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Rayleigh Wave Propagation in a Rotating Functionally Graded Fiber-Reinforced Medium Subject to Magnetic and Gravitational Fields

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Abstract

An analytical investigation is conducted on Rayleigh wave propagation in a rotating fiber-reinforced functionally graded (FG) half-space subjected to magneto-gravitational effects. The orientation of the magnetic field is configured to facilitate a two-dimensional treatment of the governing equations. A transcendental dispersion equation is established, which is shown to reduce to well-known classical solutions under limiting conditions. Finally, parametric studies are performed to quantify the influence of rotation, gravity, and material grading on the phase velocity, with the findings presented graphically.

Keywords: Rayleigh waves; Fiber-reinforced medium; Functionally graded material; Magnetic permeability; Electrical conductivity; Rotational effect; Gravity field; Wave velocity.

Introduction

The analysis of stress and deformation in fiber-reinforced composites has remained a focal point of solid mechanics research for over thirty years. Although reinforcing techniques have historical precursors, the evolution of modern materials science has necessitated more sophisticated applications for advanced engineering structures. The core design objective is the optimization of tensile properties and load-carrying capacity without incurring significant

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mass penalties. An FRC typically comprises a dispersed fiber phase and a continuous matrix phase, separated by an interphase region; the specific spatial orientation of the fibers—be it unidirectional, random, or woven—dictates the material's macroscopic anisotropy. Theoretical foundations in the continuum modeling of such media were established by Belfield et al.[1], following foundational work by Spencer [2], Pipkin [3], and Rogers [4, 5]. In many instances, these composites are modeled as transversely isotropic elastic media, a framework supported by the variational approaches of Hashin and Rosen [15] for deriving effective elastic moduli.

The mechanical performance of fiber-reinforced composites can be significantly enhanced through the incorporation of functional gradation. Over recent decades, functionally graded materials (FGMs) have undergone rapid development and are increasingly utilized across diverse engineering sectors. Unlike conventional layered laminates, FGMs are spatially heterogeneous composites characterized by a continuous and gradual variation in the volume fraction of their constituent phases. First conceptualized in 1984 by researchers in Sendai, Japan (Yamanouchi et al. [7]; Koviani [6]), these materials have since garnered extensive academic attention. By virtue of their continuously varying macroscopic properties, FGMs offer superior mechanical advantages over traditional laminates, particularly in the mitigation of interfacial thermal stress concentrations. Consequently, they are uniquely suited for extreme operating environments, with applications spanning aerospace thermal protection systems, thermoelectric generators, automotive braking components, and biocompatible implants. Notable contributions include the work of Abd-Alla et al. [37], who examined radial vibrations in rotating, functionally graded orthotropic half-spaces under gravitational influence, and Gunghas et al. [38], who explored the synergistic effects of rotation and magnetic fields on thermoelastic solids. Furthermore, Barik et al. [35, 36] addressed various contact mechanics problems within this framework. The convergence of mechanical anisotropy in fiber-reinforced media with functional gradation remains a critical frontier in modern engineering research, as underscored by the studies of Sahu [28],, Deresiewicz [30], Markham [31], and Zorammuana [32].

The propagation of mechanical disturbances in solid media represents a foundational pillar of physics and engineering. The evolution of wave dynamics is supported by a distinguished historical framework, with seminal contributions from Poisson, Cauchy, Green, Lamé, and Stokes, as documented in Love's classic treatise on the mathematical theory of elasticity. While classical elasticity provides a robust starting point, it often falls short in characterizing the elastic response of materials with complex internal microstructures. In geophysics and seismology, the study of seismic waves—energy disturbances generated by tectonic shifts or anthropogenic explosions—is essential for modeling Earth's interior and optimizing resource recovery. These waves are fundamentally categorized into body waves and

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surface waves, the latter being primarily responsible for structural damage during seismic events. Among these, Rayleigh waves (1885) are particularly significant due to their confinement to the free surface and their substantial energy density. Consequently, understanding the influence of initial stress on Rayleigh wave propagation is of paramount importance for seismic risk mitigation and structural integrity assessment. Following Rayleigh's pioneering work, extensive research has addressed wave behavior in half-spaces and stratified systems involving inhomogeneous or non-homogeneous media.

The literature regarding surface wave propagation is comprehensive [8, 9, 10, 11]. Unlike body waves, surface waves are characterized by higher energy concentrations at the interface and slower propagation speeds. Research by Acharya and Sengupta [12] and others [13, 14] has examined these characteristics within fiber-reinforced anisotropic media. Because large-scale geophysical systems are inherently non-inertial, the study of wave motion in rotating media—pioneered by Schoenberg and Censor [34] —is of paramount importance. Additionally, magneto-elastic wave propagation represents a critical area of study in earthquake science, with Acharya and Roy [33] investigating these phenomena in electrically conducting reinforced media. Theoretical developments by Jassim [16] regarding the inhomogeneous wave equation, and by Pradhan et al.[19] on anisotropic dynamics, have provided deeper insights into material reinforcement. The influence of gravitational fields and initial stresses, first identified by Bromwich [20], Love [21] and Biot [22], remains a focal point in contemporary models. Recent studies, such as those by Sethi et al. [40] and Abd-Alla et al. [41], have integrated rotation, magnetism, and gravity to evaluate Rayleigh wave behavior in orthotropic and functionally graded media [39].

This study investigates the propagation of Rayleigh waves within a rotating, fiber-reinforced, functionally graded elastic medium, accounting for the influences of a magnetic field and gravity. The magnetic field orientation is assumed to permit a two-dimensional formulation of the problem. A characteristic wave velocity equation for Rayleigh waves is derived, and numerical simulations are performed using MATLAB to illustrate the relationship between wave velocity and wave number. The influence of various governing parameters is analyzed and represented graphically. Finally, the generalized results are compared with established literature, with graphical comparisons highlighting the specific impacts of the additional parameters introduced in this model.

Formulation of the Problem:

Consider a semi-infinite elastic medium composed of a functionally graded fiber-reinforced material (FGFRM) bounded by a planar surface. A Cartesian coordinate system xyz is established such that the origin O lies on the boundary surface, with the medium occupying the region $z \geq 0$. The physical model is governed by the following assumptions:

the medium is subjected to a uniform external magnetic field $\mathbf{H}_0 = (0, H_0, 0)$ and a constant

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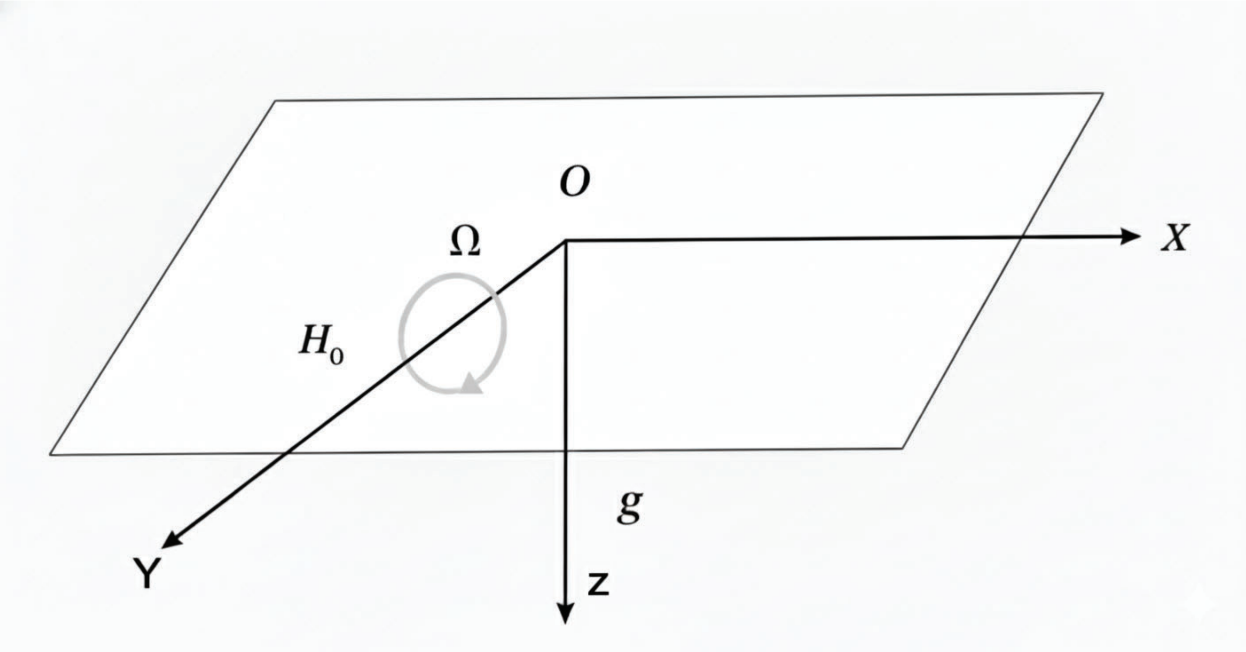

Figure 1: Geometry of the problem

gravitational force acting in the positive z-direction. Furthermore, the system undergoes uniform rotation with an angular velocity $\mathbf{\Omega} = (0, \Omega, 0)$. The reinforcing fibers are oriented parallel to the x-axis, defined by the unit vector $\mathbf{a} = (1, 0, 0)$. Finally, the analysis focuses on elastic wave propagation along the x-axis, where the disturbance is localized near the free surface $z = 0$ and vanishes asymptotically as $z \to \infty$.

Based on the aforementioned assumptions, the displacement field is uniform along any line parallel to the y-axis. Consequently, all field variables are independent of the y coordinate, rendering the problem two-dimensional in the $x - z$ plane. Since the medium is assumed to be rotating with angular velocity $\mathbf{\Omega}$ in an applied magnetic field of intensity H_0, an induced magnetic field $\mathbf{h} = (0, h, 0)$, an induced electric field $\mathbf{E}$ and a current density $\mathbf{J}$ will be developed. If μ_e is the magnetic permeability of the medium then the total magnetic field in the medium is $\mathbf{B} = \mu_e \mathbf{H}$, where $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ is the magnetic field arising from applied magnetic field $\mathbf{H}_0$ and induced field $\mathbf{h}$. Denoting the displacement vector by $\mathbf{u} = \mathbf{u}(x, t)$, the simplified system of the equations of electrodynamics for a slowly moving homogeneous electrically conducting medium, may be written as

$$
\begin{aligned}
\nabla \times \mathbf{h} &= \mathbf{J} + \epsilon_0 \dot{\mathbf{E}} \\
\nabla \times \mathbf{E} &= -\mu_e \dot{\mathbf{h}} \\
\nabla \cdot \mathbf{h} &= 0 \\
\mathbf{E} &= -\dot{\mathbf{u}} \times \mathbf{B}
\end{aligned}
\tag{1}
$$

where ∇ is the Hamilton's operator, ε_0 is the electrical permeability, and u is the dynamic displacement vector. Here we ignore the small effect of temperature radiant on the current density vector J. The deformation is supposed to be small and dynamic displacement vector is actually measured from a steady state reformed position.

Following Belfield et al. [1], the stress–strain relations for linearly fiber–reinforced elastic medium may be expressed in tensor form as

$$\begin{aligned}\tau_{ij} &= \lambda\epsilon_{kk}\delta_{ij} + 2\mu_T\epsilon_{ij} + \alpha\left(a_k a_m \epsilon_{km}\delta_{ij} + a_i a_j \epsilon_{kk}\right)\\ &\quad + 2(\mu_L - \mu_T)\left(a_i a_k \epsilon_{kj} + a_j a_k \epsilon_{ki}\right) + \beta\left(a_k a_m a_i a_j \epsilon_{km}\right)\end{aligned} \tag{2}$$

where τ_{ij} are the Cartesian components of the stress tensor, ϵ_{ij} are the strain components related to the displacement vector u_i. λ, μ_T are elastic constants, $\alpha, \beta, (\mu_L - \mu_T)$ are reinforcement parameters, and $a = (a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1$.

In the absence of body forces, the elastodynamic equations for the rotating medium take the following form:

$$\tau_{ij,j} + F_i = \rho\left(\ddot{u}_i + (\Omega \times (\Omega \times u))_i + 2(\Omega \times \dot{u})_i\right), \tag{3}$$

where ρ denotes the material density. The body force components arising from the applied magnetic field and gravitational effects are given by

$$F = J \times B + \rho g(w_{,x}, 0, -u_{,x}), \quad u = (u_1, u_2, u_3) = (u, 0, w). \tag{4}$$

By combining equations (1)–(3), and upon neglecting the cross-products of h and u (along with their derivatives), the governing elastodynamic equations for the rotating, fiber-reinforced medium subjected to magneto-gravitational effects are given by:

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} + \mu_0(J \times H)_1 + \rho g \frac{\partial w}{\partial x} = \rho\left[\ddot{u} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}\right], \tag{5}$$

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} + \mu_0(J \times H)_2 = \rho \ddot{v}, \tag{6}$$

$$\frac{\partial \tau_{31}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} + \mu_0(J \times H)_3 - \rho g \frac{\partial u}{\partial x} = \rho\left[\ddot{w} - \Omega^2 w - 2\Omega \dot{u}\right]. \tag{7}$$

To account for the functionally graded nature of the medium, the constitutive elastic parameters, the reinforcement coefficients, and the mass density are assumed to vary exponentially with the vertical coordinate z. This spatial distribution is modeled as:

$$\lambda = \hat{\lambda}e^{kz}, \quad \alpha = \hat{\alpha}e^{kz}, \quad \beta = \hat{\beta}e^{kz}, \quad \rho = \hat{\rho}e^{kz}, \quad \mu_L = \hat{\mu}_L e^{kz}, \quad \mu_T = \hat{\mu}_T e^{kz}, \tag{8}$$

where k is real constant.

Assuming the fibers are aligned with the x-axis $a = (1, 0, 0)$, the constitutive equations

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provide the relevant components of the stress tensor as follows:

$$\tau_{11} = \left[(\hat{\lambda} + 2\hat{\alpha} + 4\hat{\mu}_L - 2\hat{\mu}_T + \hat{\beta})\epsilon_{11} + (\hat{\lambda} + \hat{\alpha})(\epsilon_{22} + \epsilon_{33})\right] e^{kz}, \tag{9}$$

$$\tau_{22} = \left[(\hat{\lambda} + \hat{\alpha})\epsilon_{11} + (\hat{\lambda} + 2\hat{\mu}_T)\epsilon_{22} + \hat{\lambda}\epsilon_{33}\right] e^{kz}, \tag{10}$$

$$\tau_{33} = \left[(\hat{\lambda} + \hat{\alpha})\epsilon_{11} + \hat{\lambda}\epsilon_{22} + (\hat{\lambda} + 2\hat{\mu}_T)\epsilon_{33}\right] e^{kz}, \tag{11}$$

$$\tau_{12} = 2\hat{\mu}_L\epsilon_{12}e^{kz}, \quad \tau_{13} = 2\hat{\mu}_L\epsilon_{13}e^{kz}, \quad \tau_{23} = 2\hat{\mu}_T\epsilon_{23}e^{kz}. \tag{12}$$

For brevity, the hats on the dimensionless parameters are suppressed hereafter. Upon applying equations (9)–(12) to (5)–(7), and taking

$$h = -H_0\,(u_{,x} + w_{,z}). \tag{13}$$

the governing dynamical equations are obtained as follows:

$$\begin{aligned}(A_{11} + \mu_0 H_0^2)\,u_{,xx} + (B_2 + \mu_0 H_0^2)\,w_{,xz} + \mu_L\,u_{,zz} - \varepsilon_0\mu_0^2 H_0^2\,\ddot{u}\\ + \rho g\,w_{,x} + \mu_L k\,(w_{,x} + u_{,z}) = \rho\left(\ddot{u} - \Omega^2 u + 2\Omega\dot{w}\right)\end{aligned} \tag{14}$$

$$\mu_L\,v_{,xx} + \mu_T\,v_{,zz} + k\,\mu_T\,v_{,z} = \rho\,\ddot{v} \tag{15}$$

$$\begin{aligned}(A_{22} + \mu_0 H_0^2)\,w_{,zz} + (B_2 + \mu_0 H_0^2)\,u_{,xz} + \mu_L\,w_{,xx} - \varepsilon_0\mu_0^2 H_0^2\,\ddot{w}\\ - \rho g\,u_{,x} + k(\lambda + \alpha)\,u_{,x} + kA_{22}\,w_{,z} = \rho\left(\ddot{w} - \Omega^2 w - 2\Omega\dot{u}\right)\end{aligned} \tag{16}$$

where

$$A_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, \quad A_{22} = \lambda + 2\mu_T, \quad B_2 = \lambda + \alpha + \mu_L.$$

For Rayleigh waves we concentrate only on eqn (14) and eqn (16) which reduces to

$$\begin{aligned}(C_1^2 + C_A^2)\frac{\partial^2 u}{\partial x^2} + (C_2^2 + C_A^2)\frac{\partial^2 w}{\partial x\partial z} + c_3^2\left(\frac{\partial^2 u}{\partial z^2} + k\frac{\partial u}{\partial z} + k\frac{\partial w}{\partial x}\right)\\ = (1 + C_B^2)\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t} - g\frac{\partial w}{\partial x}\end{aligned} \tag{17}$$

$$\begin{aligned}(C_4^2 + C_A^2)\frac{\partial^2 w}{\partial z^2} + (C_2^2 + C_A^2)\frac{\partial^2 u}{\partial x\partial z} + c_3^2\frac{\partial^2 w}{\partial x^2} + \frac{k(\lambda + \alpha)}{\rho}\frac{\partial u}{\partial x} + kc_4^2\frac{\partial w}{\partial z}\\ = (1 + C_B^2)\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x}\end{aligned} \tag{18}$$

where $C_1^2 = \frac{A_{11}}{\rho}$, $C_A^2 = \frac{\mu_0 H_0^2}{\rho}$, $C_2^2 = \frac{B_2}{\rho}$, $C_3^2 = \frac{\mu_L}{\rho}$, $C_B^2 = \frac{\epsilon_0\mu_0^2 H_0^2}{\rho}$, $C_4^2 = \frac{A_{22}}{\rho}$,

C_A = alfven velocity, C_4 = P-wave velocity, C_0 = velocity of light in vacuum.

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By introducing the dimensionless parameters $\bar{x}, \bar{z}, \bar{u}, \bar{w}, \bar{t}$ such that

$$x = c_1\omega\bar{x},\ z = c_1\omega\bar{z},\ u = c_1\omega\bar{u},\ w = c_1\omega\bar{w},\ t = \omega\bar{t}$$

where ωdenotes the wavenumber, the Equations (17) and (18) for the half-space reduce to

$$\begin{aligned}(1+\chi_H)\frac{\partial^2\bar{u}}{\partial\bar{x}^2} + (c_2'^2+\chi_H)\frac{\partial^2\bar{w}}{\partial\bar{x}\partial\bar{z}} + c_3'^2\left(\frac{\partial^2\bar{u}}{\partial\bar{z}^2} + \kappa\frac{\partial\bar{u}}{\partial\bar{z}} + \kappa\frac{\partial\bar{w}}{\partial\bar{x}}\right) \\ = M\frac{\partial^2\bar{u}}{\partial\bar{t}^2} - \Gamma^2\bar{u} + 2\Gamma\frac{\partial\bar{w}}{\partial\bar{t}} - \bar{g}\frac{\partial\bar{w}}{\partial\bar{x}}\end{aligned} \tag{19}$$

$$\begin{aligned}(c_4'^2+\chi_H)\frac{\partial^2\bar{w}}{\partial\bar{z}^2} + (c_2'^2+\chi_H)\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{z}} + c_3'^2\frac{\partial^2\bar{w}}{\partial\bar{x}^2} + \beta'\frac{\partial\bar{u}}{\partial\bar{x}} + \gamma'\frac{\partial\bar{w}}{\partial\bar{z}} \\ = M\frac{\partial^2\bar{w}}{\partial\bar{t}^2} - 2\Gamma\frac{\partial\bar{u}}{\partial\bar{t}} - \Gamma^2\bar{w} + \bar{g}\frac{\partial\bar{u}}{\partial\bar{x}}\end{aligned} \tag{20}$$

where $c_4'^2 = \frac{C_4^2}{C_1^2}$, $\chi_H = \frac{C_A^2}{C_1^2}$, $\Gamma = \Omega\omega$, $c_2'^2 = \frac{C_2^2}{C_1^2}$, $c_3'^2 = \frac{C_3^2}{C_1^2}$, $\bar{g} = \frac{\omega g}{C_1}$, $\beta' = \frac{k(\lambda+\alpha)}{C_1^2\rho}$, $\gamma' = \frac{kC_4^2}{\rho C_1^2} = \frac{k}{\rho}c_4'^2$ and $M = 1 + \chi_H\ \frac{C_1^2}{C_0^2} = 1 + C_B^2$.

Boundary Conditions

Assuming that the disturbance is propagating near the surface of the half space in the form of Rayleigh waves, the boundary conditions at the free surface of the half-space can be written as:

$$\tau_{13} = 0, \quad \tau_{33} = 0 \quad \text{on } z = 0 \tag{21}$$

Solution of the Problem:

To solve the governing equations, we seek a harmonic wave solution of the form:

$$(\bar{u}, 0, \bar{w}) = \{u(z), 0, w(z)\}\exp\{i\omega(x - ct)\} \tag{22}$$

where $u(z), w(z)$ are are depth-dependent amplitudes. In this expression, ω is the wave number associated with a wave length of $\frac{2\pi}{\omega}$ and c is the wave speed. By substituting equation (22) into the equations (19) and (20), we obtain

$$\begin{aligned}\Bigg[\frac{c_3'^2}{2}\left(D^2 + kD\right) + \left\{Mc^2 - (1+\chi_H)^2 + \Gamma'^2\right\}\omega^2\big)\, u \\ + i\omega\left\{(c_2'^2+\chi_H)D + \bar{g} + k\frac{c_3'^2}{2} + 2c\,\Gamma'\right\} w\Bigg] = 0\end{aligned} \tag{23}$$

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$$\Big[(c_4'^2+\chi_H)D^2+\{M\omega^2c^2+\Gamma^2-\omega^2c_3'^2\}\Big]w+\gamma'Dw$$
$$+\Big[(c_2'^2+\chi_H)i\omega D-(\bar{g}'+2c\Gamma+\beta')i\omega\Big]u=0 \tag{24}$$

where $D\equiv\frac{d}{dz}$, $\Gamma'=\frac{\Gamma}{\omega}$ and $\bar{g}'=\frac{\bar{g}}{\omega}$.
From equations (23) and (24) we get the following equations to determining $u(z)$ or $w(z)$:

$$\{c_3'^2\left(c_4'^2+\chi_H\right)D^4+\Big[c_3'^2\omega^2+k\left(c_4'^2+\chi_H\right)\Big]D^3+\Big[c_3'^2\omega^2\left(Mc^2+\Gamma'^2-c_3'^2\right)$$
$$+\omega^2\left(c_4'^2+\chi_H\right)\left\{Mc^2-(1+\chi_H)+\Gamma'^2\right\}+k\delta'\omega^2-\omega^2\left(c_2'^2+\chi_H\right)^2\Big]D^2+\Big[\omega^4\delta'\left\{Mc^2-+\Gamma'^2\right\}$$
$$+k\left\{Mc^2+\Gamma'^2-c_3'^2\right\}+\omega^2\left(c_2'^2+\chi_H\right)\left(2\bar{g}'+4c\Gamma'+kc_3'^2+\beta'\right)\Big]D+\omega^4\left\{Mc^2-(1+\chi_H)\right.$$
$$\left.+\Gamma'^2\left\{Mc^2+\Gamma'^2-c_3'^2\right\}+\omega^2\left(\bar{g}'+kc_3'^2+2c\Gamma'\right)\left(\bar{g}'+2c\Gamma'+\beta'\right)\right\}u(z),w(z)=0 \tag{25}$$

Since u,w represent surface waves, we assume that they vanish as $z\to\infty$. Accordingly, we seek a solution of the form

$$u(z)=\left[A_1e^{-i\omega\lambda_1z}+B_1e^{-i\omega\lambda_2z}\right]e^{i\omega(x-ct)}, \tag{26}$$
$$w(z)=\left[A_2e^{-i\omega\lambda_1z}+B_2e^{-i\omega\lambda_2z}\right]e^{i\omega(x-ct)}. \tag{27}$$

Using Eqs. (26) and (27) in Eqs. (23) and (24), and equating the coefficients of $e^{-i\omega\lambda_1z}$ and $e^{-i\omega\lambda_2z}$ to zero we obtain

$$A_2=k_1A_1, \tag{28}$$

$$B_2=k_2B_1 \tag{29}$$

where
$$k_i=\frac{(1+\chi_H)+\lambda_i^2c_3'^2-Mc^2+\Gamma'^2+ik'\lambda_i}{(c_2'^2+\chi_H)\lambda_i+i\left(\bar{g}'+2c\Gamma'+k'\right)},\qquad(i=1,2). \tag{30}$$

and $k'=\frac{k}{\omega}$.
Imposing the boundary conditions from Eq. (21) at $z=0$ yields the following system of equations:

$$(\lambda_1-k_1)A_1+(\lambda_2-k_2)B_1=0 \tag{31}$$
$$\left(c_2'^2-c_3'^2-\lambda_1k_1c_4'^2\right)A_1+\left(c_2'^2-c_3'^2-\lambda_2k_2c_4'^2\right)B_1=0 \tag{32}$$

For a non-trivial solution for the constants A_1 and B_1, the determinant of the coefficients

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must vanish, leading to the characteristic equation:

$$\begin{vmatrix} \lambda_1 - k_1 & \lambda_2 - k_2 \\ c_2'^2 - c_3'^2 - \lambda_1 k_1 c_4'^2 & c_2'^2 - c_3'^2 - \lambda_2 k_2 c_4'^2 \end{vmatrix} = 0 \tag{33}$$

Equation (33) represents the dispersion relation (wave velocity equation) for Rayleigh waves propagating in a rotating, fiber-reinforced, functionally graded medium subjected to magnetic and gravitational fields. In the absence of gravitational effects, this frequency equation is consistent with the results obtained by Acharya [12]. Furthermore, by neglecting the influences of gravity, rotation, and the magnetic field, Eq. (33) reduces to the classical Rayleigh wave velocity equation for an isotropic medium:

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\left(1 - \frac{c^2}{c_2^2}\right)^{\frac{1}{2}}\left(1 - \frac{c^2}{c_1^2}\right)^{\frac{1}{2}} \tag{34}$$

Numerical Results and Discussions:

The present research examines the collective impact of functionally graded parameters, magnetic fields, rotational forces, and gravitational effects on the propagation characteristics of Rayleigh waves in fiber-reinforced media. To evaluate these effects numerically, three distinct parameter sets—designated as Fiber-1, Fiber-2, and Fiber-3—were adopted from the established literature [25, 31, 32] as detailed below.

$$\lambda = 9.4 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_T = 1.89 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_L = 2.45 \times 10^9\,\mathrm{N\,m^{-2}}, \\ \rho = 1.7 \times 10^3\,\mathrm{kg\,m^{-3}}$$

$$\lambda = 5.65 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_T = 2.46 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_L = 5.66 \times 10^9\,\mathrm{N\,m^{-2}}, \\ \rho = 2.26 \times 10^3\,\mathrm{kg\,m^{-3}}$$

$$\lambda = 7.59 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_T = 3.5 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \mu_L = 7.07 \times 10^9\,\mathrm{N\,m^{-2}}, \\ \rho = 1.6 \times 10^3\,\mathrm{kg\,m^{-3}}$$

Although the theoretical framework accommodates arbitrary propagation directions, the numerical computations are restricted to specific orientations to facilitate calculation and highlight key trends in wave velocity under fibre-reinforced functionally graded characteristics of the media. Utilizing the aforementioned parameter sets, this numerical analysis examines the propagation behavior of Rayleigh waves under varying media conditions. Figures 2–6 present the Raleigh wave velocity plotted against the wavenumber for different configurations of reinforcement and functional grading.

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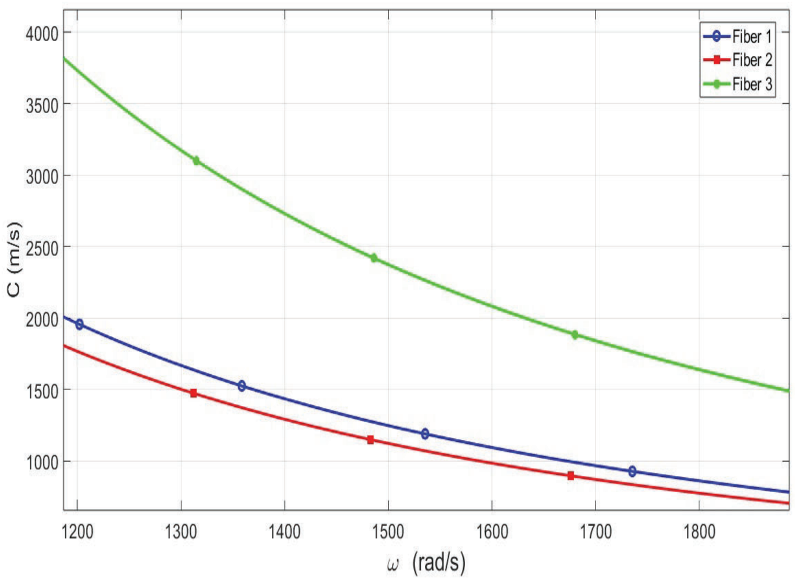

Figure 2: Variation of Rayleigh wave velocity for different fiber reinforced media with fixed value of graded parameter k

Figure 2 displays the variation Rayleigh wave velocity with respect to wavenumber for dif-ferent fibre reinforce media with a fixed functionally gradation. It indicates that the Rayleigh wave velocity decreases when the value of wave number increases. We also observe that for a fiber-reinforced medium, the Rayleigh wave velocity is affected significantly by the rein-forcing parameter. The dispersion curves indicate that the Rayleigh wave velocity vanishes in the short-wavelength limit as the wavenumber increases ($\omega \longrightarrow \infty$): the wave velocity approaches to zero. Figure 3 illustrates the influence of the magnetic field on Rayleigh wave velocity. The velocity decreases with increasing wave number. Furthermore, at a constant wave number, the Rayleigh wave velocity is found to decrease as the applied magnetic field intensity increases.

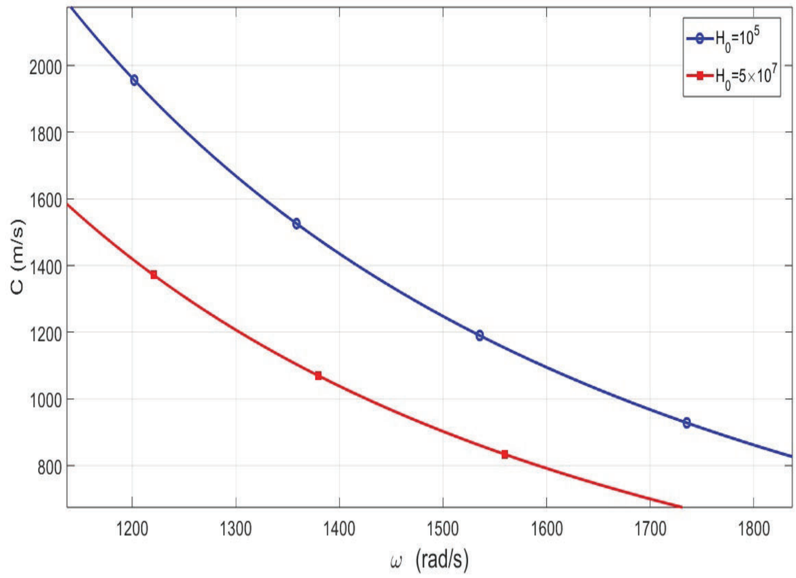

Figure 3: Effect of magnetic field on Rayleigh wave velocity.

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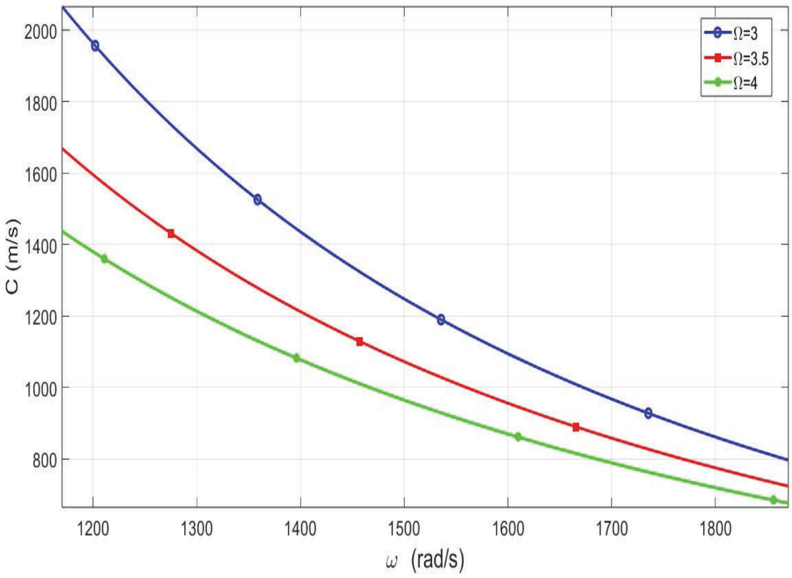

Figure 4: Effect of rotation on Rayleigh wave velocity.

The effects of rotation and material density are captured in Figures 4 and 5 respectively, where an increase in either parameter yields a reduction in wave speed. Figure 6 is plotted to observe the influence of different gravity parameter on the Rayleigh wave velocity with respect wave number. It is observed that for a particular value of wave number, Rayleigh wave velocity decreases with the increase of gravitational effects.

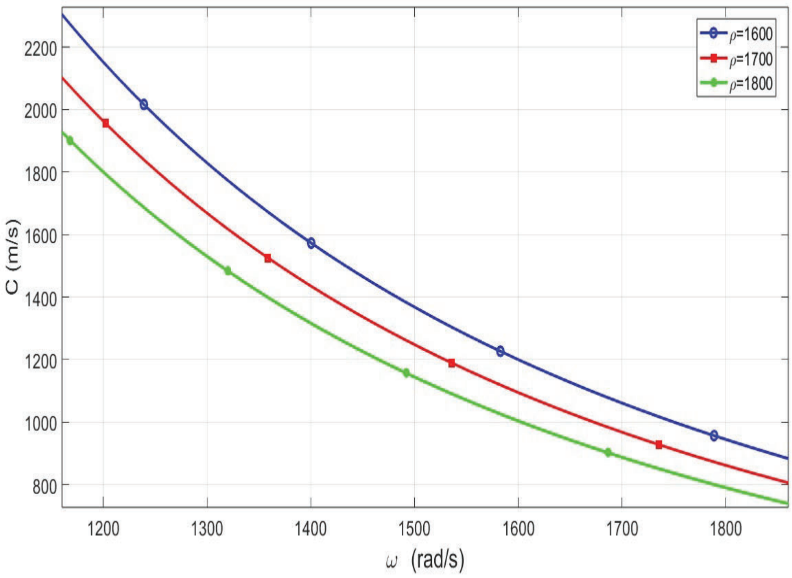

Figure 5: Effect of density on Rayleigh wave velocity.

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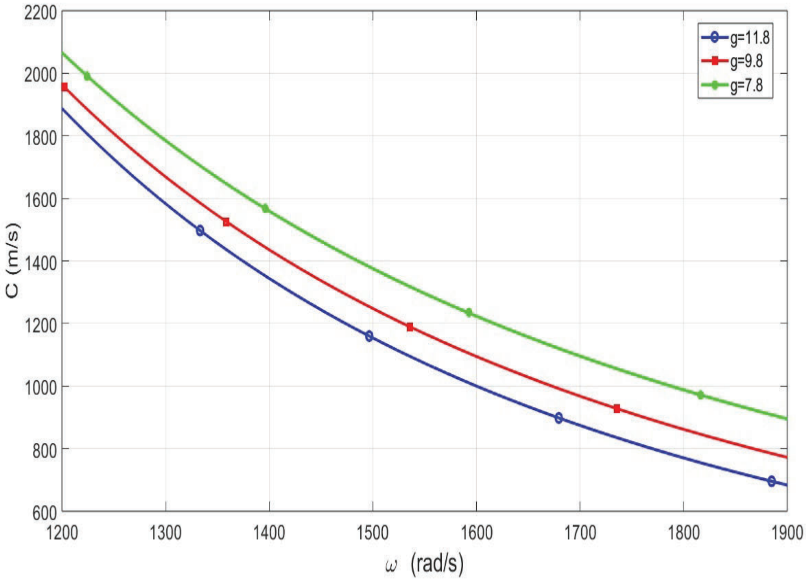

Figure 6: Effect of gravity on Rayleigh wave velocity.

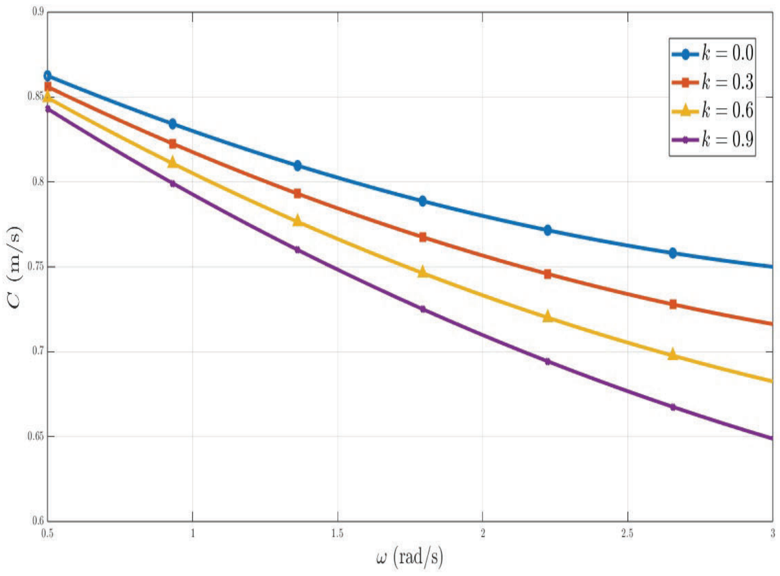

Figure 7: Effect of functionally graded parameter k on Rayleigh wave velocity

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Finally, Figure 7 demonstrates that the functional gradation parameter (k) significantly alters the velocity magnitude; specifically, the intensification of material gradation (increased k) results in a consistent attenuation of the Rayleigh wave velocity.

Conclusion:

The study focuses on how a plane surface wave propagating in a rotating fiber-reinforced functionally graded (FG) half-space is affected by an applied magnetic field, rotation, density, gravity, fiber-reinforcing and functionally gradation. A significant observation across these figures is that as the wavenumber increases (i.e., in the high-frequency limit, $\omega \longrightarrow \infty$),the wave velocity asymptotically approaches zero. The results indicate that the direct effects of different parameters on Rayleigh wave velocity are very pronounced.

Conflict of Interests:

The authors declare that there is no conflict of interests regarding the publication of this paper.

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