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Quasi-m*-open and quasi-m*-closed mappings

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Abstract

Recently, Savita B. Megalamani[11] have defined a new class of closed sets, namely, m*-closed sets in topology. And also studied the concept of m*-open mappings, m*-closed mappings etc. In this paper, we introduce and study a notion of quasi- m*-closed mappings and quasi- m*-open mappings.

Key words: m*-closed sets, m*--open sets, m*-open mappings, m*--closed mappings, γ- m*-mappings, always m*-closed mappings, m*-irresolute mappings, m*--continuous mappings.

1. INTRODUCTION

In 1963, 1982 and in 1986, N. Levine[7], A.S Mashhour et.al[8] and Andrijevic[1] have investigated the concepts of semiopen sets, semi continuity, preopen sets, pre continuity, semipreopen sets and semipreclosed sets in topological spaces.

In 1997, A.A.El.Atic [4], has introduced and studied the concept of γ-open sets in topology. Recently, in 2019[11], introduced a new class of closed sets called m*-closed sets and m*-open sets. Also, studied some of their properties. The aim of this paper is to study some more open and closed mappings using these new class of sets.

2. PRELIMINARIES

In this paper, X, Y, Z always means a topological spaces on which no separation axioms are assumed. Unless otherwise mentioned.
For a subset A of X, Cl(A) and Int(A) represents the closure of A and interior of A respectively.

The following definitions and results are useful in the sequel:

Definition 2.1: Let X be a topological space. A subset A is called
(i) semiopen[7] if $A \subset Cl(Int(A))$,

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(ii) preopen[8] if $A \subset Int(Cl(A))$,
(iii) semipreopen[1] if $A \subset Cl(Int(Cl(A)))$.
(iv) γ-open[4] if $A \subset Cl(Int(A)) \cup Int(Cl(A))$.

The complement of semiopen (resp. peropen, semipreopen, γ-open) set is called semiclosed[3](resp. preclosed[8], semipreclosed[1], γ-closed[4]).

Definition 2.2: Let A be a subset of a space X. Then,the intersection of all semipreclosed sets containing A is called semipreclosure[1] of A and is denoted by spCl(A).

Definition 2.3: Let A be a subset of a space X. Then the union of all semipreopen sets contained in A is called semipre-interior[1] and is denoted by spInt(A).

Definition 2.4 : A subset A of a space X is termed as m^* -closed set[11] if $spCl(A) \subset U$ whenever $A \subset U$ and U is γ-open set in X.

Definition 2.5: A subset A of a space X is termed as m^*-open[11] set if $F \subset spInt(A)$ whenever $F \subset A$ and F is γ-closed set in X.

The family of all m^*- open sets in topological space X is denoted by $M^*O(X)$ and that of the family of all m^*-closed sets in topological space X is denoted by $M^*F(X)$.

Definition 2.6: Let A be a subset of a space X, then the union of all m^*-open sets contained in A is called the m^*-interior[11] of A and is denoted by $m^*Int(A)$.

Definition 2.7: A set $U \subset X$ is termed as m^*-neighbourhood (in brief, m^*-nbd)[11] of a point $x \in X$ if and only if there exists $A \in M^*O(X)$ such that $A \subset U$.

Definition 2.8: A mapping $i: X \to Y$ is called semiopen[2](resply., preopen[9], semipreopen[10]), if the image of each open set of X is semiopen(resply., preopen, Semopreopen) set in Y.

Definition 2.9: A mapping $i: X \to Y$ is called m^*-open[11], if the image of each open set of X is m^*-open set in Y.

Definition 2.10: A mapping $i: X \to Y$ is called m^*s-open[11](resply., m^*p-open[11], m^*sp-open[11]), if the image of each m^*-open set of X is semiopen(resp. preopen, semipreopen) set in Y.

Definition 2.11: A mapping $i: X \to Y$ is called m^*s- closed [11](resply., m^*p- closed [11], m^*sp- closed [11]), if the image of each m^*- closed set of X is semiclosed (resp. pre closed, semipreclosed) set in Y.

Definition 2.12: A mapping $i: X \to Y$ is called always m^*-open[11], if the image of each m^*-open set of X is m^*-open set in Y.

Definition 2.13: A mapping $i: X \to Y$ is called always m^*-closed[11], if the image of each m^*-open set of X is m^*-closed set in Y.

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Definition 2.13: A mapping $i: X \to Y$ is called strongly m^*-closed[11], if the image of each m^*-closed set of X is closed set in Y.

Definition 2.14: A mapping $i: X \to Y$ is called γm^*-closed[11], if the image of each γ-closed set of X is m^*-closed set in Y.

Definition 2.15: A mapping $i: X \to Y$ is called quasi-γ-closed[5], if the image of each γ-closed set of X is closed set in Y.

3. On Quasi m^*-open mappings and quasi m^*-closed mappings

In this section we define and study the following:

Definition 3.1 : A mapping $i: X \to Y$ is termed as quasi- m^*-open if the image of each m^*-open set of X is an open set in Y.

Now we have the following characterizations:

Theorem 3.2: A mapping $i: X \to Y$ is said to be quasi- m^*-open if and only if for every subset U of X, $i(m^*Int(U)) \subset Int(i(U))$.

Proof: Let i be a quasi- m^*-open mapping. Now, we have $m^*Int(U) \subset U$ and $m^*Int(U)$ is a m^*-open set. Hence we obtain that $i(m^*Int(U)) \subset i(U)$. As $i(m^*Int(U))$ is open, $i(m^*Int(U)) \subset Int(i(U))$.

Conversely, assume that U is a m^*-open set in X. Then $i(U)= i(m^*Int(U)) \subset Int(i(U))$, but $Int(i(U) \subset i(U)$. Consequently $i(U)= Int(i(U))$, which is open and hence i is quasi- m^*-open mapping.

Lemma 3.3: If a mapping $i: X \to Y$ is quasi- m^*-open, then $m^*Int(i^{-1}(A)) \subset i^{-1}(Int(A))$ for every subset A of Y.

Proof: Let A be any subset of Y. Then, $m^*Int(i^{-1}(A))$ is a m^*-open set in X and i is quasi- m^*-open, then $i(m^*Int(i^{-1}(A))) \subset Int(i(i^{-1}(A))) \subset Int(A)$. Thus, $m^*Int(i^{-1}(A)) \subset i^{-1}(Int(A))$.

Theorem 3.4 : For a mapping $i: X \to Y$, the following are equivalent:
(i) i is quasi- m^*-open
(ii) for each subset U of X, $i(m^*Int(U)) \subset Int(i(U))$
(iii) for each $x \in X$ and each m^*-neighbourhood U of x in X, there exists a neighbourhood V of i(x) in Y such that $V \subset i(U)$.

Proof: (i)$\Rightarrow$(ii) It follows from the theorem 5.3.2.

(ii)$\Rightarrow$(iii) Let $x \in X$ and U be an arbitrary m^*-neighbourhood of x in X. Then there exist a m^*-open set V in X such that $x \in V \subset U$. Then by (ii), we have $i(V)= i(m^*Int(V)) \subset Int(i(V))$ and hence $i(V)= Int(i(V))$. Therefore, it follows that i(V) is open in Y such that $i(x) \in i(V) \subset i(U)$.

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(iii)$\Rightarrow$(i) Let U be an arbitrary m^*-open set in X. Then for each $y \in i(U)$, by (iii) there exist a neighbourhood V_y of y in Y such that $V_y \subset i(U)$. As V_y is a neighbourhood of y, there exist an open set W_y in Y such that $y \in W_y \subset V_y$. Thus $i(U) = \bigcup \{ W_y : y \in i(U)\}$ which is a open set in Y. This implies that i is quasi- m^*-open mapping.

Theorem 3.5 : A mapping $i: X \rightarrow Y$ is quasi- m^*-open if and only if for any subset B of Y and for any m^*-closed set F of X containing $i^{-1}(B)$, there exist a closed set G of Y containing B such that $i^{-1}(G) \subset F$.

Proof: Suppose i is quasi- m^*-open mapping. Let $B \subset Y$ and F be a m^*-closed set of X containing $i^{-1}(B)$. Now, put $G = Y - i(X-F)$. It is clear that $i^{-1}(B) \subset F$ implies $B \subset G$. Since i is quasi- m^*-open , we obtain G as a closed set of Y. Moreover, we have $i^{-1}(G) \subset F$.

Conversely, let U be a m^*-open set of X and put $B = Y - i(U)$. Then $X - U$ is a m^*-closed set in X containing $i^{-1}(B)$. By hypothesis, there exists a closed set F of Y such that $B \subset F$ and $i^{-1}(F) \subset X - U$. Hence, we obtain $i(U) \subset Y - F$. On the other hand, it follows that $B \subset F$, $Y - F \subset Y - B = i(U)$. Thus, we obtain $i(U) = Y - F$ which is open and hence i is a quasi- m^*-open mapping.

Theorem 3.6 : A mapping $i: X \rightarrow Y$ is a quasi- m^*-open if and only if $i^{-1}(Cl(B)) \subset m^*Cl(i^{-1}(B))$ for every subset B of Y.

Proof: Suppose that i is quasi- m^*-open mapping. For any subset B of Y, $i^{-1}(B) \subset m^*Cl(i^{-1}(B))$. Therefore, by theorem 3.5, there exists a closed set F in Y such that $B \subset F$ and $i^{-1}(F) \subset m^*Cl(i^{-1}(B))$. Therefore, we obtain $i^{-1}(Cl(B)) \subset i^{-1}(F) \subset m^*Cl(i^{-1}(B))$.

Conversely, let $B \subset Y$ and F be a m^*-closed set of X containing $i^{-1}(B)$. Put $W = ClY(B)$, then we have $B \subset W$ and W is closed set and $i^{-1}(W) \subset m^*Cl(i^{-1}(B)) \subset F$. Then by theorem 3.5, i is quasi- m^*-open mapping.

Decompositions of quasi- m^*-open mappings:

Theorem 3.7 : Let $i: X \rightarrow Y$ and $j: Y \rightarrow Z$ be two mappings. The following statements are valid:
(i) If i is quasi- m^*-open and j is preopen then $j \circ i$ is (m^*, p)-open mapping.
(ii) If i is quasi- m^*-open and j is semiopen then $j \circ i$ is (m^*, s)-open mapping.
(iii) If i is quasi- m^*-open and j is semipreopen then $j \circ i$ is (m^*, sp)-open mapping.

Proof: (i) Let V be any m^*-open set in X. Since i is quasi- m^*-open mapping, j(V) is open set in Y. Again, j is preopen mapping and j(V) is open set in Y, then $j(i(V)) = (j \circ i)(V)$ is preopen set in Z. Thus, $j \circ i$ is (m^*, p)-open mapping.

(ii) Obvious.

(iii) Obvious.

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Theorem 3.8 : Let $i: X \to Y$ be m^*-open mapping and $j: Y \to Z$ be quasi- m^*-open mapping then $j \circ i$ is open mapping.

Proof: Obvious.

Theorem 3.9 : Let $i: X \to Y$ be quasi- m^*-open mapping and $j: Y \to Z$ be m^*-open mapping then $j \circ i$ is always m^*-open mapping.
Proof: Obvious.

Theorem 3.10 : Let $i: X \to Y$ and $j: Y \to Z$ be two mappings and $j \circ i: X \to Z$ is quasi- m^*-open mapping. If j is continuous injective, then i is quasi- m^*-open.

Proof: Let U be a m^*-open set in X. Then $(j \circ i)(U)$ is open in Z, since $j \circ i$ is quasi- m^*-open. Again, j is an injective continuous mapping, $i(U)= j^{-1}((j \circ i)(U))$ is open in Y. This shows that i is quasi- m^*-open mapping.

We define the following:

Definition 3.11 : A mapping $i: X \to Y$ is termed as quasi- m^* -closed if the image of each m^*-closed set of X is closed set in Y.

Now we have the following characterizations:

Lemma 3.12 : If a mapping $i: X \to Y$ is quasi- m^*-closed, then $i^{-1}(Int(B)) \subset m^*Int(i^{-1}(B))$ for every subset B of Y.

Proof: This proof is similar to the proof of lemma 3.3.

Theorem 3.13 : A mapping $i: X \to Y$ is quasi- m^*-closed if and only if for any subset B of Y and for any m^*-open set G of X containing $i^{-1}(B)$, there exists an open set U of Y containing B such that $i^{-1}(U) \subset G$.

Proof: This proof is similar to the proof of the theorem 3.5.

Theorem 3.14 : If $i: X \to Y$ and $j: Y \to Z$ are two quasi- m^*-closed mappings, then $j \circ i: X \to Z$ is quasi- m^*-closed mapping.

Proof: Obvious.

Theorem 3.15 : Let X and Y be topological spaces. Then the mapping $j: X \to Y$ is a quasi- m^*-closed if and only if j(X) is closed in Y and $j(V) - j(X - V)$ is open in j(X) whenever V is m^*-open in X.

Proof: Necessity: Suppose $j: X \to Y$ is a quasi- m^*-closed mapping. Since X is a m^*-closed, j(X) is closed in Y and $j(V) - j(X - V) = j(V) \cap j(X) - j(X - V)$ is open in j(X) when V is m^*-open in X.

Sufficiency: Suppose j(X) is closed in Y, $j(V)- j(X - V)$ is open in j(X) when V is m^*-open in X and let C be closed in X. Then $j(C)=j(X) - j(X - C) - j(C)$ is closed in j(X) and hence closed in Y.

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Corollary 3.16 : Let X and Y be topological spaces. Then a surjective mapping $j: X \to Y$ is quasi- m^*-closed if and only if $j(V) - j(X - V)$ is open in Y whenever V is m^*-open in X.

Proof: Obvious.

Corollary 3.17 : Let X and Y be topological spaces and let $j: X \to Y$ be a m^*-continuous quasi- m^*-closed surjection mapping. Then the topology on Y is $\{ j(V) - j(X - V) : V$ is m^*-open in $X\}$.

Proof: Let W be open in Y. Then $j^{-1}(W)$ is m^*-open in X and $j(j^{-1}(W)) - j(X - j^{-1}(W))$ =W. Hence, all open sets of Y are of the form $j(V) - j(X - V)$, V is m^*-open set in X. On the other hand, all sets of the form $j(V) - j(X - V)$, V is m^*-open in X, are open in Y from corollary 3.16.

Theorem 3.18 : A mapping $i : X \to Y$ and $j : Y \to Z$ be two mappings. We have the following statements:
(i) i is γ- m^*-closed and j is quasi- m^*-closed, then $j \circ i$ is quasi- γ-closed.
(ii) i is strongly m^*-closed and j is quasi- m^*-closed then $j \circ i$ is quasi- m^*-closed.
(iii) i is γ- m^*-closed and j is strongly m^*-closed then $j \circ i$ is γ- m^*-closed.

Proof: (i) Let V be a γ-closed subset of X. Since i is γ- m^*-closed, i(V) is m^*-closed set in Y. Again j is quasi- m^*-closed and i(V) is m^*-closed set in Y, then $j(i(V))=(j \circ i)(V)$ is closed set in Z. This shows that $j \circ i$ is quasi- γ-closed.

(ii) Let V be a m^*-closed set in X. Since i is strongly m^*-closed, i(V) is m^*-closed set in Y. Again j is quasi- m^*-closed and i(V) is m^*-closed set in Y, then $j(i(V))=(j \circ i)(V)$ is closed set in Z. Hence $j \circ i$ is quasi- m^*-closed.

(iii) Let V be γ-closed set in X. Since i is γ- m^*-closed, i(V) is m^*-closed set in Y. We have, j is strongly m^*-closed and i(V) is m^*-closed set in Y, then $j(i(V))=(j \circ i)(V)$ is m^*-closed set in Z. Thus $j \circ i$ is γ- m^*-closed mapping.

Decompositions of quasi- m^*-closed mappings:

Theorem 3.19 : Let $i: X \to Y$ and $j: Y \to Z$ be two mappings. Then
(i) If i is m^*-closed and j is quasi- m^*-closed, then $j \circ i$ is closed.
(ii) If i is quasi- m^*-closed and j is m^*-closed, then $j \circ i$ is always m^*-closed.
(iii) If i is m^*-closed and j is quasi- m^*-closed, then $j \circ i$ is quasi- m^*-closed.

Proof: Obvious.

Theorem 3.20 : Let $i: X \to Y$ and $j: Y \to Z$ be two mappings such that $j \circ i: X \to Z$ is quasi- m^*-closed mapping.

(i) If i is m^*-irresolute surjective, then j is closed.
(ii) If j is m^*-continuous injective, then i is always m^*-closed.

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Proof: (i) Suppose that F is an arbitrary closed set in Y. As i is m^*-irresolute, $i^{-1}(F)$ is m^*-closed in X. Since $j\circ i$ is quasi- m^*-closed and i is surjective, $j\circ i(i^{-1}(F))=j(F)$, which is closed in Z. This implies j is a closed mapping.

(ii) Suppose F is any m^*-closed set in X. Since $j\circ i$ is quasi- m^*-closed, $(j\circ i)(F)$ is closed in Z. Again, j is a m^*-continuous injective mapping, $j^{-1}(j\circ i(F))=i(F)$, which is m^*-closed in Y. This shows that i is always m^*-closed mapping.

Theorem 3.21 : Let $i: X\to Y$ and $j: Y\to Z$ be two mappingss. Then the following statements are valid:

(i) If i is quasi- m^*-closed and j is semiclosed, then $j\circ i$ is (m^*, s) –closed mapping.
(ii) If i is quasi- m^*-closed and j is preclosed then $j\circ i$ is (m^*,p) –closed mapping.
(iii) If i is quasi- m^*-closed and j is semipreclosed then $j\circ i$ is (m^*,sp) –closed mapping.

Proof: (i) Let V be any m^*-closed set in X. Since i is quasi- m^*-closed mapping, j(V) is closed set in Y. Again, j is semiclosed mapping and j(V) is closed set in Y, then $j(i(V))=(j\circ i)(V)$ is semiclosed set in Z. Thus, $j\circ i$ is (m^*,s) –closed mapping.

(ii) Obvious.

(iii) Obvious.

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