lambda_1\lambda_3(\lambda_1+\lambda_3) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_3)(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)}\right]\frac{1}{\lambda_2} + \left[\frac{\lambda_1\lambda_2(\lambda_1+\lambda_2) - 2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_2-\lambda_3)(\lambda_1-\lambda_3)}\right]\frac{1}{\lambda_3}$$

$$+\left[\frac{2\lambda_1\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_3+\lambda_1)}\right]\frac{1}{(\lambda_1+\lambda_2+\lambda_3)}$$

The reliability function in the exponential case R is failure rate and t is the period of time over which reliability is measured. The probability of failure is equal to .

State	Component 1	Component 2	Component 3	System state
1	good	good	good	success
2	failed	good	good	success
3	good	failed	good	success
4	good	good	failed	success
5	failed	failed	good	success
6	good	failed	failed	success
7	failed	good	failed	success
8	failed	failed	failed	down

Table.1

State	Component 1	Component 2	Component 3	System state
1	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$e^{-\lambda_1 t}.e^{-\lambda_2 t}.e^{-\lambda_3 t}$
2	$1-e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$(1-e^{-\lambda_1 t})e^{-\lambda_2 t}e^{-\lambda_3 t}$
3	$e^{-\lambda_1 t}$	$1-e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$e^{-\lambda_1 t}(1-e^{-\lambda_2 t})e^{-\lambda_3 t}$
4	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$1-e^{-\lambda_3 t}$	$e^{-\lambda_1 t}e^{-\lambda_2 t}(1-e^{-\lambda_3 t})$
5	$1-e^{-\lambda_1 t}$	$1-e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})e^{-\lambda_3 t}$
6	$e^{-\lambda_1 t}$	$1-e^{-\lambda_2 t}$	$1-e^{-\lambda_3 t}$	$e^{-\lambda_1 t}(1-e^{-\lambda_2 t})(1-e^{-\lambda_3 t})$
7	$1-e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$1-e^{-\lambda_3 t}$	$(1-e^{-\lambda_1 t})e^{-\lambda_2 t}(1-e^{-\lambda_3 t})$
8	$1-e^{-\lambda_1 t}$	$1-e^{-\lambda_2 t}$	$1-e^{-\lambda_3 t}$	$(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})(1-e^{-\lambda_3 t})$

Table.2

From table.2 the probability of success is

$$P(Success) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$$

$$P(Success) = e^{-(\lambda_1+\lambda_2+\lambda_3)t} + e^{-\lambda_1 t} + e^{-\lambda_2 t} + e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_2+\lambda_3)t} - e^{-(\lambda_1+\lambda_3)t}$$

and MTTF is

$$MTTF = \int_0^\infty P(Success)\,dt$$

$$= \frac{1}{(\lambda_1+\lambda_2+\lambda_3)} + \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} - \frac{1}{(\lambda_1+\lambda_2)} - \frac{1}{(\lambda_2+\lambda_3)} - \frac{1}{(\lambda_1+\lambda_3)}$$

If all the three components have the same failure rate, we get

$$MTTF = \frac{11}{6\lambda}$$

(ii) 3-component repairable system with different failure rates and repair rates

A repairable system is a system which, after failure, can be restored to a functioning condition by some maintenance action other than replacement of the entire system. 3-component system can have distinct states. The failure rates are and the repair rates are The transition rate diagram is given below.

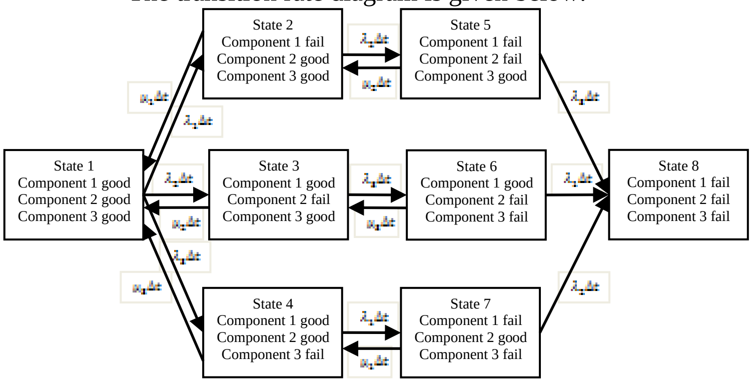

Figure 2: Repairable system state diagram

The transition matrix is

$$\begin{vmatrix} -(\lambda_1+\lambda_2+\lambda_3) & \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 0 & 0 \\ \lambda_1 & -(\lambda_2+\mu_1) & 0 & 0 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\lambda_3+\mu_2) & 0 & 0 & \mu_3 & 0 & 0 \\ \lambda_3 & 0 & 0 & -(\lambda_1+\mu_3) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & 0 & 0 & -(\lambda_3+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & -(\lambda_1+\mu_3) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -(\lambda_2+\mu_1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

From the transition matrix, the probability of the system in state one at $t+\Delta t$ time is

$$P_1(t+\Delta t) = (1-(\lambda_1+\lambda_2+\lambda_3)\Delta t)P_1(t) + \mu_1\Delta t P_2(t) + \mu_2\Delta t P_3(t) + \mu_3\Delta t P_4(t)$$

The probability of the system in state two at $t+\Delta t$ time is

$$P_2(t+\Delta t) = \lambda_1\Delta t P_1(t) + (1-(\lambda_2+\mu_1)\Delta t)P_2(t) + \mu_2\Delta t P_5(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2+\mu_1)P_2(t) + \mu_2 P_5(t)$$

Integrating above equation

$$\int_0^\infty dP_2(t) = \lambda_1\int_0^\infty P_1(t)\,dt - (\lambda_2+\mu_1)\int_0^\infty P_2(t)\,dt + \mu_2\int_0^\infty P_5(t)\,dt$$

The boundary conditions are $P_1(0)=1$ & $P_8(\infty)=1$, zero at all other conditions.

$$\lambda_1 T_1 - (\lambda_2+\mu_1)T_2 + \mu_2 T_5 = 0$$

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Where $T_1, T_2 \,\&\, T_5$ are the expected time in state one, two and five respectively.

$$T_2 = \frac{\lambda_1}{(\lambda_2+\mu_1)}T_1 + \frac{\mu_2}{(\lambda_2+\mu_1)}T_5$$

The probability of the system in state three at $t+\Delta t$ time is

$$P_3(t+\Delta t) = \lambda_2 \Delta t P_1(t) + (1-(\lambda_3+\mu_2)\Delta t)P_3(t) + \mu_3 \Delta t P_6(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - (\lambda_3+\mu_2)P_3(t) + \mu_3 P_6(t)$$

Integrating above equation

$$\int_0^\infty dP_3(t) = \lambda_2\int_0^\infty P_1(t)\,dt - (\lambda_3+\mu_2)\int_0^\infty P_3(t)\,dt + \mu_3\int_0^\infty P_6(t)\,dt$$

$$\lambda_2 T_1 - (\lambda_3+\mu_2)T_3 + \mu_3 T_6 = 0$$

Where $T_1, T_3 \,\&\, T_6$ are the expected time in state one, three and six respectively.

$$T_3 = \frac{\lambda_2}{(\lambda_3+\mu_2)}T_1 + \frac{\mu_3}{(\lambda_3+\mu_2)}T_6$$

The probability of the system in state four at $t+\Delta t$ time is

$$P_4(t+\Delta t) = \lambda_3 \Delta t P_1(t) + (1-(\lambda_1+\mu_3)\Delta t)P_4(t) + \mu_1 \Delta t P_7(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_4(t)}{dt} = \lambda_3 P_1(t) - (\lambda_1+\mu_3)P_4(t) + \mu_1 P_7(t)$$

Integrating above equation

$$\int_0^\infty dP_4(t) = \lambda_3\int_0^\infty P_1(t)dt - (\lambda_1+\mu_3)\int_0^\infty P_4(t)\,dt + \mu_1\int_0^\infty P_7(t)\,dt$$

$$\lambda_3 T_1 - (\lambda_1+\mu_3)T_4 + \mu_1 T_7 = 0$$

Where $T_1, T_4 \,\&\, T_7$ are the expected time in state one, four and seven respectively.

$$T_4 = \frac{\lambda_3}{(\lambda_1+\mu_3)}T_1 + \frac{\mu_1}{(\lambda_1+\mu_3)}T_7$$

The probability of the system in state five at $t+\Delta t$ time is

$$P_5(t+\Delta t) = \lambda_2 \Delta t P_2(t) + (1-(\lambda_3+\mu_2)\Delta t)P_5(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_5(t)}{dt} = \lambda_2 P_2(t) - (\lambda_3+\mu_2)P_5(t)$$

Integrating above equation

$$\int_0^\infty dP_5(t) = \lambda_2\int_0^\infty P_2(t)dt - (\lambda_3+\mu_2)\int_0^\infty P_5(t)\,dt$$

$$\lambda_2 T_2 - (\lambda_3+\mu_2)T_5 = 0$$

$$T_5 = \frac{\lambda_2}{(\lambda_3+\mu_2)}T_2$$

Substituting T_5 value in T_2 we get

$$T_2 = \frac{\lambda_1}{(\lambda_2+\mu_1)}T_1 + \frac{\mu_2}{(\lambda_2+\mu_1)}\frac{\lambda_2}{(\lambda_3+\mu_2)}T_2$$

$$T_2 = \left[\frac{\lambda_1(\lambda_3+\mu_2)}{\lambda_2\lambda_3+\lambda_2\mu_1+\mu_1\mu_2}\right]T_1$$

Substituting T_2 value in T_5 we get

$$T_5 = \frac{\lambda_2}{(\lambda_3+\mu_2)}\left[\frac{\lambda_1(\lambda_3+\mu_2)}{\lambda_2\lambda_3+\lambda_2\mu_1+\mu_1\mu_2}\right]T_1$$

$$T_5 = \left[\frac{\lambda_1\lambda_2}{\lambda_2\lambda_3+\lambda_2\mu_1+\mu_1\mu_2}\right]T_1$$

The probability of the system in state six at $t+\Delta t$ time is

$$P_6(t+\Delta t) = \lambda_3 \Delta t P_3(t) + (1-(\lambda_1+\mu_3)\Delta t)P_6(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_6(t)}{dt} = \lambda_3 P_3(t) - (\lambda_1+\mu_3)P_6(t)$$

Integrating the above equation

$$\int_0^\infty dP_6(t) = \lambda_3\int_0^\infty P_3(t)dt - (\lambda_1+\mu_3)\int_0^\infty P_6(t)\,dt$$

$$\lambda_3 T_3 - (\lambda_1+\mu_3)T_6 = 0$$

$$T_6 = \frac{\lambda_3}{(\lambda_1+\mu_3)}T_3$$

Substituting T_6 value in T_3 we get

$$T_3 = \frac{\lambda_2}{(\lambda_3+\mu_2)}T_1 + \frac{\mu_3}{(\lambda_3+\mu_2)}\frac{\lambda_3}{(\lambda_1+\mu_3)}T_3$$

$$T_3 = \left[\frac{\lambda_2(\lambda_1+\mu_3)}{\lambda_1\lambda_3+\lambda_1\mu_2+\mu_2\mu_3}\right]T_1$$

Substituting T_3 value in T_6 we get

$$T_6 = \frac{\lambda_3}{(\lambda_1+\mu_3)}\left[\frac{\lambda_2(\lambda_1+\mu_3)}{\lambda_1\lambda_3+\lambda_1\mu_2+\mu_2\mu_3}\right]T_1$$

$$T_6 = \left[\frac{\lambda_2\lambda_3}{\lambda_1\lambda_3+\lambda_1\mu_2+\mu_2\mu_3}\right]T_1$$

The probability of the system in state seven at $t+\Delta t$ time is

$$P_7(t+\Delta t) = \lambda_1 \Delta t P_4(t) + (1-(\lambda_2+\mu_1)\Delta t)P_7(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

$$\frac{dP_7(t)}{dt} = \lambda_1 P_4(t) - (\lambda_2+\mu_1)P_7(t)$$

Integrating the above equation

$$\int_0^\infty dP_7(t) = \lambda_1\int_0^\infty P_4(t)dt - (\lambda_2+\mu_1)\int_0^\infty P_7(t)\,dt$$

$$\lambda_1 T_4 - (\lambda_2+\mu_1)T_7 = 0$$

$$T_7 = \frac{\lambda_1}{(\lambda_2+\mu_1)}T_4$$

Substituting T_7 value in T_4

$$T_4 = \frac{\lambda_3}{(\lambda_1+\mu_3)}T_1 + \frac{\mu_1}{(\lambda_1+\mu_3)}\frac{\lambda_1}{(\lambda_2+\mu_1)}T_4$$

$$T_4 = \left[\frac{\lambda_3(\lambda_2+\mu_1)}{\lambda_1\lambda_2+\lambda_2\mu_3+\mu_1\mu_3}\right]T_1$$

Substituting T_4 value in T_7 we get

$$T_7 = \frac{\lambda_1}{(\lambda_2+\mu_1)}\left[\frac{\lambda_3(\lambda_2+\mu_1)}{\lambda_1\lambda_2+\lambda_2\mu_3+\mu_1\mu_3}\right]T_1$$

$$T_7 = \left[\frac{\lambda_1\lambda_3}{\lambda_1\lambda_2+\lambda_2\mu_3+\mu_1\mu_3}\right]T_1$$

The probability of the system in state eight at $t+\Delta t$ time is

$$P_8(t+\Delta t) = \lambda_3 \Delta t P_5(t) + \lambda_1 \Delta t P_6(t) + \lambda_2 \Delta t P_7(t)$$

Taking the Limit as $\Delta t \to 0$, and the differential equation is

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$$\frac{dP_8(t)}{dt} = \lambda_3 P_5(t) + \lambda_1 P_6(t) + \lambda_2 P_7(t)$$

$$1 = \lambda_3 T_5 + \lambda_2 T_6 + \lambda_1 T_7$$

Substituting $T_5, T_6 \,\&\, T_7$ values in above equation

$$\left[\frac{\lambda_1\lambda_2\lambda_3}{\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2}\right]T_1 + \left[\frac{\lambda_1\lambda_2\lambda_3}{\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3}\right]T_1 + \left[\frac{\lambda_1\lambda_2\lambda_3}{\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3}\right]T_1 = 1$$

$$T_1 = \frac{(\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3)}{\lambda_1\lambda_2\lambda_3\begin{bmatrix}(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + (\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2) \\ (\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + (\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)\end{bmatrix}}$$

The MTBF is the sum of the expected time in state one, two, three, four, five, six and seven.

$$MTBF = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7$$

$$= T_1\left[\begin{matrix} 1 + \left[\dfrac{\lambda_1(\lambda_3+\mu_2)}{\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2}\right] + \left[\dfrac{\lambda_2(\lambda_1+\mu_3)}{\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3}\right] + \\ \left[\dfrac{\lambda_3(\lambda_2+\mu_1)}{\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3}\right] + \left[\dfrac{\lambda_1\lambda_2}{\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2}\right] + \\ \left[\dfrac{\lambda_2\lambda_3}{\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3}\right] + \left[\dfrac{\lambda_1\lambda_3}{\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3}\right] \end{matrix}\right]$$

Substituting T_1 value in above equation, we get

$$MTBF = \frac{\begin{bmatrix}(\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + \\ (\lambda_1(\lambda_3 + \mu_2) + \lambda_1\lambda_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + \\ (\lambda_2(\lambda_1 + \mu_3) + \lambda_2\lambda_3)(\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + \\ (\lambda_3(\lambda_2 + \mu_1) + \lambda_1\lambda_3)(\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)\end{bmatrix}}{\lambda_1\lambda_2\lambda_3\begin{bmatrix}(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)(\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + (\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2) \\ (\lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\mu_3) + (\lambda_2\lambda_3 + \lambda_2\mu_1 + \mu_1\mu_2)(\lambda_1\lambda_3 + \lambda_1\mu_2 + \mu_2\mu_3)\end{bmatrix}}$$

If all the 3-components have the same failure rate and repair rate, we get

$$MTBF = \frac{7\lambda^2 + 4\lambda\mu + \mu^2}{3\lambda^3}$$

Mean time to repair (**MTTR**) is a basic measure of the maintainability of repairable items. It represents the average time required to repair a failed component or device.

$$MTTR = \int_0^\infty [1 - M(t)]dt$$

Where

$$M(t) = 1 - e^{-\int_0^t \mu(t)dt}$$

If the repair rate $\mu(t)$ is constant and is equal to μ then

$$MTTR = \frac{1}{\mu}$$

In this case the repair rates $\mu_1, \mu_2 \,\&\, \mu_3$ are constants and $\mu_1 = \mu_2 = \mu_3 = \mu$, then

$$MTTR = \frac{1}{3\mu}$$

The steady state availability is

$$A(t \to \infty) = \frac{MTBF}{MTBF + MTTR}$$

On simplification, we get

$$A = \frac{3\mu(7\lambda^2 + 4\lambda\mu + \mu^2)}{3\mu(7\lambda^2 + 4\lambda\mu + \mu^2) + 3\lambda^3}$$

Numerical results:

3-component non-repairable system with different failure rates

λ_1	λ_2	λ_3	t	$R(t)$	$MTTF$
0.001	0.02	0.05	40	0.988642	1041.831
0.009	0.02	0.05	40	0.909736	155.7947
0.017	0.02	0.05	40	0.848541	105.835
0.025	0.02	0.05	40	0.800984	88.94737
0.033	0.02	0.05	40	0.763956	80.88555
0.041	0.02	0.05	40	0.735073	76.37223
0.049	0.02	0.05	40	0.712504	73.60144
0.057	0.02	0.05	40	0.694843	71.79583
0.065	0.02	0.05	40	0.681005	70.5698

λ_1	λ_2	λ_3	t	$R(t)$	$MTTF$
0.03	0.001	0.04	40	0.986559	1030.164
0.03	0.009	0.04	40	0.893242	144.128
0.03	0.017	0.04	40	0.820961	94.16836
0.03	0.025	0.04	40	0.764867	77.2807
0.03	0.033	0.04	40	0.721257	69.21889
0.03	0.041	0.04	40	0.687296	64.70556
0.03	0.049	0.04	40	0.660808	61.93477
0.03	0.057	0.04	40	0.640123	60.12916
0.03	0.065	0.04	40	0.623952	58.90313

λ_1	λ_2	λ_3	t	$R(t)$	$MTTF$
0.02	0.06	0.001	40	0.987867	1041.975
0.02	0.06	0.009	40	0.903527	155.3059
0.02	0.06	0.017	40	0.838035	104.8716
0.02	0.06	0.025	40	0.787066	87.61905
0.02	0.06	0.033	40	0.747312	79.27058
0.02	0.06	0.041	40	0.716239	74.52798
0.02	0.06	0.049	40	0.691902	71.57095
0.02	0.06	0.057	40	0.672804	69.61199
0.02	0.06	0.065	40	0.657792	68.25818

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λ_1	λ_2	λ_3	t	$R(t)$	$MTTF$
0.01	0.02	0.05	10	0.99638	145
0.01	0.02	0.05	20	0.978602	145
0.01	0.02	0.05	30	0.945621	145
0.01	0.02	0.05	40	0.901213	145
0.01	0.02	0.05	50	0.849698	145
0.01	0.02	0.05	60	0.794635	145
0.01	0.02	0.05	70	0.738605	145
0.01	0.02	0.05	80	0.683349	145
0.01	0.02	0.05	90	0.629983	145

3-component repairable system with same failure rate and repair rates

λ	μ	$A(t \to \infty)$	$MTBF$
0.02	0.1	0.996169	866.6667
0.04	0.1	0.983087	193.75
0.06	0.1	0.964798	91.35802
0.08	0.1	0.944299	56.51042
0.1	0.1	0.923077	40
0.12	0.1	0.901863	30.63272
0.14	0.1	0.881027	24.68416
0.16	0.1	0.860756	20.60547
0.18	0.1	0.841142	17.64975

λ	μ	$A(t \to \infty)$	$MTBF$
0.05	0.1	0.974359	126.6667
0.05	0.12	0.981706	149.0667
0.05	0.14	0.98647	173.6
0.05	0.16	0.989704	200.2667
0.05	0.18	0.99198	229.0667
0.05	0.2	0.993631	260
0.05	0.22	0.994857	293.0667
0.05	0.24	0.995787	328.2667
0.05	0.26	0.996506	365.6

CONCLUSION:

Reliability and MTTF have been derived for the non-repairable parallel redundant system with different failure rate and MTBF has been derived for repairable systems with different failure rate and repair rate. And also steady- state availability has been computed for repairable system with same failure rate and repair rate. It is observed from the computations that the reliability and MTTF decreases as λ_1, λ_2 & λ_3 increases and reliability decreases as t increases for the non repairable system and as λ increases steady state availability & MTBF decreases, as μ increases steady- state availability & MTBF increases for the repairable system.

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