Reduced Order Modelling using PSO and Discrete Robust Controller Design

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***Abstract*--The objective of this paper is to reduce higher order discrete system to lower order discrete system using the PSO algorithm. Then, the discrete time PID controller has been designed using the same principle to improve peak overshoots and steady state responses of the reduced order model. Finally, the robust controller has been designed by taking another design example. The discrete H_∞ controller is designed using the mixed sensitivity H_∞ control method, based on 2-Riccati state space approach of Glover and Doyle. The proposed methods are illustrated with the help of typical design problems considered from literature**.

Keywords--PSO optimization, order reduction, transfer function, discrete systems, robust control, mixed sensitivity, H_∞ controller synthesis.

I. INTRODUCTION

The model order reduction is an area of control system theory, which studies properties of dynamical systems in application for reducing their complexity, while preserving their input-output behaviour. Modelling physical systems usually results in system of higher order whose order is greater than two. Design of controllers for the working system becomes tedious when the system order is high. Thus the purpose of Model Order Reduction (MOR) is to replace a large system of equations by a smaller one, which preserves the essential properties of the original model.

One of the most important step in the design of reduced order model is to get the optimal value of the coefficients of transfer function. This is not an easy procedure and often needs many iterations. The PSO algorithm is used to obtain these values that lead to obtain the reduced order model by minimizing the integral square error of the output. Then, using the same optimization technique, the discrete PID controller has been designed for the same reduced order model. The PSO method is used because of its simplicity and ease of implementation.

Generally, the model has many uncertainties due to disturbances, parametric variations, noises and so on. Therefore, several robust control methods are available to solve above problems[3][20]. H_∞ control theory can be one of the most powerful solutions for such problems. It is the method in control theory for optimal controller design. It is known that the H_∞ control is an effective method for attenuating disturbances and noise that appear in the system. The "H" stands for Hardy space. "Infinity" means that it is designed to accomplish minmax restrictions in the frequency domain[3][11]. The H_∞ norm of a dynamic system is the maximum amplification that the system can make to the energy of the input signal[12]. In this paper, a method for discrete robust controller for an electromechanical fin servo system of a missile is presented.

II. PARTICLE SWARM OPTIMIZATION ALGORITHM (PSO)

PSO is one of a powerful optimization methods with high efficiency in comparison to other methods. It is a stochastic Evolutionary Computation technique based on the movement and intelligence of swarms. The PSO mechanism is initialized with a population of random solutions and searches for optima by updating generations [18]. A swarm consists of N particles that are moving around in a D dimensional search space. Each particle keeps track of its coordinates in the space of the problem, which are associated with the best solution (best fitness) it has achieved so far. The best particle in the population is denoted by (gbest), while the best position that has been visited by the current particle is denoted by (pbest). The global best individual connects all members of the population to one another. That is, each particle is influenced by every best performance of any member in the entire population. The local best individual is seen as the ability for particles to remember past personal success. The particle swarm optimization concept involves, at each time step, changing the velocity of each particle towards its global best and local best locations.[20] The particles are manipulated according to the following equations of motion.

$$v_{id}^{k+1} = v_{id}^{k} + c_1 * r_1 * (p_{id}^{k} - x_{id}^{k}) + c_2 * r_2 * (p_{gd}^{k} - x_{id}^{k}) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k} \quad (2)$$

Where v is the velocity, x is the position, p_{id} and g_{id} are the pbest and gbest ,k is iteration and c_1,c_2 are the cognition and social parameter. These parameters are variable or constant. Generally these values are '2' and r_1,r_2 are the random numbers in the range (0,1).

III. MODEL ORDER REDUCTION OF DISCRETE TIME SYSTEM

In this section, the proposed model order reduction technique and the design of PID controller for the reduced models have been represented. Consider the following transfer function of discrete time system of n^{th} order:

$$G(z) = \frac{b_1 z^{n-1} + b_2 z^{n-2} + \dots\dots\dots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \dots\dots\dots + a_{n-1} z + a_n} \quad (3)$$

The objective is to find a reduced r^{th} order transfer function such that r < n:

$$G_r(z) = \frac{d_1 z^{n-1} + d_2 z^{n-2} + \dots . + d_{r-1} z + d_r}{z^r + c_1 z^{r-1} + \dots\dots c_{r-1} z + c_r} \quad (4)$$

The reduced model $G_r(z)$ retains the important characteristics of the original model G(z) and the transient responses of this reduced plant should be as close as possible to that of original plant for similar input [1]. The original higher order model is reduced by using the Evolutionary Technique such as Particle Swarm optimization (PSO). In this paper, this method has been used. The objective function is defined as Integral squared error (ISE) of difference between the responses given by the expression:

$$J = \int_0^\infty [y_0(t) - y_r(t)]^2 \, dt \quad (5)$$

Where $y_0(t)$ is the step response of the plant. $y_r(t)$ is the step response of the reduced plant.

There are two approaches for the reduction of discrete system, namely the indirect method and direct method. The indirect method uses some transformation and then reduction is carried out in the transformed domain. First the z- domain transfer functions are converted into s-domain by the bilinear transformation and then after reducing them in s-domain they are converted back into z-domain.
In the direct method the higher order z-domain transfer functions are reduced to a lower order transfer function in the same domain without any transformation. In this paper, the direct method is used for model order reduction of the discrete-time systems.
During the last decade, evolutionary algorithms (EAs) have shown its usefulness for solving optimization problems. This is one of the most promising research fields, an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Therefore the original higher order discrete system is reduced to a lower order model employing Evolutionary technique[18].
The discrete-time system i.e. the original higher order model is reduced to lower order model (2^{nd} order) using optimization techniques such as PSO [1][13] and then a PID controller is designed for the reduced order model using the same optimization techniques of PSO. The same PID controller as designed for reduced model with the same parameters is then used for the higher order model to obtain the desired specifications.

The model order reduction can be done by choosing the structure of the reduced order and then defining objective function using original system transfer function and reduced order model transfer function. This objective function may be representation of difference between two models or the difference between responses of the two models. The objective function so obtained can be minimized by using Evolutionary Algorithms to get the parameters of the chosen reduced order model. The reduced r^{th} order model should satisfy some or all of the following requirements:
1. The approximation error must be small, and a global error bound should exist. Usually this means that the output error $[y_o(t) - y_r(t)]$ should be minimized for some or even all inputs u(t) in an appropriate norm.
2. The order of the reduced system is much smaller than the order of the original system.
3. Preservation of (physical and numerical) properties such as stability and passivity.
4. The procedure must be computationally stable and efficient.
5. The procedure must be based on some error tolerance (automatically) and a cheap measurement for the error is desired.
6. Need to be able to create parameterized reduced models.
Since non-reduced models may have millions of unknowns,
the algorithm must be efficient.

IV. SYSTEM IDENTIFICATION

A. Design example for reduced order model

In this section, example is taken for model order reduction of the discrete-time system[1]. Firstly, discrete-time model of 8^{th} order is taken which is reduced to 2^{nd} order by using optimization techniques PSO. Then a PID controller is designed for the reduced order model by using same optimization techniques and Integral Square Error (ISE) is used as the objective function. The same PID controller is then used for the original higher order model with the same structure and same controller parameters. Consider the transfer function of the plant:

$$G(z) = \frac{0.1625z^7 + 0.125z^6 - 0.0025z^5 + 0.00525z^4 - 0.02263z^3 - 0.00088z^2 + 0.003z - 0.000413}{z^8 - 0.6307z^7 - 0.4185z^6 + 0.0078z^5 - 0.057z^4 + 0.1935z^2 + 0.09825z^2 - 0.0165z + 0.00225} \quad (6)$$

The objective is to find out the 2nd order reduced model by using Particle Swarm Optimization (PSO) of the form[1]:

$$G_r = \frac{A_r z + B_r}{C_r z^2 + D_r z + E_r} \quad (7)$$

To reduce the higher order model in to a lower order model firstly PSO is employed[1][2]. The objective function *J* defined as an integral squared error of difference between the responses given by the equation (8) is minimized by firstly PSO.

$$J = \int_0^\infty [y_0(t) - y_r(t)]^2 \, dt \quad (8)$$

where yo(t) is the step response of the original plant and yr(t) is the step response of the reduced plant.

The transfer function for reduced model obtained from the technique PSO is:

$$\frac{0.0050898z - 0.0024351}{0.032614z^2 - 0.0573272z + 0.027169} \quad (9)$$

The ISE (integral square error) is 0.000741. The results and simulations are explained in later sections.

B. *Discrete PID Controller Design using PSO*

The PID (Proportional Integral Derivative) is one of the earlier control strategies. The PID controller is the most common form of feedback. According to a survey for process control systems conducted in 1989, more than 95% of the control loops are of PID type. In this paper, the discrete PID controller has been developed. The integral term is used to reduce the overshoots whereas the derivative term is used to reduce the steady state error. The k_p ,k_i and k_d are tuned by using PSO algorithm. The integral square error i.e the objective function is minimized as explained in below given block diagram.

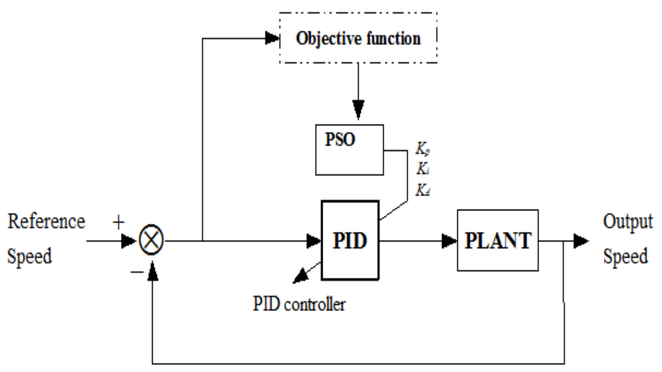

Fig. 1 Block diagram of plant with discrete time PID controller tuned by PSO algorithm

The resulting PID parameters for reduced order model obtained are shown in table I:

TABLE I. PID PARAMETERS OBTAINED WITH PSO

Controller	K_p	K_i	K_d
PID (PSO)	2.7674	0.636	2.7133

The simulation of plant with and without PID controller is shown in later section.

C. *Electromechanical fin servo system*

The electro-mechanical fin actuation servo system, which is the object of this research, is composed of a DC Motor, a gearing mechanism, sensors, and a Microprocessor-controlled PWM servo-amplifier. This system is taken for discrete H∞ controller design. Consequently, the state equation of the fin servo system is taken from [10].

D. *H∞ controller design*

In this paper, the mixed sensitivity H∞ design method, which utilizes the sensitivity function S(z) and the complementary sensitivity function T(z). The mixed sensitivity H∞ design method is expressed as:

$$S(z) + T(z) = I \quad (10)$$

Our aim is to find the discrete controller K that minimize the weighted sensitivity and weighted complementary sensitivity function over different frequency ranges.[10] This can be expressed in terms of linear fractional transformation $F_l(P,K)$ where P is generalized plant i.e nominal plant and their weighting functions and K is controller. The block diagram is shown in Fig. 2.

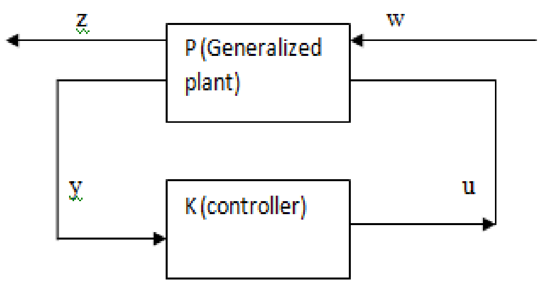

Fig. 2 Block diagram of linear fractional transformation

Here, the generalized plant comprises of nominal plant and its associated weighting functions. The sensitivity weighting function given by W_1. This weighting function is used to limit the magnitude of the sensitivity function within a particular frequency range. This is performance function (measure) in the controller synthesis.[3][9]
Whereas the complimentary sensitivity weighting function given by W_2 , which is used to limit the magnitude of T within particular frequency range. This is robustness weighting function in the controller synthesis.[3][9][12]

The weights chosen are such that the weighted sensitivity function is reduced below 3rad/sec and weighted complementary sensitivity function above 20 rad/sec[20]. The weighting functions are given by equations (11) and (12) :

$$W_1 = \frac{(s/1.2+3)}{(s+3*0.01)} \tag{11}$$

$$W_2 = \frac{(s+20/3)}{(0.01s+20)} \tag{12}$$

So, the definition is modified as : to find the controller k such that the H∞ norm of weighted sensitivity and weighted complementary functions are minimized over some frequency range[3] i.e.

$\left\| \begin{matrix} W_1 S \\ W_2 T \end{matrix} \right\|_\infty = F_l(P,K)$ is to be minimized.

IV. SIMULATIONS & RESULTS

A. Model order reduction of discrete time system

In this figure, both the reduced and original plant responses are similar and overlapping with same steady state and settling time .

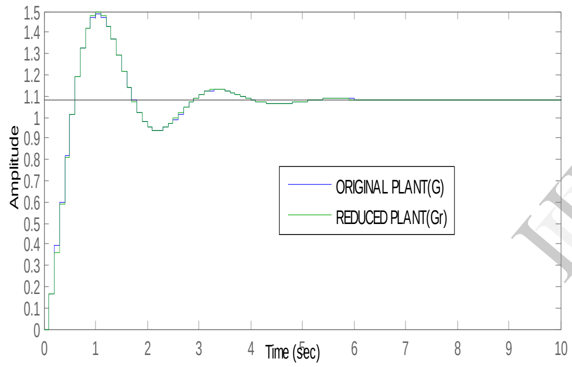

Fig. 3 Step responses of original and its reduced order model with sampling time 0.1 sec.

B. Discrete PID Controller of Reduced Order Model

The response of reduced plant with and without PID controller using PSO technique is shown below. This shows that the introduction of PID controller to plant has reduced the overshoots and settling time significantly.

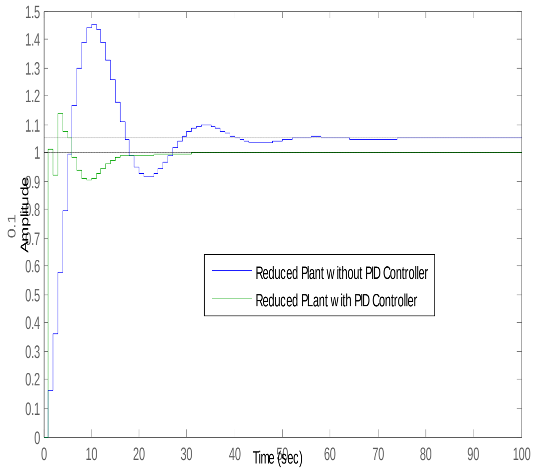

Fig. 4 Step responses of reduced plant with and without PID controller with sampling time 0.1 sec.

B. Discrete H∞ controller of electromechanical fin actuator

The discrete H∞ controller transfer function is coming out to be using hinfsyn command and then discretizing it at sampling time=0.1s:

$$k = \frac{45.21z^3 - 45.21z^2 - 1.427e-012z + 8.044e-030}{z^4 - 0.997z^3 - 1.155e-026z^2 + 1.733e-053z - 1.659e-084} \tag{13}$$

The step response of plant with the discrete H∞ controller is shown in Fig. 4. This shows the difference in the two plots for the same closed loop system due to the different values of damping factors. The table II shows the variation in performance characteristics of the closed loop system with the change in damping factor. It is clear that the overshoots are reduced from 46.7% to 11.783%.

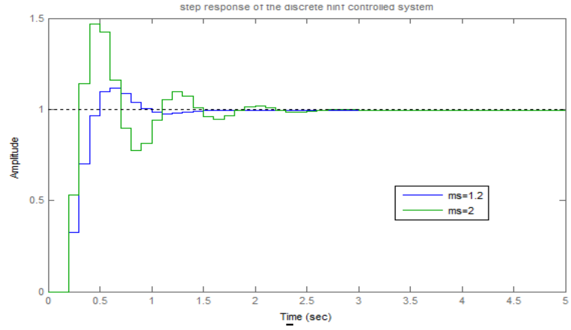

Fig. 4 Variation in Step responses of electromechanical fin actuator with H∞ controller with the change in damping factors and with sampling time 0.1 sec.

TABLE II. PERFORMANCE SPECIFICATIONS

Characteristics	**Ms=2(damping factor low)**	**Ms=1.2(damping factor large)**
Rise time	0.1seconds	0.2seconds
Settling time	1.8seconds	1.3 seconds
Overshoot	46.7%	11.7830%
Peak	1.46	1.1175
Peak time	0.4seconds	0.6seconds

V. CONCLUSION

In this paper, firstly the reduced order model is obtained from original higher order model using PSO technique. Then, the discrete PID controller is obtained using the same technique. This method guarantees stability of the reduced model if the original high order system is stable and which exactly matches the steady state value of the original system. PSO method is based on the minimization of the integral squared error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. From this method, it is concluded that the proposed method is simple; computer oriented and achieves better approximations than the other existing methods. The design of discrete PID controller improves the settling time as well as peak overshoots.

In next part of the paper, a discrete robust controller for a fin position servo system of a missile has been designed. This structure uses discrete H∞ controller design. The performance of the designed controller has been examined through simulations and experiments.

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