Rayleigh-Bénard-Taylor Convection in Temperature-sensitive Newtonian Liquid with Heat Source/Sink

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***Abstract*— The present paper deals with a linear stability analysis of Rayleigh-Bénard convection in a rotating Newtonian fluid with heat source/sink confined between two parallel, infinitely extended horizontal surfaces. It is proved that internal Rayleigh number; Thermo rheological parameter and the Taylor number influence the onset of convection. It is found that the effect of increasing the strength of rotation is to stabilize the system whereas the increase of internal heat source and variable viscosity parameter is to destabilize the system. The result has possible applications in the astrophysical and geophysical context.**

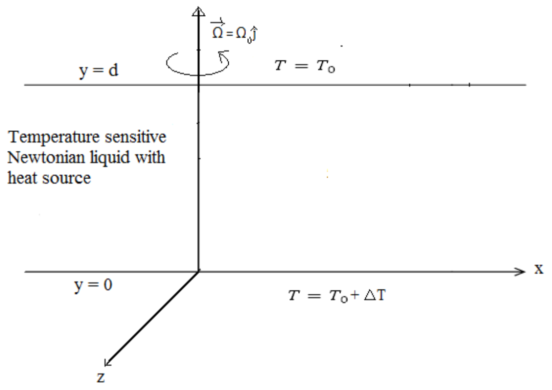
Keywords*— Rayleigh-Benard convection; Taylor number; variable heat source; temperature-dependent viscosity.

I. INTRODUCTION

The Rayleigh-Benard instability problem with internal heat generation, thermo rheological effect and rotation have been received great attention due to their implications in heat and mass transfer. The rotation is one of the important external mechanisms in controlling the onset of convection. The effect of non-inertial acceleration on the stability analysis of buoyancy driven convection is discussed in several books and monographs such as Chandrasekhar [1961], Platten and Legros [1984] and Drazin and Reid [2004].

Thermal convection occurs when a fluid is heated from below. Rayleigh-Bénard convection as it is called is discussed in two excellent books Chandrasekhar [1] and Drazin and Reid [2]. Over several decades, many researchers are trying to control the convective phenomena in Rayleigh-Bénard convection by imposing external constraints such as temperature, Magnetic field, rotation etc. Rotation plays a very important role in controlling the convection. Motivated by the experiments of Donnelly [3] on the effect of rotation on the onset of instability in a fluid flow between two concentric cylinders, Venezian [4] performed a linear analysis under temperature modulation with free-free surfaces. He obtained an expression for critical Rayleigh number and found that by suitably tuning the frequency of rotation one can regulate the heat transport effectively. Later on, Rayleigh–Bénard problem under many constraints were studied by various researchers, considering different physical models. Kloosterziel and Carnevale [5] investigated the effect of rotation on the stability of a thermally modulated system, and determined analytically the critical points on the marginal stability boundary above which an increment either in viscosity or in diffusivity is destabilizing the system. Finally, they showed that when the fluid has zero viscosity the system is always unstable, in contradiction to Chandrasekhar's [1] conclusion. Siddheshwar and Vanishree [7] studied the effect of rotation on thermal convection in an anisotropic porous medium with temperature dependent viscosity. The Galerkin variant of the weighted residual technique is used to obtain the Eigen value of the problem. They presented some new result on the parameters' influence on convection in the presence of rotation, for both high and low rates. Bhadauria [8] investigated rotational influence on Darcy convection and found that both rotation and permeability suppress the onset of thermal instability. The effect of non-uniform temperature gradient on Rayleigh-Bénard convection in micro polar fluid has been studied by Siddheshwar and Pranesh [9]. The Eigen value is obtained for free-free, rigid-free and rigid-rigid boundary conditions on the spin vanishing boundaries. They presented some important results on the micro polar fluid parameters and the internal heat generation on the onset of convection. Bhadauria et al. [10] investigated the non-linear thermal instability in a rotating viscous fluid layer under temperature/gravity Modulation. They found that by suitably adjusting the frequency or amplitude of modulation one can control the convective flow. Recently Siddheshwar and Chan [11] investigated the thermo rheological effect on Benard and Marangoni convection in an anisotropic porous media. They showed that the effect of increasing thermo rheological parameter is to destabilize the system. A brief study of the combined effect of thermal modulation and rotation on the onset of convection in a rotating fluid layer was made by Rauscher and Kelly [12]. It is clear most of the available studies considered are not involving all the three effects such as internal heat generation, rotation and the temperature-sensitivity of the liquid on the onset of convection. However there are only few studies available, This paper deals with the study of convective non-linear Rayleigh-B\'{e}nard-Taylor instability problem in an electrically conducting temperature-sensitive Newtonian liquid with heat source; we use the truncated representation for half range Fourier cosine series expansion to represent the basic non-uniform temperature gradient and basic viscosity. The Galerkin procedure is utilized to discover the systematic articulation for the thermal Rayleigh number as the function of Internal heat source parameter, thermo rheological parameter and Taylor number.

II. MATHEMATICAL FORMULATION



Consider a Newtonian liquid confined between two infinite, parallel horizontal planes of depth d between $y = 0$ and $y = d$. The lower and the upper plates are maintained at different temperatures, the lower plate is hotter than the upper plate (see Fig. 1). The system is rotated about y axis. We assume the Oberbeck-Boussinesq approximation is valid and consider only small-scale convective motions (Lorenz) and the boundaries are assumed to be to be stress-free and isothermal. Density, dynamic viscosity and heat source are assumed to be temperature-dependent. The governing equations for the study of Rayleigh-Bénard convection in variable viscosity Newtonian liquids with the Coriolis force are given by

$$\nabla.\vec{q} = 0 \qquad (1)$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} + 2(\vec{\Omega} X \vec{q})\right] = -\nabla p + \rho(T)\vec{g} +$$

$$\nabla.\left[\mu_f(T)(\nabla\vec{q} + \nabla\vec{q}^{Tr})\right], \qquad (2)$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi\nabla^2 \mathrm{T} + \mathrm{Q}_1(\mathrm{T} - \mathrm{T}_0), \qquad (3)$$

$$\rho(T) = \rho_0[1 - \beta(T - T_0)], \qquad (4)$$

$$\mu_f(T) = \mu_0 e^{-\delta(T - T_0)}. \qquad (5)$$

To make a finite amplitude analysis, we consider the following perturbations:

$$\left.\begin{aligned}\vec{q} = q_b + \vec{q}', T = T_b + T', \rho = \rho_b + \rho',\\ p = p_b + p', \ \ \mu_f = \mu_{f_b} + \mu_f{}'\end{aligned}\right\}, \qquad (6)$$

where the primes indicates perturbed quantities. The basic state quantities $T_b(y)$, $\mu_{f_b}(y)$, $\rho_b(y)$ have the forms

$$\left.\begin{aligned}T_b = T_0 + \Delta T f\left(\tfrac{y}{d}\right), \ \rho_b = \rho_0\left(1 - \beta\nabla T f\left(\tfrac{y}{d}\right)\right)\\ p_b = -\int \rho_b\left(\tfrac{y}{d}\right) g d\left(\tfrac{y}{d}\right) + C_0, \mu_{f_b} = \mu_0 e^{-\beta f\left(\frac{y}{d}\right)}\end{aligned}\right\}. \qquad (7)$$

Where, $\left(\frac{y}{d}\right) = \frac{\sin\left[\sqrt{R_I}\left(1 - \frac{y}{d}\right)\right]}{\sin\left[\sqrt{R_I}\right]}$, C_0 is the constant of integration and $R_I = \frac{Q_1 d^2}{\chi}$ is the internal Rayleigh number. Since the flow considered is two dimensional, we introduce the stream function ψ' as follows:

$$u' = \frac{\partial \psi'}{\partial y}, \qquad v' = -\frac{\partial \psi'}{\partial x} \qquad (8)$$

Eliminating the pressure in the equation (2) and using the equations (8) in the resulting equation, we get the following equations

$$\begin{aligned}\rho_0 \frac{\partial}{\partial t}(\nabla^2\psi') &= \frac{\partial \mu_{f_b}}{\partial y}\frac{\partial}{\partial y}(\nabla^2\psi') + \mu_{f_b}\nabla^4\psi'\\ &+ \frac{\partial \mu_f{}'}{\partial y}\frac{\partial}{\partial y}(\nabla^2\psi') + \mu_f'\,\nabla^4\psi'\\ &+ \frac{\partial \mu_f{}'}{\partial x}\frac{\partial}{\partial x}(\nabla^2\psi') - \rho_0 \alpha g \frac{\partial T'}{\partial x}\\ &-2\rho_0\Omega_0\frac{\partial w'}{\partial y} + \frac{\partial(\psi', \nabla^2\psi')}{\partial(x,y)},\end{aligned} \qquad (9)$$

$$\frac{\partial T'}{\partial t} = -\frac{\partial \psi'}{\partial x}\frac{dT_b}{dy} + \chi\nabla^2 T' + Q_1 T' + \frac{\partial(\phi', T')}{\partial(x,y)}, \qquad (10)$$

Further from the momentum equation we can write

$$\rho_0\frac{\partial w}{\partial t} = \mu_{f_b}\nabla^4\psi' + 2\rho_0\Omega_0\frac{\partial w}{\partial y} + \frac{\partial(\psi', w)}{\partial(x,y)}. \qquad (11)$$

We non-dimensionalize the equations (12), (13) and (14) using the following definitions:

$$\left.\begin{aligned}(X, Y, \zeta) = \left(\frac{x}{d}, \frac{y}{d}, \frac{w}{d}\right), \tau = \frac{\chi}{d^2}t, \Psi = \frac{\psi'}{\chi},\\ \Theta = \frac{\mathrm{T}'}{\Delta \mathrm{T}}\end{aligned}\right\}, \qquad (12)$$

and obtain

$$\begin{aligned}\frac{1}{Pr}\frac{\partial}{\partial \tau}(\nabla^2\Psi) &= \frac{\partial \mu_{f_b}}{\partial Y}\frac{\partial}{\partial Y}(\nabla^2\Psi) + \mu_{f_b}\nabla^4\Psi\\ &+ \frac{\partial \mu_f{}'}{\partial Y}\frac{\partial}{\partial Y}(\nabla^2\Psi) + \mu_f'\,\nabla^4\Psi\\ &+ \frac{\partial \mu_f{}'}{\partial X}\frac{\partial}{\partial X}(\nabla^2\Psi) + R_E\frac{\partial \Theta}{\partial X}\\ &-\sqrt{Ta}\frac{\partial \zeta}{\partial Y} + \frac{1}{Pr}\frac{\partial(\Psi, \nabla^2\Psi)}{\partial(X,Y)},\end{aligned} \qquad (13)$$

$$\frac{\partial \Theta}{\partial \tau} = -\frac{\partial \Psi}{\partial X}f'(Y) + \nabla^2\Theta + R_I\Theta + \frac{\partial(\Theta, \Psi)}{\partial(X,Y)}, \qquad (14)$$

$$\frac{1}{Pr}\frac{\partial \zeta}{\partial \tau} = \mu_{f_b}\nabla^2\zeta + \sqrt{Ta}\frac{\partial \zeta}{\partial Y} + \frac{\partial(\Psi, \zeta)}{\partial(X,Y)}, \qquad (15)$$

where, $Pr = \frac{\nu}{\chi}$ is Prandtl number, $R_E = \frac{\alpha g \Delta T d^3}{\chi\nu}$ is the thermal Rayleigh number, $Ta = \left(\frac{2\Omega_0 d^2}{\nu}\right)^2$ is Taylor number and $f'(Y) = 1 + 2\sum_{n=1}^{\infty}\frac{R_I}{R - n^2\pi^2}\cos(n\pi Y)$,

The boundaries are assumed to be stress free and isothermal, hence the boundary conditions for solving the equations (13), (14) and (15) are

$$\Psi = \nabla^2\Psi = D\zeta = \Theta = 0 \text{ at } Y = 0{,}1 \qquad (16)$$

In the next section, the linear stability analysis is performed using fourier series and the Galerkin technique is used to find out the analytical expression for the critical Rayleigh number which is of great utility in performing the linear stability analysis.

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III. LINEAR STABILITY ANALYSIS

In order to study the linear theory, the linearized version of equations (13),(14) and (15) is considered along with the boundary conditions (16). This means that the Jacobeans $\frac{\partial(\Psi,\zeta)}{\partial(X,Y)}$ and $\frac{\partial(\Theta,\Psi)}{\partial(X,Y)}$ in equations. (13), (14) and (15) is neglected. The solution of the linearized system is assumed to be periodic waves of the form:

$$\left.\begin{aligned}\Psi(X,Y) &= \Psi_0 \sin(\pi\alpha X)\sin(\pi Y)\\ \Theta(X,Y) &= \Theta_0 \cos(\pi\alpha X)\sin(\pi Y)\\ \zeta(X,Y) &= \zeta_0 \sin(\pi\alpha X)\cos(\pi Y)\end{aligned}\right\}, \quad (17)$$

Where, $r_E = \frac{R_E}{R_{E_c}}$, $\eta_1^2 = \pi^2(1+\alpha^2)$

$$R_E = \frac{\eta_1^2(\eta_1^2 - R_I)(4\pi^2 - R_I)}{4\pi^2\alpha^2}\left(\frac{1+\alpha^2}{2} + \frac{1-\alpha^2}{2}a_2 - \frac{2Ta}{\eta_1^4\left(\frac{2a_2}{3} + \frac{2a_4}{15} - a_0\right)}\right), \quad (18)$$

$$a_0 = 2\mu_0 \int_0^1 e^{\frac{V}{\sin[\sqrt{R_I}]}\sin[\sqrt{R_I}(Y-1)]}\, dY,$$

$$a_2 = 2\mu_0 \int_0^1 e^{\frac{V}{\sin[\sqrt{R_I}]}\sin[\sqrt{R_I}(Y-1)]}\cos(2\pi Y)\, dY,$$

$$a_4 = 2\mu_0 \int_0^1 e^{\frac{V}{\sin[\sqrt{R_I}]}\sin[\sqrt{R_I}(Y-1)]}\cos(4\pi Y)\, dY,$$

In equation (18), α is the scaled horizontal wave number. The quantities Ψ_0 , Θ_0 and ζ_0 are respectively, amplitudes of the stream function, temperature, and the vorticity function. Substituting equation (17) into the linear version of equations (13) to (15) and integrating the above equation with respect to X in $\left[0, \frac{2\pi}{\pi\alpha}\right]$ and also with respect to Y in $[0, 1]$, a set of homogeneous equations in Ψ_0 , Θ_0 and ζ_0 is obtained. In obtaining the non-trivial solution of the linear system, the above expression of the critical Rayleigh number is obtained.

It is now clear that R_E defined by equation (20) is the critical Rayleigh number of the marginal stationary state. The scaled critical wave number for the preferred mode satisfies the following equation:

$$\left.\begin{aligned}&(a_0 - a_2)\pi^4\alpha_c^6 + \frac{1}{2}\begin{pmatrix}3a_0\pi^4 - a_2\pi^4\\ -R_I a_0\pi^2 + R_I a_2\pi^2\end{pmatrix}\alpha_c^4\\ &-\frac{1}{2}\begin{pmatrix}(a_0 + a_2)\pi^4 - R_I a_0\pi^2 - R_I a_2\pi^2\\ +\frac{30Ta(R_I - \pi^2)}{\eta_1^2(10a_2 + 2a_4 - 15a_0)}\end{pmatrix} = 0\end{aligned}\right\}, \quad (22)$$

IV. RESULTS AND DISCUSSIONS

The effect of a variable heat source (sink) and rotation on Rayleigh-Bénard convection is studied. The effect of heat source (sink) appears in the equation in the form of an internal Rayleigh number R_I and rotation in the form of Taylor number Ta. The external Rayleigh number R_E is the Eigen value of the problem.

The highlights of the linear study are

1. Derivation of a useful analytical expression for the stationary critical Rayleigh number by using the half-range Fourier cosine series expansion for the basic non uniform temperature gradient and for the basic viscosity.
2. Discounting the possibility of oscillatory motions

The focus in the paper is on Rayleigh-Benard convection influenced by a heat source (sink) and for this reason only those values of R_I that do not allow dominance of heat source (sink) over buoyancy in effecting convection are considered. In order to understand better the results arrived at in the problem we analyze the nonlinear basic state temperature distribution, which throws light on the observed effect of heat source (sink) on the stability. A scaled, dimensionless temperature distribution is considered in the following form:

$$\theta_b(Y) = \frac{T_b - T_0}{\Delta T} = \frac{\sin[\sqrt{R_I}(Y-1)]}{\sin[\sqrt{R_I}]}$$

Fig.2 is the plot of the non uniform basic temperature gradient $\theta_b(Y)$ versus Y for different values of R_I. It is evident that the plots are not symmetric about the line $\theta = 1 - Y$, which is the basic temperature distribution when there is no heat source (sink).

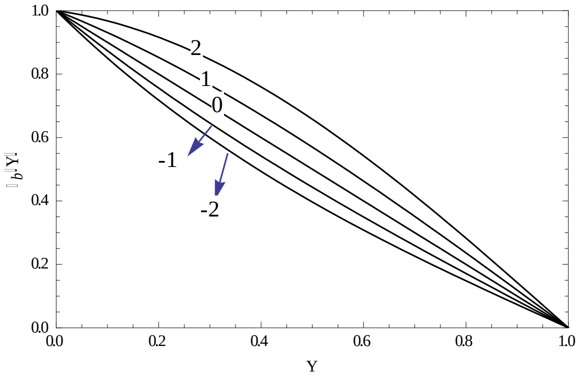

Fig.2: Temperature profile of quiescent basic state for different values of R_I

This asymmetry due to temperature-dependent heat source/sink helps in making the inference that the results on the problem with a heat sink cannot be obtained from that of a heat source through a suitable transformation as can be done in the case of a constant heat source.

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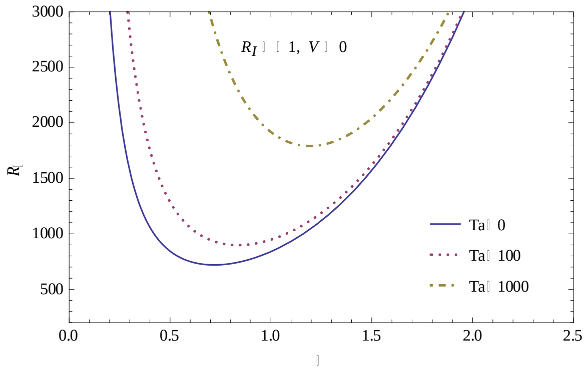

Fig.3 (a)

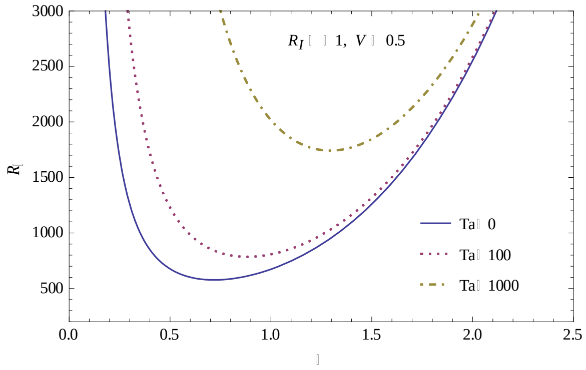

Fig.3 (b)

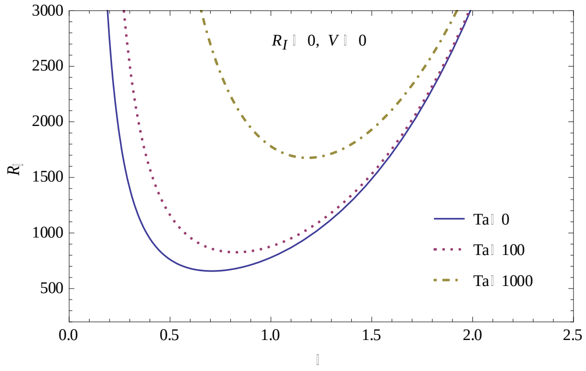

Fig.3 (c)

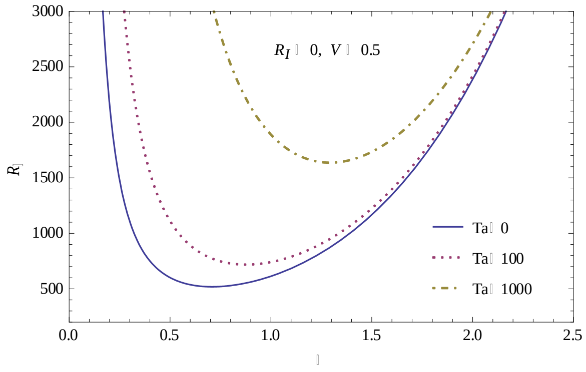

Fig.3 (d)

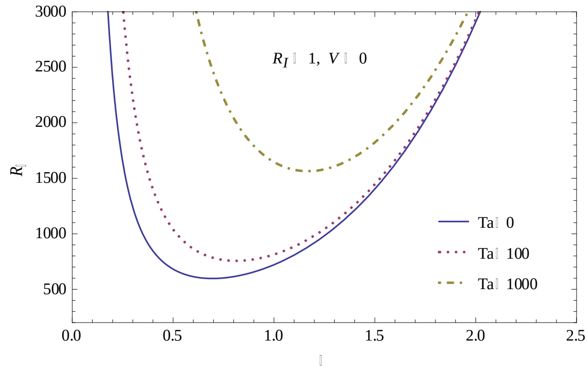

Fig.3 (e)

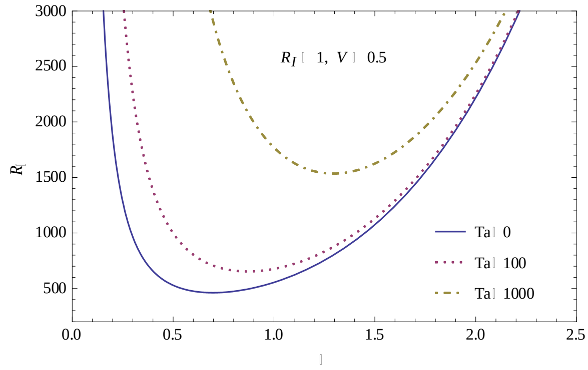

Fig.3 (f)

Figs. 3(a),(b),(c),(d),(e) and (f) illustrate the variation of Rayleigh number with respect to the wave number for different parameters' combinations. It is clear that as an increase in Taylor number, Ta, results in an increase in the value of critical Rayleigh number, R_{E_c}, and the critical wave number, α_c, and also it is clear that if V increases the critical Rayleigh number R_{E_c} and the critical wave number α_c decreases. The negative value of R_I represents the heat sink and the positive value corresponds to the heat source. Clearly, the wave number decreases with increase in R_I, for both the heat source and heat sink.

V. CONCLUSION

The paper presents an analytical study of Rayleigh-Benard convection with variable heat source. Closed form expressions for α_c and R_{E_c} are obtained as functions of the parameters of the problem. It is found that one can regulate the flow by suitably tuning the parameters' involved in the problem. The effect of increasing the strength of rotation is to stabilize the system where as the increasing the Thermo rheological parameter and the internal Rayleigh number lead to the destabilization of the system.

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NOMENCLATURE

LATIN SYMBOLS

$\boldsymbol{d}$ depth of the fluid layer
$\vec{g}$ acceleration due to gravity
$\hat{\imath}$ unit normal in x direction
$\hat{\jmath}$ unit normal in y direction
Pr Prandtl number
$\boldsymbol{p}$ pressure
$\vec{q}$ velocity of the fluid, (u, v)
$\boldsymbol{Q_1}$ heat source / sink
$\boldsymbol{R_I}$ internal Rayleigh number
$\boldsymbol{R_E}$ external Rayleigh number
Ta Taylor number

GREEK SYMBOLS

$\boldsymbol{\pi\alpha}$ wave number
$\boldsymbol{\beta}$ thermal expansion coefficient
$\boldsymbol{\chi}$ constant thermal diffusivity
$\boldsymbol{\mu}$ constant of dynamic viscosity
$\boldsymbol{\nabla^2}$ two dimensional Laplacian
$\boldsymbol{\gamma}$ kinematic viscosity
$\boldsymbol{\phi}$ phase angle
$\boldsymbol{\psi}$ stream function
$\boldsymbol{\Psi}$ perturbed stream function
$\boldsymbol{\rho}$ density
$\boldsymbol{\Theta}$ perturbed temperature

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