

# Reduction of PAPR of MIMO-OFDM using Non-Linear Companding Transform

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**Abstract:** MIMO-OFDM has gained popularity for high rate data services. It effectively combats the multipath fading channel and improves the band width efficiency. At the same time, it also increases system capacity so as to provide a reliable transmission. In MIMO-OFDM system, the output is the superposition of multiple sub-carriers. In this case, when the phases of these carriers are same, instantaneous power outputs increases and are higher than then the mean power of the system. This is large peak-to-Average power Ratio (PAPR). OFDM has the specific drawback of increase in PAPR. Various methods are in existence to counteract this drawback and as each generation of applications demand betterment, new methods for improvement are under consideration. In this paper, we propose a novel method based on non-linear companding transform (NCT) scheme for PAPR reduction in (STBC) MIMO-OFDM systems. The key idea for proposed method is to detect and compand the least probable and high PAPR OFDM symbols, leaving rest of the transmitted data unchanged. In this procedure, we can reduce the number of symbols to be operated which lowers the computational complexity. The methodology involves saving their pattern and employing the NCT algorithm to reduce the PAPR on the specific symbols. The modified symbols replace the original index in OFDM symbol matrix. The index is transmitted as side information for recovery OFDM symbols at the receiver. This method attains improved the PAPR performance over existing methods. Simulation results show a significant improvement in this method over the conventional STBC MIMO-OFDM systems.

**Keywords:** PAPR, CCDF, STBC, MIMO-OFDM, Non-linear Companding Transform.

## I. INTRODUCTION

Multiple-Input Multiple-output with orthogonal frequency division multiplexing (MIMO-OFDM) system has been receiving great attention in 4G wireless networks for achieving higher speeds, efficiency and better quality of service. Orthogonal frequency division multiplexing (OFDM) and its variants are gaining tremendous interest in modern wireless and wire line communication systems due to OFDM's ability to effectively mitigate multipath fading. More specifically, OFDM is immune to delay spread that is smaller than the length of the cyclic-prefix (CP), making single-tap frequency domain equalization optimal i.e. simplifies the equalization process by elimination the Inter Symbol Interference (ISI). This immunity makes the combination of OFDM and Multiple-Input and Multiple-Output (MIMO) technology to increase diversity gain and

enhance system capacity on the wireless channel, which results in high spectral efficiencies.

More importantly high peak-to-average-power ratio (PAPR) due to coherent combining of subcarriers results in the saturation of high power amplifier (HPA). As a result, the digital-to-analog converter (DAC) and HPA with extremely large dynamic range are required to avoid the nonlinear distortion. Due to its practical importance, OFDM PAPR reduction has been treated extensively in the literature [1], clipping and filtering [2], selective mapping (SLM) [3], partial transmit sequence (PTS) [4], active constellation extension (ACE) [5], and companding transform [6,7].

In MIMO-OFDM systems, PAPR reduction techniques focused on known single antenna methods optimized for multiple transmission antennas were proposed. These include variation of SLM [8, 9] and PTS [10], where PAPR is reduced for each transmit antenna. NCT technique was first described in [11], which employed a logarithmic-based  $\mu$ -law companding. NCT is effective than the 'hard' clipping PAPR reduction technique. Researchers have addressed designing the desirable distribution form of the transformed signals, viz. the exponential companding (EC) [12], piecewise companding (PC), and the trapezoidal companding (TC). Nevertheless, EC scheme is suitable under limited BER conditions only, since it greatly increases the distribution of large amplitude signals. Another issue of the EC and PC scheme is the lack of necessary flexibility in the tradeoff between PAPR and BER performance. In other words, the NCT approach may obtain a significant PAPR reduction, but at the price of a reduced BER performance.

In this paper, we proposed a novel method based on non-linear companding transform (NCT) scheme [13] for PAPR reduction in (STBC) MIMO-OFDM systems. The key idea for proposed method is to detect and compress the least probable and high PAPR OFDM symbols, leaving rest of the transmitted data unchanged. In this procedure, we can reduce the number of symbols to be operated which lowers the computational complexity. The paper is organized as follows: section II describes the basics of the STBC MIMO-OFDM system and PAPR, in section III, we present the proposed method, simulation results are presented in section IV, and section V contains discussion and conclusions.

## II. STBC MIMO-OFDM SYSTEM AND PAPR

Figure 1 shows the block diagram of STBC MIMO-OFDM system using novel method based on Non-linear Companding Transform. Baseband modulated symbols are passed through serial-to-parallel (S/P) converter which generates complex vector of size N. We represent the complex vector of size N as  $X = [X_0, X_1, X_2, \dots, X_{N-1}]^T$ . The complex vector, X is then passed through the STBC encoder (2 X 2) which generates two sequences:  $\hat{X}_1 = [\hat{X}_{1,0}, \hat{X}_{1,1}, \hat{X}_{1,2}, \dots, \hat{X}_{1,N-1}]^T$  and  $\hat{X}_2 = [\hat{X}_{2,0}, \hat{X}_{2,1}, \hat{X}_{2,2}, \dots, \hat{X}_{2,N-1}]^T$ . Both these sequences are then passed through IFFT blocks for antenna 1 and antenna 2 respectively.

The complex baseband STBC MIMO-OFDM signal for antenna I with N subcarriers can be written as:

$$\hat{x}_{ni}[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{X}_{k,i} e^{j2\pi mk/N}, n = 0, 1, 2, \dots, N-1 \quad (1)$$

Where (i = 1, 2) denote the antenna number,  $j = \sqrt{-1}$ , l is over sampling factor.

### A. Alamouti Space-Time Block Code (STBC)

The 2x2 orthogonal STBC can be defined as

$$X = \begin{bmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{bmatrix} \quad (2)$$

Alamouti encoded signal is transmitted from the two transmit antennas over two symbol periods.

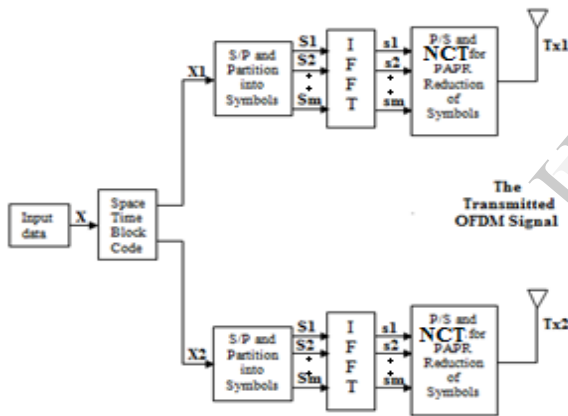


Figure 1 Block diagram of STBC MIMO-OFDM system using novel method based on Non-linear Companding Transform method.

### B. Peak-to-Average-Power Ratio (PAPR)

PAPR is the ratio of peak power to average power of OFDM signal, for the STBC MIMO-OFDM signal  $\hat{x}_{n,i}[n]$  of  $i^{th}$  antenna in (1), can be written as:

$$PAPR(\text{dB}) = 10 \log_{10} \left( \frac{\max\{|\hat{x}_{n,i}[n]|^2\}}{E\{|\hat{x}_{n,i}[n]|^2\}} \right) \quad (3)$$

Where  $E[\cdot]$  denotes expectation. If a discrete-time STBC MIMO-OFDM signal ( $\hat{x}_{n,i}[n]$ ) is over sampled by a factor  $l \geq 4$ , then its PAPR is a good approximation of the continuous time OFDM signal.

It can be proved that  $X_1$  and  $X_1^*$  (i = 1, 2) have the same PAPR properties. After performing the PAPR reduction on  $X_1$  and  $X_2$ , we obtain two modified sequences with good PAPR properties  $\hat{X}_1$  and  $\hat{X}_2$ , which will be transmitted during the first symbol period. Then, during the second symbol period

$-\hat{X}_1^*$  and  $\hat{X}_2^*$  are transmitted, which have the same good PAPR properties as  $\hat{X}_1$  and  $\hat{X}_2$ , respectively.

## III. THE PROPOSED PAPR REDUCTION METHOD

We consider a MIMO-OFDM system with N subcarriers. The proposed method is to detect less probable and high PAPR yielding OFDM symbols and process them only from the data. In this way, we can reduce the number of symbols to be operated and computational complexity. The methodology involves saving the index of high PAPR symbols, and then employing the NCT method on those specific symbols to reduce the PAPR. Now these modified symbols are placed in their original index in OFDM symbol matrix and transmit. The index is included for transmission as side information for performing expanding operation at the receiver to recover that original OFDM symbols. The companding method used is Non-linear companding method, which transforms the original OFDM signals into a specific static form, by introducing the variable transform parameters and inflexion point in the target probability density function (PDF), while remaining an unchanged output power level, this method can achieve an effective PAPR reduction as well as an improved overall performance of OFDM system simultaneously. This method enables more flexibility and freedom in the companding form so that a favorable tradeoff between the PAPR reduction and BER performance can be offered.

### NCT Technique and Analysis:

Based on central limit theorem, when N is large (e.g.  $N \geq 64$ ), the real and imaginary parts of  $x_n$  become Gaussian distributed, each with zero mean and variance  $\sigma^2$ . Thus, the signal amplitude  $|x_n|$  follows Rayleigh distributed, with the PDF as

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} \exp\left(-\frac{x^2}{\sigma^2}\right), x \geq 0 \quad (4)$$

The cumulative distribution (CDF) of  $|x_n|$  can be derived as

$$F_{|x_n|}(x) = \text{Prob}\{|x_n| \leq x\} = \int_0^x \frac{2y}{\sigma^2} \exp\left(-\frac{y^2}{\sigma^2}\right) dy = 1 - \exp\left(-\frac{x^2}{\sigma^2}\right), x \geq 0 \quad (5)$$

Where  $\text{Prob}\{A\}$  is the probability of the event A.

This NCT method can transform the original Gaussian-distributed signal  $\hat{x}_{n,i}[n]$  into a specific statistics form defined by a piecewise function in the interval  $[0, A]$  ( $A > 0$ ). The target PDF of  $|y_n|$  is simply defined as

$$f_{|y_n|}(x) = \begin{cases} kx^m, & 0 \leq x \leq cA \\ k(cA)^m, & cA \leq x \leq A \end{cases} \quad (6)$$

Where  $cA (0 < c < 1)$  is the inflexion point. The parameters  $k$  and  $m$  are variable positive numbers and can be used to specify the ultimate companding form along with adjusting the average output power in the transform. Especially, when  $m$  is equal to zero, the statistics form of  $|y_n|$  is similar to the uniform-distributed schemes. According to the definition of PDF i.e.  $\int_{-\infty}^{+\infty} f_{|y_n|}(x) dx = 1$ , we have

$$K = \frac{m+1}{c^m A^m (m+1-mc)} \quad (7)$$

The CDF of  $|y_n|$  can then be given by

$$F_{|y_n|}(x) = \begin{cases} \frac{kx^{m+1}}{m+1}, & 0 \leq x \leq cA \\ k(cA)^m x - \frac{km(cA)^{m+1}}{m+1}, & cA \leq x \leq A \\ 1, & x > A \end{cases} \quad (8)$$

The inverse function of CDF is therefore

$$F_{|y_n|}^{-1}(x) = \begin{cases} \left(\frac{(m+1)x}{k}\right)^{\frac{1}{m+1}}, & x \leq \frac{k(cA)^{m+1}}{m+1} \\ \frac{x}{k(cA)^m} + \frac{mcA}{m+1}, & x > \frac{k(cA)^{m+1}}{m+1} \end{cases} \quad (9)$$

Additionally, in order to keep an unchanged average power level in this transform, we let  $E\{|y_n|^2\} = E\{|x_n|^2\}$ . Making appropriate substitutions, we obtain

$$A = (3\sigma^2 \frac{m+3}{m+1} * \frac{m(1-c)+1}{m(1-c^3)+3})^{\frac{1}{2}} \quad (10)$$

Given that  $h(x)$  is a strictly monotone increasing function, it can be calculated according to the following identity.

$$h(x) = \text{sgn}(x) * F_{|y_n|}^{-1}[F_{|x_n|}^{-1}(x)] \quad (11)$$

where  $\text{sgn}(x)$  denotes the sign function, which returns the sign of variable  $x$ .

In the NCT approach, the original signal  $x_n$  is transformed according to the companding function  $h(\cdot)$ , which only changes the amplitude of input signal. The transformed signal  $y = [y_0, y_1, y_2, \dots, y_{N-1}]^T$  can be expressed as  $y_n = h(x_n)$ ,  $n = 0, 1, 2, \dots, N-1$  (12)

This operation transforms each OFDM signal sample one at time. At the receiver side, the inverse function  $h^{-1}(\cdot)$  can be used as the corresponding de-companding function.

Substituting (5) and (9) into (11), we obtain  $h(x)$  as

$$h(x) = \begin{cases} \text{sgn}(x) \left(\frac{m+1}{k} (1 - \exp(-\frac{|x|^2}{\sigma^2}))\right)^{\frac{1}{m+1}}, & |x| \leq \gamma_0 \\ \text{sgn}(x) \left(\frac{1 - \exp(-\frac{|x|^2}{\sigma^2})}{k(cA)^m} + \frac{mcA}{m+1}\right), & |x| > \gamma_0 \end{cases} \quad (13)$$

where  $\gamma_0 = (-\ln(1 - \frac{k(cA)^{m+1}}{m+1}))^{\frac{1}{2}}$ . At the receiver, the corresponding de-companding function is

$$h^{-1}(x) = \begin{cases} \text{sgn}(x) \sigma \left(-\ln\left(1 - \frac{k|x|^{m+1}}{m+1}\right)\right)^{\frac{1}{2}}, & |x| \leq cA \\ \text{sgn}(x) \sigma \left(-\ln\left(1 - k(cA)^m \left(|x| - \frac{mcA}{m+1}\right)\right)\right)^{\frac{1}{2}}, & |x| > cA \end{cases} \quad (14)$$

The transfer curves of (13) with various parameters are depicted in Figure 2.

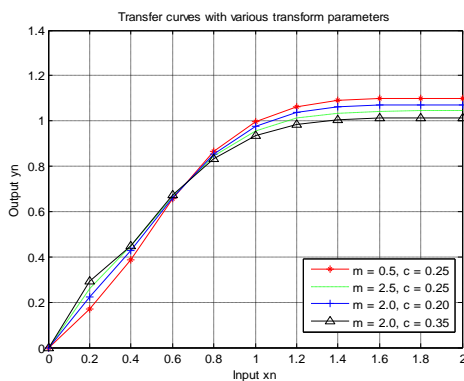


Figure 2 Transfer curves with various transform parameters.

It can be seen from Figure 2 that this method compresses large signals while partially enlarging small ones simultaneously. Besides, it also has the advantage of maintaining a constant average power level in the transform. As a result, not only is the PAPR reduced more effectively, but the immunity of small signals from the channel noise can also be achieved.

#### IV. SIMULATION RESULTS

The CCDF of the PAPR for Non-linear Companded MIMO-OFDM signal is used to express the probability of exceeding a threshold  $PAPR_0$  ( $CCDF = \text{Prob}(PAPR > PAPR_0)$ ). The simulation results of the proposed system with fully companded STBC MIMO-OFDM and conventional STBC MIMO-OFDM systems are compared. Rectangular window and with  $N=1024$  system subcarriers are considered for PAPR reduction analysis of the proposed system in MATLAB. All the simulations have been performed with 1048576 random data. Simulation parameters used are given in Table. 1.

Table 1: SYSTEM PARAMETERS

Oversampling Factor	4
System Subcarriers	1024
OFDM symbols	4370
MIMO Scheme	2x2
Companding	Non-Linear Transform
Modulation	16-QAM
CCDF Clip Rate	$10^{-3}$

Figure 3 shows that CCDF based comparison of PAPR of the proposed system, companded STBC MIMO-OFDM and conventional STBC MIMO-OFDM systems, with  $N = 1024$  for 16-QAM modulation. At clip rate of  $10^{-3}$ , the PAPR gains are 3.0dB, 8.4dB with respective to NCT and Proposed methods respectively.

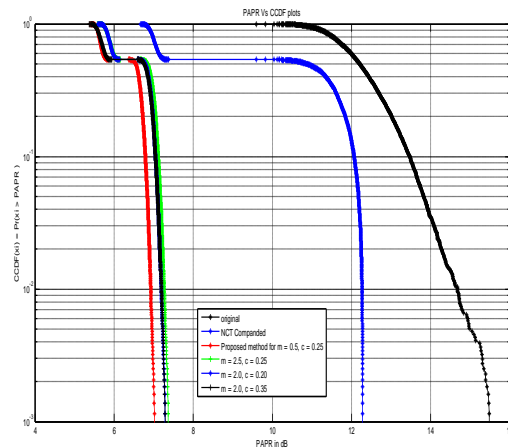


Figure 3 CCDF comparison of PAPR of the proposed system with fully companded STBC MIMO-OFDM and conventional STBC MIMO-OFDM systems, with  $N = 1024$  for 16-QAM.

## V. CONCLUSION

In this paper, we proposed a new PAPR reduction method for STBC MIMO-OFDM Systems. The method is based on Non-linear companding transform scheme. The proposed method is to detect less probable, high PAPR OFDM symbols and then processing only them, leaving rest of the data to be transmitted unchanged. The new method exhibits significant PAPR reduction results since a small number of OFDM symbols are considered for companding process. From the Simulation results it was shown that the proposed method has PAPR gain of 3.0dB and 8.4dB from NCT companded and conventional STBC MIMO-OFDM systems. Furthermore, the proposed system requires less side information than PTS, SLM methods.

## REFERENCES

- [1] T. Jiang and Y. Wu, "An Overview: Peak-to-average power ratio reduction techniques for OFDM signals," *IEEE Trans, on Broadcasting*, Vol. 54, no. 2, pp. 257-268, 2008.
- [2] Wang, L., & Tellambura, C, (2005). A simplified clipping and filtering technique for PAR reduction in OFDM systems, *IEEE Signal Processing Letters*, 12(6), 453-456.
- [3] Le Goff, S. Y., Khoo, B. K., Tsimenidis, C. C., & Sharif, B. S. (2008). A novel selected mapping technique for PAPR reduction in OFDM systems. *IEEE Transactions on Communications*, 56(11), 1775-1779.
- [4] Ghassemi, A., & Gulliver, T. A. (2008). A low-complexity PTS-based radix FFT method for PAPR reduction in OFDM systems. *IEEE Transactions on Signal Processing*, 56(3), 1161-1166.
- [5] Krongold, S., & Jones, D. L. (2003). PAR reduction in OFDM via active constellation extension. *IEEE Transactions on Broadcasting*, 49(3), 258-268.
- [6] Wang, X. B., Tjhung, T. T., & Ng, C. S. (1999). Reduction of peak-to-average power ratio of OFDM system using a companding technique. *IEEE Transactions on Broadcasting*, 45(3), 303-307.
- [7] Jiang, Y. (2010). New companding transform for PAPR reduction in OFDM. *IEEE Communications Letters*, 14(4), 282-284.
- [8] Y. Lee, Y. You, W. Jeon, J. Paik and H. Song, "Peak-to-average power ratio in MIMO-OFDM systems using selective mapping," *IEEE Commun. Lett.*, vol. 7, no. 12, pp. 575-577, 2003.
- [9] J. Gao, J. Wang and Z. Xie, "Peak-to-average power ratio reduction for MIMO-OFDM systems with decomposed selected mapping," *International Journal of Information and Systems Science*, vol. 3, no. 3-4, pp. 572-580, 2009.
- [10] M. Tan, Z. Latinovic and Y. Bar-Ness, "STBC MIMO-OFDM peak-to average ratio reduction by cross-antenna rotation and inversion," *IEEE Commun. Lett.*, vol. 9, no. 7, pp. 592-594, 2005.
- [11] Wang, X. B., Tjhung, T. T., & Ng, C. S. (1999). Reduction of peak-to-average power ratio of OFDM system using a companding technique. *IEEE Transactions on Broadcasting*, 45(3), 303-307.
- [12] Jiang, T., Yang, Y., & Song, Y. (2005). Exponential companding transform for PAPR reduction in OFDM systems. *IEEE Transactions on Broadcasting*, 51(2), 244-248.
- [13] Yong Wang, Jianhua Ge, Lihua Wang, Jing Li "Reduction of PAPR of OFDM Signal Using Nonlinear Companding Transform" Springer Sciences + Business Media, LLC. 2012, Wireless Per Communications.