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Regular pre semi I Separation axioms in Ideal Topological Spaces

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Abstract

The authors introduced $rpsI$-closed sets and $rpsI$-open sets in ideal topological spaces and established their relationships with some generalized sets in ideal topological spaces. The aim of this paper is to introduce $rpsI - T_0$, $rpsI - T_1$, $rpsI - T_2$, $rpsI - T_{\frac{1}{4}}$, $rpsI - Q_1$ spaces and characterize their basic properties.

Keywords:$rpsI - T_0$ spaces, $rpsI - T_1$ spaces, $rpsI - T_2$ spaces, $rpsI - T_{\frac{1}{4}}$ spaces, $rpsI - Q_1$ spaces

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1 Introduction

Separation axioms in topological spaces play a dominated role in analysis. Recently general topologists concentrate on separation axioms between T_0 and T_1. In this paper the concepts of $rpsI - T_0$, $rpsI - T_1$, $rpsI - T_2$, $rpsI - T_{\frac{1}{4}}$, $rpsI - Q_1$ spaces are introduced, characterized and studied their relationships with $\alpha - T_1$ space and semi-I-T_2

2 Preliminaries

For a subset A of an ideal topological space(X, τ, I), $cl^*(A)$ and $int^*(A)$ denote the closure of A and interior of A respectively. $X - A$ denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1. *A subset A of an ideal topological space (X, τ, I) is called*

i) semi-I-open [6] if $A \subseteq cl^*(int(A))$.

ii) semi pre I-open [6] if $A \subseteq cl(int(cl^*(A)))$.

iii) αI- open [6] if $A \subseteq int(cl^*(int(A)))$.

iv) regular I-open [5] $A = int(cl^*(A))$.

Definition 2.2. *A subset A of an ideal topological (X, τ, I) is called*

i) regular generalized I-closed (rgI-closed) [2] if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is regular I-open.

ii) regular pre semi I-closed [2] (rps$I$-closed) if $spIcl(A) \subseteq A$ whenever $A \subseteq U$ and U is regular generalized I-open.

The complement of $rpsI$-closed set is $rpsI$-open set. $rpsIcl(A)$ is the smallest $rpsI$-closed set containing A.

Theorem 2.3. *[2]*

i) Every semi-I-closed set is rps$I$-closed.

ii) Every αI-closed set is rps$I$-closed.

Definition 2.4. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is called*

i) rps$I$-irresolute [3] if $f^{-1}(A)$ is rps$I$-closed in X, for every rps$I$-closed subset A of Y.

ii) rps$I$-open map [3] if for every rps$I$-open subset F of X, then the set $f(F)$ is rps$I$-open in Y.

Theorem 2.5. *If $f : (X, \tau, I) \to (Y, \sigma)$ is rps$I$-irresolute, then the inverse image of every rps$I$-open set in Y is also a rps$I$-open set in X.*

Definition 2.6. *An ideal topological space (X, τ, I) is said to be regular pre semi I-T_0 (briefly $rpsI - T_0$ space) if for each pair of distinct points x, y of X, there exists an rps$I$-open set containing one point but not the other.*

Theorem 2.7. *Every subspace of an $rpsI - T_0$ space is a $rpsI - T_0$ space.*

Proof. Let X be a $rpsI - T_0$ space and Y be a subspace of X. Let x, y be two distinct points of Y. Since $Y \subseteq X$, we have x and y are distinct points of X. Since X is $rpsI - T_0$ space, there exists an $rpsI$-open set G such that $x \in G$ but $y \notin G$. Then there exists an $rpsI$-open set $G \cap Y$ in Y which contains x but does not contain y. Hence Y is a $rpsI - T_0$ space. □

Example 2.8. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space X is a $rpsI - T_0$ space because $\{b, c, d\}$ is $rpsI$-open set with $a \neq b$ containing b but not a.*

Theorem 2.9. *Given a map $f : (X, \tau, I) \to (Y, \sigma)$ is bijective and $rpsI$-open, X is $rpsI - T_0$ space, then Y is $rspI - T_0$ space.*

Proof. Let Y be an ideal topological space and let u, v be two distinct points of Y. Since f is a bijection, we have $, y \in X$ such that $f(x) = u, f(y) = v$. Since X is $rpsI - T_0$ space, there exists $rpsI$-open set G in X such that $x \in G$ but $y \notin G$. Also, f is $rpsI$-open, $f(G)$ is $rpsI$-open in Y containing $f(x) = u$ but not containing $f(y) = v$. Thus there exists a $rpsI$-open set $f(G)$ in Y such that $u \in f(G)$ but $v \notin f(G)$ and hence Y is a $rpsI - T_0$ space. □

Theorem 2.10. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be an $rpsI$- irresolute bijective map. If Y is $rpsI - T_0$ space, then X is $rpsI - T_0$ space.*

Proof. Assume that Y is a $rpsI - T_0$ space. Let u, v be two distinct points of Y. Since f is a bijection, we have $x, y \in X$ such that $f^{-1}(u) = x; f^{-1}(v) = y$. since Y is a $rpsI - T_0$ space, there exists $rpsI$-open set H in Y such that $u \in H$ but $v \notin H$. Since f is $rpsI$-irresolute, $f^{-1}(H)$ is $rpsI$-open in X containing $f(x) = u$ but not containing $f(y) = v$. Thus there exists a $rpsI$-open set $f^{-1}(H)$ in X such that $x \in f^{-1}(H)$ but $y \notin f^{-1}(H)$ and hence X is a $rpsI - T_0$ space. □

Definition 2.11. *An ideal topological space (X, τ, I) is said to be $rpsI - T_1$ space if for each pair of distinct points x, y of X, there exists a pair of $rpsI$-open sets, one containing x but not y and the other containing y but not x.*

Example 2.12. *Consider the ideal topological space (X, τ, I), where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal topological space X is $rpsI - T_1$ space because $\{b, c, d\}$ and $\{a, d\}$ are $rpsI$-open sets with $a \neq c$, $c \in \{b, c, d\}$ but $a \notin \{b, c, d\}$ and $a \in \{a, d\}$ but $c \notin \{a, d\}$.*

Theorem 2.13. *Every subspace of a $rpsI - T_1$ space is also a $rpsI - T_1$ space.*

Proof. Let X be a $rpsI - T_1$ space and let Y be a subspace of X. Let $x, y \in Y \subseteq X$ such that $x \neq y$. By hypothesis X is a$rpsI - T_1$ space, hence there exists $rpsI$-open set U, V in X such that $x \in U$ and $y \in V$, $x \notin V$ and $y \notin U$. By definition of subspace, $U \cap Y$ and $V \cap Y$ are $rpsI$-open sets in Y. Further $x \in U, x \in Y$ implies $x \in U \cap Y$ also $y \in V, y \in Y$ implies $y \in V \cap Y$. Thus there exists $rpsI$-open sets $U \cap Y$ and $V \cap Y$ in Y such that $x \in U \cap Y, y \in V \cap Y$ and $x \notin V \cap Y, y \notin U \cap Y$. Hence Y is a $rpsI - T_1$ space. □

Theorem 2.14. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be an $rpsI$-irresolute, injective map. If Y is $rpsI - T_1$ space, then X is $rpsI - T_1$ space.*

Proof. Assume that Y is a $rpsI - T_1$ space. Let $x, y \in Y$ such that $x \neq y$. Then there exists a pair of $rpsI$-open sets U, V in Y such that $f(x) \in U$ and $f(y) \in V$, $f(x) \notin V$, $f(y) \notin U$ which implies $x \in f^{-1}(U), y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$ and $y \notin f^{-1}(U)$. Since f is $rpsI$-irresolute, we have X is $rpsI - T_1$ space. □

Theorem 2.15. *If $\{x\}$ is $rpsI$-closed in X, for every $x \in X$. Then X is $rpsI - T_1$ space.*

Proof. Let x, y be two distinct points of X such that $\{x\}$ and $\{y\}$ are $rpsI$-closed. Then $\{x\}^c$ and $\{y\}^c$ are $rpsI$-open in X such that $y \in \{x\}^c$ but $x \notin \{x\}^c$ and $x \in \{y\}^c$ but $y \notin \{y\}^c$. Hence X is $rpsI - T_1$ space. □

Definition 2.16. *An ideal topological (X, τ, I) is said to be $\alpha I - T_1$ space if for each pair of distinct points x, y of X, there exists a pair of αI-open sets one containing x but not y and the other containing y but not x.*

Theorem 2.17. *Every $\alpha I - T_1$ space is a $rpsI - T_1$ space.*

Proof. Let (X, τ, I) be an $\alpha I - T_1$ space and $x \neq y$ of X. Since X is $\alpha I - T_1$ space, there exists a pair of αI-open sets U, V in X such that $x \in U$ and $y \in V$, $x \notin V, y \notin U$. We know that every αI-open set is $rpsI$-open. Therefore X is $rpsI - T_1$ space. □

Definition 2.18. *A ideal topological space (X, τ, I) is said to be $rpsI - T_2$ space if for each pair of distinct points x, y of X, there exists disjoint $rpsI$-open sets U and V such that $x \in U$ and $y \in V$.*

Remark 2.19. *Every $rpsI - T_2$ space is $rpsI - T_1$ space. The converse need not be true as seen from the following example.*

Example 2.20. *Consider the ideal topological space (X, τ, I), where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space, X is $rpsI - T_1$ space but not $rpsI - T_2$ space. That is there exists a pair of $rpsI$-open sets $U = \{b, c, d\}$ and $V = \{a, b\}$ such that $c \in U$ and $a \in V$ but $c \notin V$ and $a \notin U$. Here the intersection of $rpsI$-open sets is non-empty.*

Theorem 2.21. *Every subspace of a $rpsI - T_2$ space is also a $rpsI - T_2$ space.*

Proof. Let X be a $rpsI - T_2$ space and let Y be a subspace of X. Let $a, b \in Y \subseteq X$ with $a \neq b$. By hypothesis there exists $rpsI$-open sets G, H in X such that $a \in G$ and $b \in H$, $G \cap H = \phi$. By definition of subspace $G \cap Y$ and $H \cap Y$ are $rpsI$-open sets in Y. Further $a \in G, a \in Y$ implies $a \in G \cap Y$ and $b \in H, b \in Y$ implies $b \in H \cap Y$. Since $G \cap H = \phi$, $(Y \cap G) \cap (Y \cap H) = Y \cap (G \cap H) = Y \cap \phi = \phi$. Therefore $G \cap Y$ and $H \cap Y$ are disjoint $rpsI$-open sets in Y such that $a \in G \cap Y$ and $b \in H \cap Y$. Thus Y is $rpsI - T_2$ space.

Theorem 2.22. *Given a map $f : (X, \tau, I) \to (Y, \sigma)$ is bijection, $rpsI$-open and X is $rpsI - T_2$ space. Then Y is $rpsI - T_2$ space.*

Proof. Given f is a bijection and $rpsI$-open. Let X be a $rpsI - T_2$ space and Y be any ideal space. Let $a, b \in Y$ with $a \neq b$. Since f is a bijection, there exists distinct points $x, y \in X$ such that $f(x) = a$ and $f(y) = b$. Since X is a $rpsI - T_2$ space, there exists disjoint $rpsI$-open sets U and V of X such that $x \in U$ and $y \in V$. Also f is $rpsI$-open, we have $f(U)$ and $f(V)$ are $rpsI$-open sets containing $f(x) = a$ and $f(y) = b$. Thus there exists disjoint $rpsI$-open sets $f(U)$ and $f(V)$ in Y such that $a \in f(U)$ and $b \in f(V)$. Hence Y is $rpsI - T_2$ space. □

Theorem 2.23. *If $\{x\}$ is $rpsI$-closed in X, for every $x \in X$, then X is $rpsI - T_2$ space.*

Proof. Let x and y be two distinct points of X such that $\{x\}$ and $\{y\}$ are $rpsI$-closed set. Then $\{x\}^c$ and $\{y\}^c$ are $rpsI$-open in X such that $x \in \{y\}^c$ and $y \in \{x\}^c$. Hence X is $rpsI - T_2$ space. □

Theorem 2.24. *If X is $rpsI - T_2$ space, then for $y \neq x \in X$, there exists an $rpsI$-open set G such that $x \in G$ and $y \notin rpsIcl(G)$.*

Proof. Let $x, y \in X$ such that $y \neq x$. Since X is $rpsI - T_2$ space, there exists disjoint $rpsI$-open sets G and H in X such that $x \in G$ and $y \in H$. Therefore H^c is $rpsI$-closed set such that $rpsIcl(G) \subseteq H^c$. Since $y \in H$, we have $y \notin H^c$. Hence $y \notin rpsIcl(G)$. □

Definition 2.25. *An ideal topological space (X, τ, I) is $rpsI - Q_1$ space, if for $x, y \in X$ with $rpsIcl(\{x\}) \neq rpsIcl(\{y\})$, then there exists disjoint $rpsI$-open sets U and V such that $rpsIcl(\{x\}) \subseteq U$ and $rpsIcl(\{y\}) \subseteq V$.*

Theorem 2.26. *If (X, τ, I) is $rpsI - T_2$ space, then it is $rpsI - Q_1$ space.*

Proof. Let $\{x\}, \{y\}$ be two distinct closed set in X such that for every $x, y \in X$ with $rpsIcl(\{x\}) \neq rpsIcl(\{y\})$. We know that, every closed set is $rpsI$-closed and so $\{x\} = rpsIcl(\{x\})$,$\{y\} = rpsIcl(\{y\})$. Since X is $rpsI - T_2$ space, there exists disjoint $rpsI$-open sets U and V in X such that $x \in U$ and $y \in V$. Therefore $rpsIcl(\{x\}) \subseteq U$ and $rpsIcl(\{y\}) \subseteq V$. Hence X is $rpsI - Q_1$ space. □

Definition 2.27. *An ideal topological space (X, τ, I) is called $rpsI - T_{\frac{1}{4}}$ space if for any $rpsI$-closed set $F \subseteq X$ and any point $x \in X - F$ there exists disjoint open sets $U, V \subseteq X$ and $x \in U$ and $F \subseteq V$.*

Theorem 2.28. *For any ideal space (X, τ, I), if $x \in G \subseteq X$ and G is a $rpsI$-open set, there exists a $rpsI$-open set $H \subseteq X$ such that $x \in H \subseteq \overline{H} \subseteq G$. Then X is $rpsI - T_{\frac{1}{4}}$ space.*

Proof. Let $F \subseteq X$ be $rpsI$-closed set with $x \in F^c$. Since F^c is $rpsI$-open by our assumption choose a $rpsI$-open set H with $x \in H \subseteq \overline{H} \subseteq X - F$. Let $K = X - \overline{H}$ and so W is $rpsI$-open. Further, $F \subseteq X - \overline{H} = W$ and $H \cap K = \phi$. Hence X is $rpsI - T_{\frac{1}{4}}$ space. □

Remark 2.29. *We have the following diagram,*

$$rpsI - T_2 \Longrightarrow rpsI - T_1 \Longrightarrow rpsI - I_0$$

Definition 2.30. *An ideal space (X, τ, I) is said to be semi-I-T_2[4], if for each pair of distinct points x and y in X, there exists two semi-I-open sets U and V in X such that $x \in U$ and $y \in V$, $U \cap V = \phi$.*

Remark 2.31. *Every semi I-T_2 space is a $rpsI - T_2$ space but the converse is not true because, every semi I-open set is $rpsI$-open and $rpsI$-open set need not be semi-I-open [2].*

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