Reconfigurable controller design for continuous stirred tank reactor

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***Abstract*-In this paper, an optimized Eigenstructure Assignment based reconfigurable Linear Quadratic Regulator (LQR) controller is designed for linear discrete Continuous Stirred Tank Reactor (CSTR). The servo operation is achieved by augmenting the linear discrete state and Proportional-Integral (PI) control action is incorporated in the design. LQR parameters are optimized using Particle Swarm Optimization (PSO) and Parallel implementation of Genetic Algorithm (PI-GA). The closed-loop performance of the normal (Fault-free) system obtained using PSO and PI-GA are compared. Then, the best optimized technique is used to reconfigure the controller parameters against partial actuator failure in the process.**

Key Words-Eigenstructure Assignment, Reconfigurable controller, LQR, PSO, PI-GA, PI, CSTR.

I. INTRODUCTION

With the growing complexity of modern engineering and ever increasing demand for safety and reliability there has been great interest in the development of Fault Tolerant Control Systems (FTCS) [1, 2, 6, 11, 13]. Thus FTCS are closed-loop control systems which can tolerate system component faults/failures, and are capable of maintaining stability and a certain degree of performance of the system. It can be divided into passive FTCS and active FTCS. Passive methods are essentially robust control techniques which are suitable for dealing with certain types of structure failures that can be modeled as uncertainty regions around a nominal model. In contrast with passive FTCS, active FTCS react to the system component failures actively either by selecting a pre computed control law (projection-based approaches) or by synthesizing a new control strategy on-line (on-line automatic controller redesign approaches [1].

Typically, an active FTCS consists of three parts: a reconfigurable controller, a Fault Detection and Diagnosis scheme, and a control law reconfiguration mechanism. Key issues are how to design: 1) a robust reconfigurable controller, 2) an FDD scheme with high sensitivity to faults and robustness to model uncertainties and external disturbances, and 3) a reconfiguration mechanism which can organize the reconfigured controller such that the eigenstructure of the reconfigured faulty closed-loop system should recover its eigenstructure as close as possible to normal closed-loop system. Several reconfigurable control techniques have been proposed in the literatures. However, a very few techniques have dealt with multi-input and multi-output (MIMO) systems. Linear Quadratic Regulator (LQR) and Eigenstructure Assignment (EA) are the popular techniques to design controller for MIMO systems. In general, the LQR-based control design guarantees the closed-loop stability and certain degrees of robustness, but may not easily achieve specific system performances due to the difficulty in the selection of the weighting matrices. EA-based reconfigurable controller (RC) design can be found in the literatures [4,6].The advantage of EA is that if the specifications are given in terms of system eigenstructure, the eigenstructure can be achieved exactly for the desired stability and dynamic performance. Here EA is used for RC system design while LQR is used for normal system.

This paper focuses on design of an integrated design of reconfigurable control for Multi-Input and Multi-output (MIMO) systems. The scheme uses Proportional-integral (PI) control structure in the reconfigurable controller so as to recover both the dynamic and steady-state performance of the pre-fault systems. An optimized eigenstructure assignment based reconfigurable state feedback controller is developed to achieve automatic redesign of the controller that compensates partial actuator failures in the Continuous Stirred Tank Reactor (CSTR).Here reconfiguration is carried out against the availability of a perfect Fault Detection and Diagnosis (FDD) and detects the fault after a delay time of 1 sampling instant [1,4,6].

The paper is organized as follows. In Section 2, Continuous Stirred Tank Reactor is described, actuator fault is presented. Section 3 presents the problem formulation. Section 4 and 5 discusses the reconfigurable controller design and simulation respectively.

II. SYSTEM MODELING

A. Continuous Stirred Tank Reactor

The continuous stirred tank reactor (CSTR), also known as vat- or back mix reactor is a common ideal reactor type in chemical engineering. The CSTR process assumed here is the linear time invariant multi input – multi output process shown Fig 1. It is non-isothermal first order process with an irreversible exothermic reaction (A -> B) that occurs in constant volume. The heat of the reaction is removed by a coolant medium that flows around the reactor. It has two controlled variable namely outlet concentration (C_A) and outlet temperature (T) and manipulated variable are coolant flow rate (qc) and reactant flow rate (q).

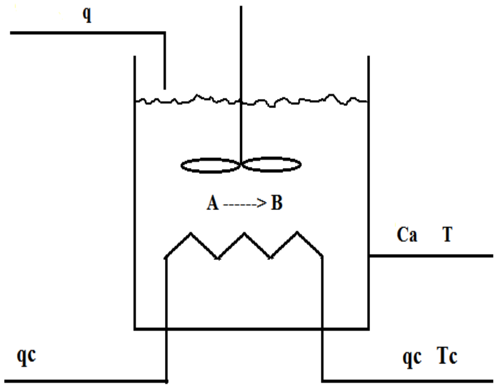

Figure 1. CSTR schematic diagram.

The schematic diagram of the CSTR is depicted in Figure 1 as defined by Sentil et al. (2007) [8].The dynamic behavior of CSTR can be described by mass balance and energy balance. Using mass balance and energy balance, the process is modeled by:

$$\frac{dC_A(t)}{dt} = \frac{q}{v}\left(C_{A0}(t) - C_A(t)\right) - k_0 C_A(t)\exp\left(\frac{-E}{R\overline{T}(t)}\right)$$

$$\frac{dT(t)}{dT} = \frac{q}{v}\left(T_0(t) - \overline{T}(t)\right) + \left(\frac{\Delta Hk_o}{\rho c_\rho} C_A(t)\exp\left(\frac{-E}{RT(t)}\right)\right)$$

$$+ \frac{q_c \rho_C C_{\rho C}}{\rho C_\rho v} \times \left(T_{CO}(t) - T(t)\right)\left(1 - \exp\left(\frac{-hA}{\rho q_c c_\rho}\right)\right)$$

In order to obtain a linear discrete model, the nonlinear system is linearized around the steady-state operating points. The linearization procedure is based on the expansion of the nonlinear function into Taylor series around the operating point and the retention of only the linear term. The physical parameters of the continuous stirred tank reactor are shown in Table 1.

B. Modeling of system

The discrete state space representation for the fault-free model (nominal) is given by

$$x(k+1) = A\,x(k) + B\,u(k)$$

$$y(k) = C\,x(k) \qquad (1)$$

where

$$A = \begin{bmatrix} 0.2248 & -0.0034 \\ 133.3 & 1.5010 \end{bmatrix} \quad B = \begin{bmatrix} 0.0007 & 0.0001 \\ -0.0423 & -0.0934 \end{bmatrix}$$

$x(k) = [C_A \quad T]^T$, $u(k) = [q \quad q_C]^T$, $y(k) = [C_A \quad T]^T$ are the state, input, and output vectors respectively where $[q \quad q_C]^T = [100 \quad 103]^T$ l/min and $[C_A \quad T]^T = [0.0989 \quad 438.7763]$

TABLE 1
Physical Parameters of the CSTR

parameters	***value***
Feed Concentration (C_{AO})	1 mol/l
Feed temperature (To)	350 K
Inlet coolant temperature (Tco)	350 K
CSTR volume (V)	100 l
Heat transfer term (hA)	7 x 10^5 cal/(min K)
Reaction rate constant (ko)	7.2 x 10^{10} min^{-1}
Activation energy term (E/R)	1 x 10^4 k
Heat of reaction (-ΔH)	-2 x 10^5 cal/mol
Liquid density (ρ,ρ_c)	1 x 10^3 g/l
Specific heats (C_P ,C_{PC})	1 cal/(g K)

C. Modelling of faulty system

Modelling of Actuator Faults as Loss in Control Effectiveness Factor:

In this paper, abrupt type bias fault has been considered and it is represented by control effectiveness factor (γ) reduction. During normal operation actuator could deliver control signal without any loss. When fault occurs in the actuator due to partial blockage or aging, the actuator cannot deliver control signal without loss. Such a failure condition is represented by reduced control effectiveness factor [11].The faulty model given by.

$$x_f(k+1) = A_f\,x_f(k) + B_f\,\gamma\,u_f(k) \qquad (2)$$

$$y_f(k) = C\,x_f(k)$$

III. PROBLEM FORMATION.

A. Augmented Dynamic System

To solve the command tracking in the presence of noise, disturbance and uncertainty, PI control is incorporated in the proposed scheme [11]. To incorporate the integral action into the controller design, one popular approach is to expand an integrator vector as an augmented state. In the discrete domain, the state-space realization of the integrator vector can be described in:

$$e^I(k+1) = e^I(k) + T_S e(k) \qquad (3)$$

Where e(k) is the tracking error, Ts is the sampling time.

The augmented system is derived from Equations (1) and (3) and is represented by

$$\tilde{x}(k+1) = \overline{A}\,\tilde{x}(k) + \overline{B}u(k) + Er(k)$$

$$y(k) = \overline{C}\tilde{x}(k)$$

Where $\tilde{x}(k+1)=\begin{bmatrix} x(k+1)\\ s(k+1)\end{bmatrix}$ $\tilde{x}(k)=\begin{bmatrix}x(k)\\ s(k)\end{bmatrix}$ and $\bar{A}=\begin{bmatrix}A & 0\\ T_sC & I\end{bmatrix}$

$$\bar{B}=\begin{bmatrix}B\\ 0\end{bmatrix} \quad E=\begin{bmatrix}0\\ T_sI\end{bmatrix} \quad \bar{C}=\begin{bmatrix}C & 0\\ 0 & I\end{bmatrix}$$

The design of state feedback controllers based on the augmented system can allow the computation of proportional and integral controller gains for the system. PI control incorporated Reconfigurable control illustrated in Figure 2.

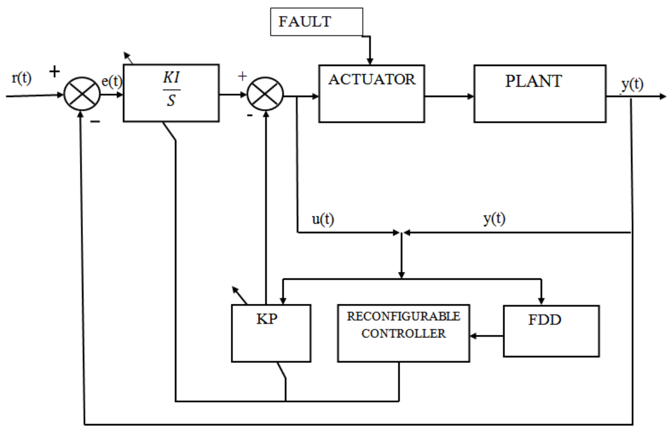

Figure 2 PI Integrated Reconfigurable control

B. Mathematical formulation of the ea and the lqr problems

In this work, the natural algorithm (PI-GA and PSO) can be used to search the weighting matrices Q and R with Q is a [n x n] matrix and R is a [m x m] matrix. The design problem is described as follows:

The LQR is produced by the minimization of the following cost function:

$$J = \int (x^T Q x + u^T R u)\, dt \qquad (4)$$

in which $Q \geq 0$ and $R > 0$ that defining respectively, the state and the input weighting matrices of the LQR optimization problem. According to the LQR technique, the system (Eq. (1)) is controlled by the state feedback

$$u = K_R^Q u \qquad (5)$$

The closed-loop system representation is given by:

$$\dot{X}(t) = A_C x(t) = (A + B\, K_R^Q)\, x(t) \qquad (6)$$

Via the following Algebraic Ricatti Equation (ARE)

$$A^TP + PA - PBR^{-1}B^TP+Q= 0 \qquad (7)$$

We can determine the solution P and the gain K_R^Q

$$K_R^Q= R^{-1}B^TP \qquad (8)$$

A suitable choice of the Q and R leads to the computation of the gain K_R^Q. Therefore, the eigenvalues and eigenvectors of the closed-loop system can be chosen and the system performances are provided. The weighting matrices Q an R may be manipulated by natural algorithm in order to force the system to behave in the required manner by the designer, in this case to force an eigenstructure upon the closed loop system. Provided that the controller is a solution of the Algebraic Ricatti Equation (Eq. (7)), then the performance and robustness of the LQR will be maintained. The main aim is to derive a cost function value from the difference between desired and achieved eigenstructure determined by their eigenvalues $\{\lambda_{di}, \lambda_{ai}\}$ and their corresponding eigenvectors $\{v_{di}, v_{ai}\}$, $\forall\, i$ =1,2,…..n.

For this, we define the following criteria:

(i) $C_1 = \mathrm{Max}_i\{|\lambda_{di} - \lambda_{ai}|\}, \forall i = 1,2,.n$ the maximum norm of the difference between the desired and achieved eigenvalues.

(ii) $C_2 = \mathrm{Max}_i\{\| v_{di} - v_{ai}\|^2\}, \forall i = 1,2,......n$, the Euclidean distance between $(\vec{v}_{ai})$ and $(\vec{v}_{di})$

Then, objective is to minimize the global criterion Cg defined by

$$C_g = \max\{C_1, C_2\}$$

Thus, a zero cost function value represents the ideal solution. The minimization of the cost function is achieved through manipulation of these Q and R matrices by the natural algorithm. Based on the above criteria, an algorithm for the optimized closed loop eigenstructure assignment can be given.

IV. RECONFIGURABLE CONTROL TECHNIQUE:

The Optimized Eigen structure Assignment based LQR controller discussed in previous section can be extended to recover the performance of the closed-loop system, when the system is affected by actuator fault. Faulty system can be recovered by reconfiguring the controller gain of the nominal system [11]. State feedback controller gain (K_f) is determined by replacing $(\bar{A}_f, \bar{B}_f)$ with augmented faulty system $(\bar{A}, \bar{B})$ in the Eq.(1) where

$$\bar{A}_f = \begin{bmatrix}A_f & 0\\ -T_sC & I\end{bmatrix} \quad \text{and} \quad \bar{B}_f = \begin{bmatrix}B_f\\ 0\end{bmatrix}$$

A. Particle Swarm Optimisation Via Lqr Method

Considering the social behaviour of swarm of fish, bees and other animals, the concept of PSO is developed. PSO is a robust stochastic evolutionary computation method based on the movement of swarms looking for the most fertile feeding location [5].In general, PSO implementation is easier than GA. Indeed, PSO only has one operator; velocity calculation, so the computation time is decreased significantly and has the ability to control convergence through manipulating of the inertial weight.

All solutions in PSO can be represented as particles in a swarm. Each particle has a position and velocity vector and each position coordinate represents a parameter value. Similar to the most optimization techniques, PSO requires a fitness evaluation function relevant to the particle's position. X_{PB} and X_{GB} are the personal best (Pbest) position and global best (Gbest) position of the ith particle. Each particle is initialized with a random position and velocity. The velocity of each particle is accelerated toward the global best and its own personal best based on the following equation [8].

$$V_{i\,(new)} = w \times V_{i\,(old)} + c_1 \times rand() \times (X_{PB} - X_i) + c_2 \times Rand() \times (X_{GB} - X_i)$$

Here rand() and Rand() are two random numbers in the range [0,1]; c_1 and c_2 are the acceleration constants and w is the inertia weight factor. The parameter *w* helps the particles converge to G_{best}, rather than oscillating around it.In general, *w* is set according to the following equation :

$$w = 0.5(1 + rand(0,1))$$

The positions are updated based on their movement which is given as follows:

$$X_i = X_i + V_i \tag{6}$$

Then the fitness at each position is reevaluated. If any fitness is greater than G_{best}, then the new position becomes G_{best}, and the particles are accelerated toward the new point. If the particle's fitness value is greater than P_{best}, then P_{best} is replaced by the current position. Here PSO is used to optimize the LQR tuning matrices (Q and R) in order to achieve desired eigenstructure assignment. PSO algorithm parameters are set based on trial and error as follows: Population size = 50 particles, Generation number =100 and acceleration constants: C_1=1.5, C_2=1.5.

B. Parallel implementation of genetic algorithm via lqr method

The Genetic Algorithm (G.A) is an optimisation scheme with an unconventional approach and is robust over a large range of non-convex problems**.** In normal genetic algorithm, the fitness and constraint functions are evaluated separately at each member of a population. But in Parallel-implementation of genetic algorithm, the fitness and constraint evaluations are done parallel at each member of a population, which occurs once per iteration. The motivation of parallelisation of the algorithm is speed-up of processing time [7]. The parameters used in PI-GA algorithm are: Population size=50**,** Generation number = 50, Crossover rate **=**0.8**,** Mutation rate = 0.005, Elitism number = 2**.**

V.SIMULATION RESULTS AND PERFORMANCE EVALUATION.

The desired eigenvalues and vectors are designed in terms of the time domain specifications .The desired eigenvalues (λ_{di}) and vectors (v_{di}) are computed as:

$$\lambda_{di} = \begin{bmatrix} 0.8590 + 0.222i \\ 0.8590 - 0.222i \\ 0.8800 \\ 0.900 \end{bmatrix}$$

$$v_{di} = \begin{bmatrix} 0.0049 - 0.0057i & 0.0049 + 0.0057i & 0.0032 & -0.0017 \\ -0.9882 & 0.9882 & -0.6536 & 0.009 \\ 0.0011 - 0.0004i & 0.0011 + 0.0004i & 0.0037 & -1.0 \\ -0.1347 - 0.0730i & -0.1347 + 0.0730i & -0.7568 & -0.0046 \end{bmatrix}$$

case i:Normal condition:

Optimized Eigenstructure Assignment based LQR controller using Particle Swarm Optimization and Parallel implementation-Genetic Algorithm is used to control the normal system. The achieved eigenvalues (λ_{ai}) and eigenvectors (v_{ai}) obtained through Particle Swarm Optimization and Parallel implementation-Genetic Algorithm are given Table 1.

TABLE 2. Performance comparison between PI-GA and PSO

	LQR - PI-GA	LQR - PSO
ISE	1.161*10-05	7.7202*10-06
CONTROL EFFORT	9.976*e+007	9.978*e+007

Case II Abnormal condition:

Here control effectiveness factor (30%) is taken as γ=0.7.From Table 2, the closed-loop performance of the normal system obtained using PSO and PI-GA**:** PSO gives better performance than PI-GA. So the PSO is used for recovery of the abnormal system. The state feedback gain matrix of the abnormal system is same as normal system**.**

Case III Recovered condition:

Optimized Eigenstructure Assignment based LQR controller using Particle Swarm Optimization is used to solve the reconfiguration problem. The achieved eigenvalues (λ_{ai}) and eigenvectors (v_{ai}) and the recovered gain (K_f) obtained through Particle Swarm Optimization for reconfigurable control is given by:

$$\lambda_{ai/PSO} = \begin{bmatrix} 0.8568 + 0.2243i \\ 0.8568 - 0.2243i \\ 0.9075 \\ 0.989 \end{bmatrix}$$

$$v_{ai/PSO} = \begin{bmatrix} -0.0046+0.0016i & -0.0046-0.0016i & -0.0036 & 0.0014 \\ 0.9539 & 0.9539 & 0.7183 & 0.0011 \\ -0.0012-0.0009i & -0.0012+0.0009i & -0.0035 & 0.7951 \\ 0.1618+0.2527i & 0.1618-0.2527i & 0.6957 & 0.6065 \end{bmatrix}$$

$$K_{f(LQR-PSO)} = 1.0e+003 * \begin{bmatrix} 0.0277 & -0.0013 & -0.0028 & 0.0001 \\ -1.4506 & -0.0126 & 0.0002 & 0.0059 \end{bmatrix}$$

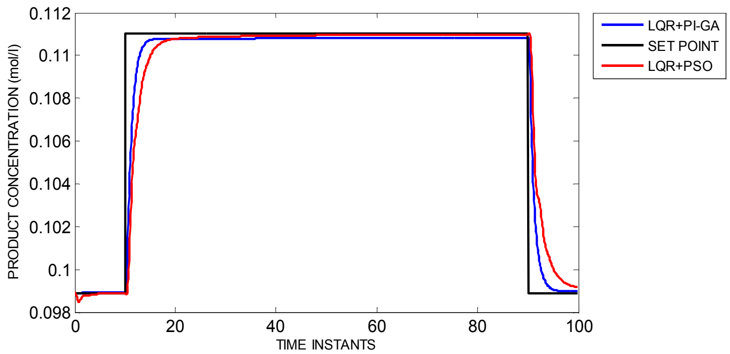

Figure 3. Reactor concentration (mol/l) of the normal.

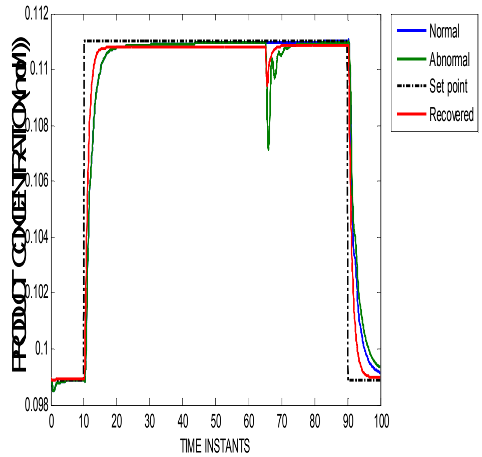

Figure 5. Reactor Concentration for 30% actuator failure introduced at 65th sampling instants.

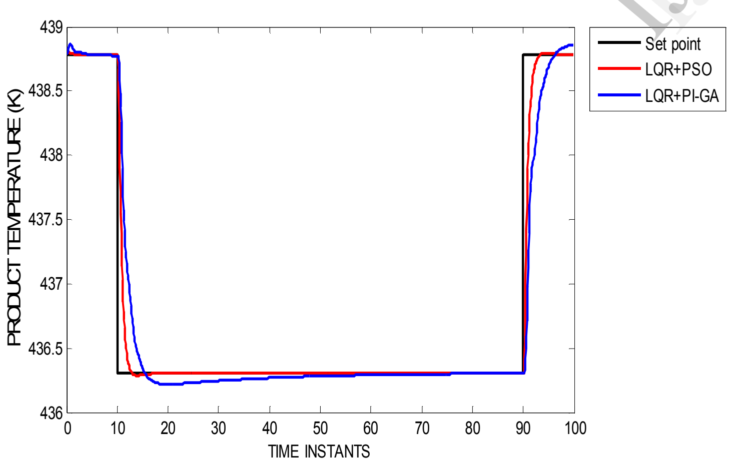

Figure 4. Reactor Temperature (K) of the normal.

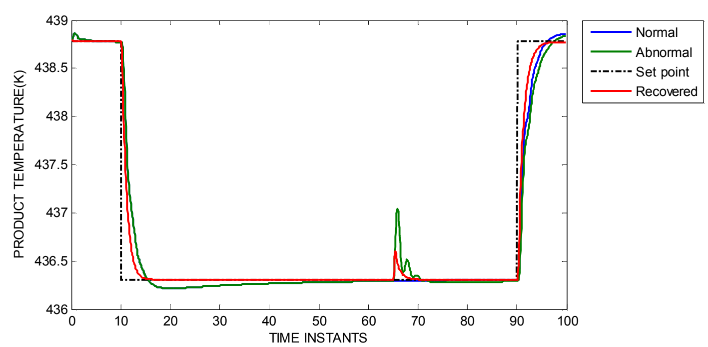

Figure 6. Reactor Temperature for 30% actuator failure introduced at 65th sampling instants.

	ACHIEVED EIGEN VALUE AND EIGEN VECTOR	GAIN
LQR+PSO	$\lambda_{ai\,/\,PSO} = \begin{bmatrix} 0.8580 + 0.2219i \\ 0.8580 - 0.2219i \\ 0.9159 \\ 0.989 \end{bmatrix}$ $v_{ai/PSO} = \begin{bmatrix} 0.0046 - 0.0016i\ \ 0.0046 & -0.0046 - 0.0017i & -0.0038 & 0.0008 \\ -0.9557 & -0.9557 & 0.7443 & 0.0006 \\ -0.0012 - 0.0009i & -0.0012 + 0.0009i & -0.0034 & 0.7792 \\ 0.1631 + 0.2449i & 0.1631 - 0.2449i & 0.6678 & 0.6268 \end{bmatrix}$	$K = \begin{bmatrix} 77.7744 & 0.3809 & -17.6834 & -0.3070 \\ -116.7231 & -0.7949 & -109.6617 & 0.0383 \end{bmatrix}$
LQR+PI-GA	$\lambda_{ai\,/\,PI-GA} = \begin{bmatrix} 0.8517 + 0.2377i \\ 0.8517 - 0.2377i \\ 0.9755 \\ 0.9999 \end{bmatrix}$ $v_{ai/PI\text{-}GA} = \begin{bmatrix} -0.0048 + 0.0020i & -0.0048 + 0.0020i & -0.0044 & 0.004 \\ 0.9659 & 0.9659 & 0.8321 & 0 \\ 0.0012 + 0.0006i & 0.0012 - 0.0006i & -0.0029 & 1.00 \\ 0.1659 + 0.1989i & 0.1659 - 0.1989i & 0.5546 & 0.6357 \end{bmatrix}$	$K = \begin{bmatrix} 52.43 & 0.2873 & -15.6774 & -0.3194 \\ -75.453 & -0.535 & -97.45 & 0.52373 \end{bmatrix}$

TABLE 3. ACHIEVED EIGEN STRUCTURE BY PSO AND PI-GA FOR NORMAL SYSTEM

In Figure 5 and 6, the response shows that under normal condition the process variables are able to track the set point changes for given step input. To demonstrate the performance of Reconfigurable state feedback controller optimized by the Particle Swarm Optimization technique, the fault is introduced at 65^{th} sampling instant in terms of loss in the control effectiveness factor (30%) i.e. γ=0.7. The faulty system response (represented in Green line) shows the presence of undershoot and oscillation which is reduced in the Recovered system response (represented in Red line).

CONCLUSION

In this thesis the design of an optimized Eigenstructure Assignment based reconfigurable LQR control is designed for linear discrete Continuous Stirred Tank Reactor (CSTR) against partial actuator failure and is investigated. To perform the servo operation linear discrete state is augmented and Proportional-Integral (PI) control action is incorporated in the design. LQR parameters (Q-state weighting matrix and R-input weighting matrix) are optimized using Particle Swarm Optimization (PSO) and Parallel implementation of Genetic Algorithm (PI-GA). The closed-loop performance of the normal (Fault-free) system obtained using PSO and PI-GA are compared. The best closed-loop performance of the nominal system obtained using PSO technique is used to reconfigure the controller parameters against partial actuator failure in the process.

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