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Reactive Power Optimization by using Sequential Quadratic Programming

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Abstract— **Electrical power transmission losses in a power system are minimized by adjusting the reactive power control variables like transformer tap ratio, generator voltage magnitude and reactive power compensation devices such as capacitor banks. Also it results in improved voltage profiles, system security and power transfer capability. In this article, reactive power optimization, including comprehensive concern of the practical constraints was considered. The reactive power optimization problem was formulated and solved by using a Sequential Quadratic Programming method. To study the application of this method, a simulation test was carried out on a 3-bus & 6-bus system. It was found that the optimization problem was solved while the satisfying all the constraints imposed.**

Keyterms—Reactive Power Optimization, Optimal Power Flow, Active Power Loss, Sequential Quadratic Programming.

I Introduction

The OPF problem was defined in the early 1960's [1] as an extension of conventional economic dispatch to determine the optimal control settings in a power system networks with various constraints [2]. Today any problem that involves the determination of the instantaneous "optimal" steady state of an electric power system is referred to as an optimal power flow problem. The OPF function can be designed to operate in real time or study mode: to schedule active /or reactive-power controls and to optimize a defined operational objective function [3].

OPF is a static constrained non-linear optimization problem, whose development has closely followed advances in numerical optimization techniques and computer technology. It has been a traditional optimization problem in power system control/planning. It schedules the power system so as to optimize a given objective function subjected to a set of non-linear equality and inequality constraints [4]. In general, the equality constraints are the real and reactive power balance at each bus of the system; the inequality constraints are the physical and operational limits on the control and state variables of the system.

Reactive power optimization (RPO) is one of the important optimization in power system optimization. The RPO is a large-scale highly constrained nonlinear optimization problem. The main task here is to minimize the power losses of transmission and maintain the bus voltage magnitudes within the designated limits for secure and efficient operation of power system.

Gradient-based search algorithms and various mathematical programming methods have been proposed to deal with Optimal Power Flow (OPF) problems.-A number of techniques such as dynamic programming [5], linear programming [6] and interior point methods [7] has been proposed and used to solve reactive power optimization problem. However these techniques have several limitations in handling nonlinear, discontinuous objective functions and constraints [8]. The further discussions on these techniques will be discussed in [9].

II Problem Formulation

The objective of reactive power optimization is to minimize system active power losses, to improve the voltage profile, and to determine optimal reactive power compensation placement keeping the values of various operational parameters within their limits. To achieve these objectives a set of control variables such as generator voltages magnitude, shunt capacitor banks, transformer tap ratio are chosen. The real power loss is a nonlinear function of bus voltage magnitude and phase angles which are the functions of control variables. The power loss function can be expressed as

$$\min P_{loss} = \sum_{\substack{k \in N_l \\ k \in (i,j)}} g_k \left(V_i^2 + V_j^2 - 2V_i V_j \cos\theta_{ij} \right) \qquad (1)$$

where,

is the total real power loss
is the number of branches in the system
is the conductance of branch *k*
is the voltage magnitude at bus
is the voltage magnitude at bus *j*
is the voltage angle difference between *i* and *j*

The real power loss function, given by equation (1), is subjected to equality and inequality constraints. The equality constraints are the real and reactive power balance at each node of power system.

A. Equality constraints

In power system, the total active power balance is expressed as

$$P_i - V_i \sum_{j=1}^{N_B} V_i\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) = 0 \qquad (2)$$

$i = 1,2, \ldots \ldots . N_B - 1$

Similarly, the total reactive power balance is expressed as

$$Q_i - V_i \sum_{j=1}^{N_B} V_i\left(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}\right) = 0 \qquad (3)$$

$i = 1,2, \ldots \ldots . N_{PQ}$

where,

is the real power injected into network at bus *i*
is the reactive power injected into network t bus *i*
is the mutual conductance between bus *i* and *j*
is the self-conductance of bus *i*
is the mutual susceptance between bus *i* and *j*
is the self susceptance of bus *i*
is the total number of buses
is the total number of buses excluding slack bus
is the number of *PQ* buses

The inequality constraints are as follows:

B. Voltage Constraints

Generator voltages outputs are restricted by their lower and upper limits as follows:

$$V_i^{min} \le V_i \le V_i^{max} \quad ; \quad i \epsilon N_B \qquad (4)$$

is the generator minimum voltage magnitude.
is the generator maximum voltage magnitude.

C. Generator reactive power capability limit

Generator reactive power outputs are restricted by their lower and upper limits are as follows:

$$Q_{gi}^{min} \le Q_{gi} \le Q_{gi}^{max} \quad ; \quad i \epsilon N_g \qquad (5)$$

is the minimum generator reactive power.
is the maximum generator reactive power.
is the number of generator buses.

D. Reactive power compensation limits

Capacitor reactive power outputs are restricted by their lower and upper limits are as follows:

$$Q_{ci}^{min} \le Q_{ci} \le Q_{ci}^{max} \quad ; \quad i \epsilon N_c \qquad (6)$$

is the reactive power generated by capacitor bank
is the number of capacitor banks

E. Transformer tap ratio

Transformer tap ratios are bounded as follows:

$$t_k^{min} \le t_k \le t_k^{max} \quad ; \quad k \in N_T \qquad (7)$$

is the number of tap ratio transformer branches
is the tap ratio of transformer at branch *k*

III. Sequential Quadratic Programming

Sequential Quadratic Programming (SQP) [10] is a form of Non Linear Programming (NLP). The objective function of SQP optimization model is quadratic and the constraints are in linear form. SQP has higher accuracy than Linear Programming (LP)-based approaches. The NLP having lot of practical importance and is referred to as quadratic optimization. Derivation of the sensitivity method is aimed at solving the NLP on the computer. SQP is also very important because many of the problems are often solved as series SQP problems.

ALGORITHM

A quadratic programming is a special case of NLP problem wherein the objective function is quadratic and the constraints are linear. Both the quadratic approximation of the constraints are based on Taylor series expansion of the nonlinear functions around the current iterate

Step 1: The objective function is replaced by a quadratic approximation as

$$q^k(D) = \nabla f(X^k) + \frac{1}{2} D^T \nabla_{zz}^2 L(X^k, \lambda^k) \qquad (8)$$

Step 2: The step is calculated by solving the following quadratic programming sub problem.

$$\text{minimize} q^k(D) \qquad (9)$$

Step 3: Subject to

$$G(X^k) + J(X^k)D = 0 \qquad (10)$$

$$H(X^k) + I(X^k)D \le 0 \qquad (11)$$

where*J* and *I* are the jacobian matrices corresponding to the constraint vectors *G* and *H*, respectively.

Step 4: The Hessian of the Lagrangian that appears in the objective function,equation (9), is computed using a quasi-Newton approximation. Once is computed by solving equations (9) to(11),X is updated using

$$X^{k+1} = X^k + \alpha^k D^k \qquad (12)$$

where is the step length.

IV. Results And Discussion

To evaluate the effectiveness and efficiency of SQP based reactive power optimization approach, the numerical experiments are made in the standard 3 bus and standard IEEE 6-bus systems.

Case (1) 3-Bus System

In this section performance of SQP for reactive power optimization was evaluated on standard IEEE 6-bus system with simulation parameter in table III and IV.

The system has two generator at buses 1 (slack bus) and 2 (PV bus). The lower voltage magnitude limits at all buses are 0.90 pu and the upper limits are 1.10pu.

The SQP based optimization procedure was tested on the 3-bus system and results provided in the Table I and Table II. The control variables, for this problem, before and after optimization is given in the Table I. The optimized case the voltage magnitude is maintained within the limits only. The optimized voltage magnitude values are minimized when compared to the base case results.

TABLE I

CONTROL VARIABLES OF 3-BUS SYSTEM BEFORE AND AFTER OPTIMIZATION

Control variables	V_{min}	V_{max}	Base Case	Optimized Case
$\|V_1\|$	0.90	1.10	1.10	1.0337
$\|V_2\|$	0.90	1.10	0.90	1.1000

The state variables of the 3-bus system results are provided in Table II. The results are compared with the base case studies. It is seen that the constraints on the control is enforced while the objective function is minimized.

TABLE II
STATE VARIABLE OF 3-BUS SYSTEM BEFORE AND AFTER OPTIMIZATION

State variables	Base Case	Optimized Value
θ_2	21.308°	9°
θ_3	0.828°	-0.83°
$\|V_3\|$	0.857	0.9241
P_{g1}	93.53	59.19
Q_{g1}	273.17	10.782
Q_{g2}	-64.10	34.79

TABLE III

ACTIVE POWER LOSS BEFORE AND AFTER OPTIMIZATION FOR STANDARD 3-BUS SYSTEM

Active Power Loss	P_L(MW)
Base Case	63.528
Optimized Value	29.17

The transmission real power losses are reduced from a base case value of 63.528 MW to 29.17 MW while satisfying equality and inequality constraints. The reduction is about 34.36 MW and is significant. Table III gives the active power losses in system at base case and after optimization

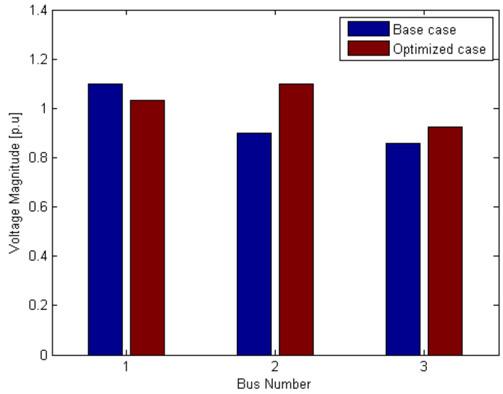

Fig.1 voltage profile of 3-bus system

In Fig 1 shows the voltage profile of the test system before and after optimization. It is observed that the voltage profile improved after the optimization.

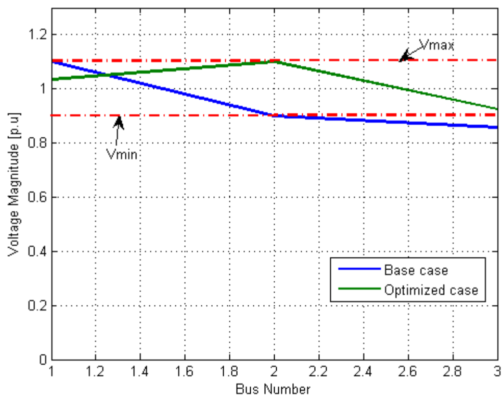

Fig.2 voltage profile of 3-bus system

In Fig 2 shows the voltage profile of sample 3-bus system. It shows the variations of bus voltage magnitude before and after optimization along with minimum and maximum limit on the variation. From Fig 2 it is clear that the bus voltage magnitudes are maintained within the limit while reducing transmission loss.

The bus voltage angle profile of 3-bus system in base case and after optimization is shown in the Fig 3. The voltage angle at bus-1 is reduced to great extent and thereby reducing the angular displacement between bus-1 and bus- 2. The reduction in angle implies reduced real power flow and hence the power loss.

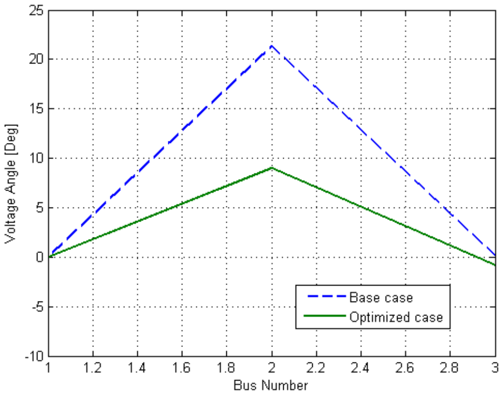

Fig 3 Voltage Angle Profile of 3 bus system

Case (2) Study of 6 bus system

In this section performance of SQP for reactive power optimization was evaluated on 6-bus system with simulation parameter in Table IV and V. The system has 2 generators at buses 1, 2 and two transformers with off nominal tap ratio and one switchable VAR source. The capacitor will be added at the 4th bus. The lower voltage magnitude limits at all buses are 0.95p.u and the upper limits are 1.05p.u for all buses.

TABLE IV

CONTROL VARIABLES OF 6-BUS SYSTEM BEFORE AND AFTER OPTIMIZATION

Control variables	V_{min}	V_{max}	Base Case	Optimized Value
$\|V_1\|$	0.90	1.05	1.0500	1.1000
$\|V_2\|$	0.90	1.10	1.1000	1.1000
tap_{6-5}	0.90	1.10	1.1000	1.0265
tap_{4-3}	0.90	1.10	1.0250	1.0037
$Q_{c,4}$	0	5.5	1.000	5.5000
$Q_{c,6}$	0	5.5	0.000	5.5000

The control variables before and after optimization along with their lower and upper limits is shown the Table IV. It is seen that the optimization procedure enforces these limit while minimizing the objective function. Some control variables are on the limits known as binding constraints.

The SQP based optimization procedure was tested on the 6-bus system and results provided in the Table IV and Table V. It is seen that the constraints on the control is enforced while the objective is minimized. The dependent variables i.e., state variables of the system before and after optimization is given in the Table V.

TABLE V

STATE VARIABLES OF 6-BUS SYSTEM BEFORE AND AFTER OPTIMIZATION

State variables	Base Case	Optimized Value
θ_2	-7.181°	-3.26
V_3	0.881	0.9827
θ_3	-14.494	-20.53
V_4	0.872	1.0005
θ_4	-10.545	-15.32
V_5	0.850	0.9669
θ_5	-14.089	-19.37
V_6	0.764	0.9806
θ_6	-13.138	-19.37
P_1	101.83	94.05
Q_1	90.08	38.8
Q_2	40.99	12.24

TABLE VI

ACTIVE POWER LOSS BEFORE AND AFTER OPTIMIZATION FOR STANDARD 6-BUS SYSTEM

Active Power Loss	P_L(MW)
Base Case	16.827
Optimized Value	9.05

The transmission real power losses are reduced from a base case value of 16.827 MW to 9.05 MW while satisfying equality and inequality constraints. The reduction is about 7.777 MW and the loss reduction is significant. Table VI gives the power losses in system at base case and after optimization.

The single line diagram of 6-bus system is shown in Appendix A. SQP is applying in standard 6 bus system bus systems and the result showing in the table IV and table V with graph.

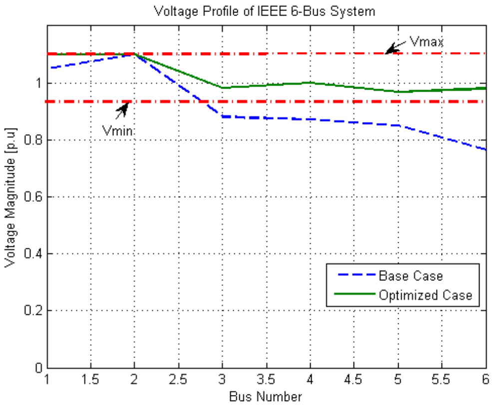

Fig 4 Voltage Magnitude Profile of 6 bus system

In Fig 4 shows the voltage profile of sample 6-bus system. It shows the variations of bus voltage magnitude before and after optimization along with minimum and maximum limit on the variation. From Fig 5 it is clear that the bus voltage magnitudes are maintained within the limit while reducing transmission loss.

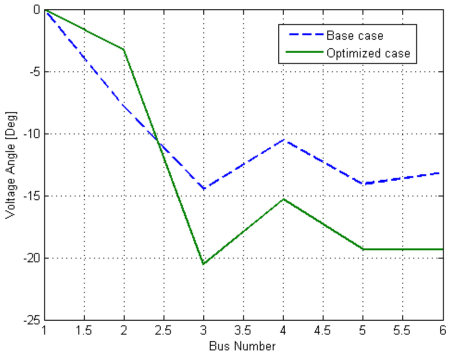

Fig 5 Voltage Angle Profile of 6 bus system

The bus voltage angle profile of 6-bus system in base case and after optimization is shown in the Fig 5. The voltage angle at bus-1 is reduced to great extent and thereby reducing the angular displacement between bus-1 and bus- 2. The reduction in angle implies reduced real power flow and hence the power loss

CONCLUSION

In the present work, the formulation of reactive power optimization to minimize the transmission real power loss was presented. The optimization problem was solved using a sequential quadratic programming method. The application of SQP to the test systems gives the optimized result satisfying all the constraints. The active power losses, in case 3-bus test system, were reduced by 34.36 MW i.e., by a significant 54.09%. It was found that losses are reduced by 46.18% in case 6-bus test system. In both test system losses are minimized while satisfying all the imposed constraints.

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