Quality Assessment of GPS Smoothed Codes for Different Smoothing Window Sizes and Times

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***Abstract*— Smoothing of GPS code observations is usually performed using traditional parameters which are obtained by increasing the weight of phase measurements by 1% between consecutive epochs. This implies a smoothing window size of 100 times the used sampling rate. Increasing the smoothing window size beyond this value is expected to produce smoother and largely biased codes and vice versa. So, it is of great importance to study the quality of the smoothed codes before using it in further position estimation. In this paper, two new indicators are established to evaluate the quality of the smoothed codes. The first indicator (I_{rel}) is responsible for the measurement of the error budget that is contained by these smoothed codes. On the other hand, the second indicator (I_{sm}) is used to describe the degree of smoothness of the resulted smoothed codes. The two indicators are computed firstly for the smoothed codes obtained by applying the traditional parameters. Reliability and smoothness indicators of 17.09m and 1.99m are obtained. The smoothed codes exhibited a tendency to be fitted better using 5th degree polynomial. The obtained two indicators are used as reference values in all the sub-sequent tests.**

To study the effect of the smoothing window size on the quality of the resulted smoothed codes, twelve smoothing window sizes were tested starting from 5 minutes up to one hour. For each used window, the two indicators were computed and the order of the best fitting polynomial for the obtained codes is deduced. Results showed that duplication of the window size degrades the reliability with a ratio of 17% and increases the smoothness by a ratio of 29.1%. In addition, changing the size of the smoothing window proved to have no effect on the order of the best fitting polynomial of the smoothed codes. At the end, one complete day of GPS data were processed and the established two indicators are computed every one hour. Results showed that the reliability of the smoothed codes at night hours is three times better than that obtained at day hours. Also, it was found that the smoothness indicator is independent on the time of the smoothing process within the day. The same independency was observed concerning the order of the best fitting polynomial of the smoothed codes.

***Keywords*-Code smoothing; Hatch filter; Reliability indicator; Smoothness indicator; Ionospheric activity.**

I. INTRODUCTION

It is well known that the accuracy of the carrier phase measurements is higher than that of the code measurements. Positional accuracy of few meters can be achieved using code data, whereas centimeter level accuracy can be obtained using phase data [1]. So, all high-accuracy GPS geodetic applications should be based on carrier phase observations [2].

In spite of the low accuracy of GPS code measurements, it has two main advantages over the carrier phase measurements. Such two advantages are its immunity against cycle slips and its simplicity in further GPS processing procedure as it does not need to go through an ambiguity resolution process [3]. Such two advantages of the code observations along with the high accuracy of the phase observations were the main motivations behind the establishment of the concept of code smoothing using phase data [4]. The idea of code smoothing using phase measurements is based on using the relatively accurate phase measurements in the smoothing of the unambiguous code measurements to increase its accuracy [5]. This idea is presented in Fig. 1. By observing this figure it can be seen that, the accuracy of the code observations is relatively low. This is presented by the high random fluctuations of curve A. On the other hand, the accuracy of the phase observations (curve B) is much higher. However, it is biased (shifted) by the value of the integer phase ambiguity. By combining the two types of observations, the smoothed codes are resulted (curve C). Such smoothed codes combine the advantages of both code and phase observations [6].

Several previous researches dealt with the issue of combining phase and code data. The majority of such researches dealt with the smoothing of code observations using phase measurements [e.g. 6, 7, 8 and 9]. In such works, the algorithm of the classical approach of code smoothing was established and the accuracy of the derived positions was analyzed and compared with other classical solutions in different observational conditions. On the other hand, some other researches dealt with the direct combination of code and phase data using least squares principles [10]. In all cases, no attention was paid to the assessment of the quality of the computed smoothed codes. Such quality is thought to be judged from two aspects. The first aspect is its accuracy, whereas the second aspect is its smoothness. This evaluation process is considered the main objective of the current paper.

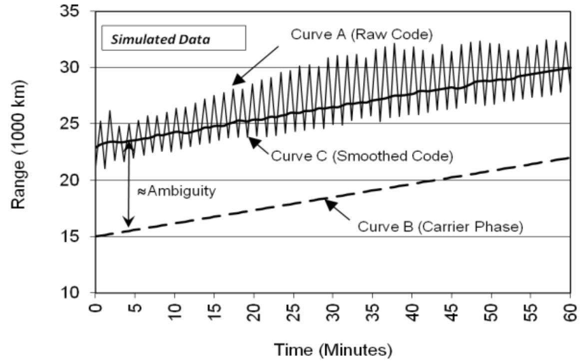

Fig. 1. Concept of Code Smoothing Using Phase Measurements

In this paper, the quality of the smoothed codes will be analyzed on two scales. The first one will be concerned with the degree of confidence (reliability) of these codes, whereas the other will be concerned with the tendency of these codes to be fitted mathematically (smoothness). To achieve this goal, two new indicators will be introduced and established mathematically to express both the reliability and smoothness of the smoothed codes. Then, both indicators will be computed for code observations that were smoothed using traditional smoothing parameters. At this stage, different smoothing trials will be applied using different smoothing parameters. For each applied trial, the quality of the produced smoothed codes will be evaluated (concerning both its reliability and smoothness). Finally, the efficiency of the smoothing process will be evaluated, each one hour, along one complete day.

II. Classical Approach of Code Smoothing (Hatch Filter)

The basic equation of code smoothing reads [4]:

$$P(t_i)_{sm}=\omega P(t_i)+(1-\omega)[P(t_{i-1})_{sm}+\phi(t_i)-\phi(t_{i-1})] \qquad (1)$$

Where:

$P(t_i)_{sm}$	Smoothed code at the epoch (i)
$P(t_i)$	Measured code at the epoch (i)
$P(t_{i-1})_{sm}$	Smoothed code at the epoch (i-1)
$\phi(t_i)$	Measured phase at the epoch (i)
$\phi(t_{i-1})$	Measured phase at the epoch (i-1)
ω	Time dependent weight factor

For the first epoch (i = 1), the weight factor is set to unity (i.e. ω=1). This corresponds to putting the full weight on the measured code pseudo range. For consecutive epochs, the value of the weight factor is continuously reduced and thus, emphasizes the influence of the carrier phases. A reduction of the weight factor by 0.01 from epoch to epoch is commonly used [11]. This means that after 100 epochs only the smoothed value of the previous epoch added to the measured phase difference is taken into account. Of course, this smoothing algorithm would fail in the case of cycle slip occurrence.

III. Application of the Classical Hatch Smoothing Algorithm

Based on the above discussion it is very evident that, the classical approach of the code smoothing suffers from many problems and biases. To clarify such problems, the classical Hatch filter (1) should be applied using GPS real data to visualize the main problems facing it. In this paper, GPS real dual frequency data will be used. Such data was collected in one long session lasted for more than one day, with a sampling rate of 15 seconds. Here, the satellite (SN 13) will be used for the application of the smoothing process. This will result in only 7 hours of data will be considered (from 7:00 to 14:00).

Based on the above mentioned fact that any smoothing algorithm will certainly fail if any cycle slips occurred, the used GPS data should be tested against the existence of any cycle slips before using it in any further computations. This is to grantee that the used data, and consequently the output smoothing results, are representing the reality of the classical Hatch filter. So, and as a pre-requisite quality assurance step, the GPS data will go through a cycle slip detection process.

A. *Validation of the Used Data Against Cycle Slips*

The considered data are tested, against the existence of cycle slips, using the Difference between Change in Phase and Code (DCPC) values as a test quantity. Such test quantity can be computed as [12]:

$$DCPC = \Delta P_{12} - \Delta\phi_{12} \qquad (2)$$

Where:

DCPC	Used test quantity
ΔP_{12}	Change in the measured codes between the two consecutive epochs t_1 and t_2
$\Delta\phi_{12}$	Change in the measured phases between the two consecutive epochs t_1 and t_2 (computed using either L_1 or L_2)

Cycle slips may occur for one carrier wave only or for the two carrier waves simultaneously [3]. So, both carrier waves (L_1 and L_2) should be tested against cycle slips throughout the entire considered seven hours of data. To achieve this goal, DCPC values and its changes between each two consecutive epochs are computed for both carriers L_1 and L_2. Also, changes in DCPC values are drafted against time for both carriers (Fig. 2 and 3).

By noticing Fig. (2 and 3) and referring to [12] it can be deduced that, the considered seven hours of GPS observations (for the satellite SN 13) are free of cycle slips for both carrier waves L_1 and L_2. This is due to the fact that no sparks were observed all over the entire two curves in Fig. 2 and 3. More details concerning the detection of cycle slips using DCPC can be found in [12].

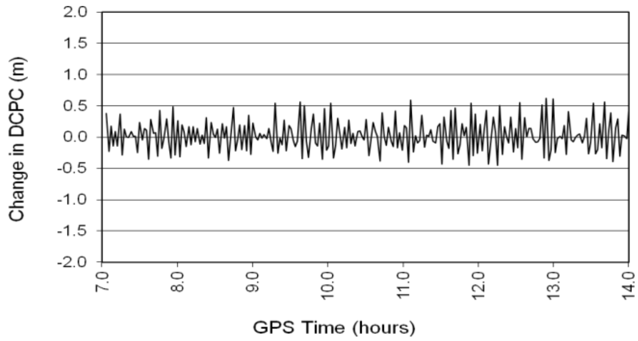

Fig. 2. Changes of DCPC Values (Considering L_1)

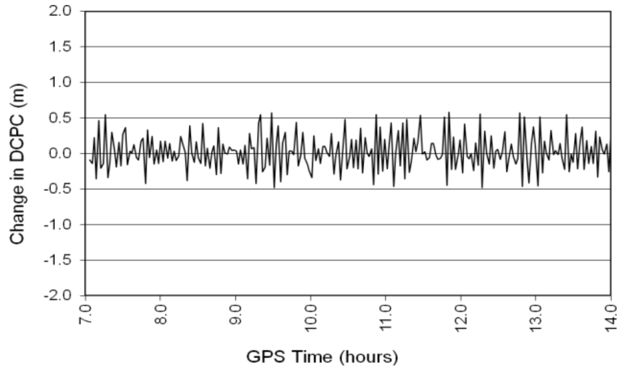

Fig. 3. Changes of DCPC Values (Considering L_2)

B. Numerical Application of Classical Hatch Smoothing Algorithm

After validating the used GPS data by checking it against the existence of cycle slips, the classical smoothing algorithm is applied using the most commonly used smoothing parameters. Such parameters are setting the weight factor to unity for the first epoch and decreasing it by an amount of 1% for each consequent epoch [3]. By substituting by these parameters in (1), the smoothed codes are computed. Finally, discrepancies between the raw codes and the smoothed ones are computed as:

$$\delta_{P(ti)} = P(t_i) - P(t_i)_{sm} \qquad (3)$$

Where:

$\delta_{P(ti)}$	Smoothing discrepancy at the epoch (i)
$P(t_i)$	Measured code at the epoch (i)
$P(t_i)_{sm}$	Smoothed code at the epoch (i)

Here, the classical Hatch filter is applied for only one hour of data (from 12:00 to 13:00), considering only one satellite (SN 13). Using (3), the smoothing discrepancies are computed for the considered one hour. Results are depicted in Fig. 4.

By observing Fig. 4 it can be seen that, the smoothing discrepancies started by zero value and increased with time. Such increase can be categorized into three patterns [11]. At the beginning of the smoothing process (region A), the smoothing discrepancies are very small (almost zero) due to the relatively high weight of the measured code in the computed smoothed code. At the middle of the smoothing process (region B), the weight factor decreases with time. So, the contribution of the measured code in the computed smoothed code decreased. Consequently, the smoothing discrepancies started to be relatively significant. Finally in region C (which starts at epoch no. 101), the weight factor already vanished from the beginning of the region, So, the measured code had no contribution into the computed smoothed code. As a result, the discrepancies are increasing faster in this region resulting in a nearly linear trend. This observed linear trend is a direct reflection to the difference in accuracy between code and phase measurements, which is nearly constant [6].

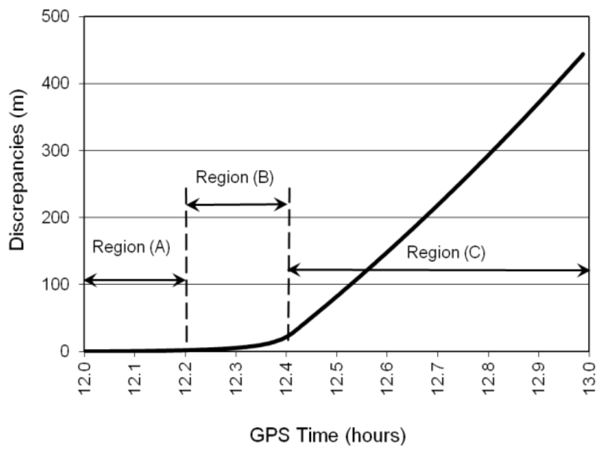

Fig. 4. Code Smoothing Discrepancies

Of course, by elongating the time needed to reach the full weight of phases (over than 100 epochs), smoother codes will be obtained. However, such smoother codes will be contaminated by larger biases [6]. On the other hand, shrinking of phases full weight time can produce more reliable codes with a lower degree of smoothness. So, it is of great importance to study the effect of the size of the smoothing window on the quality of the smoothed codes, which is the main objective of this paper. Such quality will be expressed in two ways. The first way is by studying the error budget of the resulted smoothed codes. This will reflect the external consistency (accuracy) of the smoothed codes. On the other hand, the second way is by studying the degree of smoothness of the resulted smoothed codes, which will reflect the internal consistency (precision) of such codes.

IV. External Consistency (Reliability) of Smoothed Codes

The reliability of the smoothed codes can be expressed inversely by the error budget contained by these codes. Of course, the smoothed codes should have the highest possible level of reliability. This high level of reliability will be certainly reflected on the accuracy of the 3-D positions obtained using such smoothed codes. All previous works that dealt with the issue of the code smoothing were based on using the measured codes as reference values. However, this approach has a great deficiency which is the very low accuracy of the measured codes [2]. So, such codes are not preferred to be used as a reference to judge the reliability of the resulted smoothed codes.

To overcome the above mentioned problem, another approach is used in this paper to evaluate the reliability of the computed smoothed codes. This approach is based on the computation of the difference between the change in the smoothed codes, between each two consecutive epochs, with the most reliable change in the spatial distance between the receiver and the satellite between the same two epochs. Such most reliable change will be considered here as the scaled phase difference between the considered two epochs. So, each two consecutive epochs (within the considered smoothing epochs) will result in a smoothing discrepancy. Such smoothing discrepancy will be denoted here as (Δ_{Sm}) and it can be expressed between the two epochs t_i and t_{i+1} as:

$$\Delta Sm_{i}^{i+1} = \lambda * \Delta\varphi_{i}^{i+1} - \Delta PSm_{i}^{i+1} \qquad (4)$$

Where:

ΔSm_{i}^{i+1}	Smoothing discrepancy between the two epochs t_i and t_{i+1}
$\Delta\varphi_{i}^{i+1}$	Phase difference between the considered two epochs
λ	Wave length of the considered carrier wave
ΔPSm_{i}^{i+1}	Change in the smoothed code between the considered two epochs

The resulted smoothing discrepancies will be used to judge the reliability of the smoothed codes by computing a reliability indicator (which will be denoted here as I_{rel}). Such reliability indicator will be expressed as the average of the computed smoothing discrepancies along the smoothing window only. This is due to the fact that after reaching the full weight of phases the smoothing discrepancies will be vanished. So, the reliability indicator can be given as:

$$I_{rel} = \frac{\sum_{i=1}^{i=n_{sm}} \Delta_{Sm}}{n_{sm}} \qquad (5)$$

Where:

I_{rel}	Reliability indicator
n_{sm}	Number of smoothing epochs

In (5), the number of smoothing epochs (n_{sm}) can be computed as:

$$n_{sm} = \frac{D_{sm}}{S.R} + 1 \qquad (6)$$

Where:

D_{sm}	Duration (size) of smoothing window
S.R	Sampling rate

In (5), another simpler approach can be followed to compute the numerator of the derived reliability indicator (which is the sum of the smoothing discrepancies along the used smoothing window). Here, it is very evident that the accumulation of the smoothing discrepancies between each two consecutive epochs, within the used smoothing window, will result in the difference between the change in the smoothed codes between the start and end of the smoothing process and the scaled phase difference between the same two epochs. So, (5) can be re-written as:

$$I_{rel} = \frac{\lambda * (\varphi_{last} - \varphi_{first}) - (P_{sm\text{-}last} - P_{sm\text{-}first})}{n_{sm}} \qquad (7)$$

Where:

φ_{last}	Measured phase at the end of the smoothing process.
φ_{first}	Measured phase at the beginning of the smoothing process.
$P_{sm-last}$	Last smoothed code (full weight of phases).
$P_{sm\text{-}first}$	First smoothed code (Same as the measured one).

In all sub-sequent parts of this paper, the reliability indicator (I_{rel}) will be used to evaluate the reliability of the different resulted smoothed codes instead of comparing it with the highly biased measured codes.

V. Internal Consistency of Smoothed Codes

The degree of smoothness of the computed smoothed codes is very important. Such importance can be interpreted by the quality control process of any GPS processing software package which usually rejects any GPS data that exhibit a relatively high fluctuation between consecutive epochs. This will result in a lower number of the accepted observations. Consequently, the quality of the derived positions will be certainly degraded. So, after the application of any smoothing algorithm, it is of great importance to study the internal consistency (smoothness) of the resulted codes. This is to grantee that the resulted smoothed codes have a sufficient degree of smoothness that is acceptable by the adopted GPS processing software.

In this paper, the main used concept to evaluate the internal consistency of any group of smoothed codes is by checking its degree of fitting any mathematical pattern. In other words, the smoothed codes are fitted using the highest possible best fitting pattern. Then, deviations between the actual smoothed codes and the fitted ones are computed and averaged (along the considered smoothing window) to derive out a quantity that can express the degree of internal consistency of the smoothed codes. Such quantity will be attributed as smoothing indicator (I_{sm}). Such indicator can be expressed as:

$$I_{sm} = \frac{\sum_{i=1}^{n_{sm}} P(t_i)_{sm} - P(t_i)_{fit}}{n_{sm}} \qquad (8)$$

Where:

I_{sm}	Smoothing indicator
$P(t_i)_{sm}$	Smoothed code at the epoch (i)
$P(t_i)_{fit}$	Fitted smoothed code at the epoch (i)

Here, many polynomials (with different orders) should be tried in fitting the computed smoothed codes. Then, discrepancies between the smoothed and fitted codes are computed and averaged for each tested order. Finally, the smoothing indicator will be computed using the best fitting polynomial which produces the minimum smoothing indicator. Of course, the degree of the best fitting polynomial is variable (among different times and satellites) depending on the pattern of spatial movement of the considered satellite with respect to the ground receiver.

VI. Assessment of External and Internal Consistencies of Smoothed Codes Using Traditional Smoothing Parameters

To study the reliability and smoothness of the smoothed codes, resulted by applying the traditional parameters, both the reliability and smoothness indicators are computed for that codes which were previously computed (from 12:00 pm to 13:00 pm), considering the satellite (SN 13). Concerning the computation of the reliability indicator (I_{rel}), the smoothing discrepancies are computed and averaged within the considered smoothing window. Here, the used smoothing window is 25 minutes (applying the traditional parameters and a sampling rate of 15 sec.). Also, and for validating the used algorithm, the reliability indicator is re-computed using the concept of accumulating the smoothing discrepancies, described by (7). Both scenarios resulted in the same reliability indicator with a value of 1.99m. All the described computational steps are programmed using C^{++}.

On the other hand, the computation of the smoothness indicator (I_{sm}) requires the computation of the fitted smoothed codes ($P(t_i)_{fit}$) at different smoothing epochs. This necessitates firstly the determination of the order of the best fitting polynomial of the computed smoothed codes. To achieve this goal, several polynomial orders should be tried to find out the best fitting one. Here, polynomial orders started from one (linear) to ten were tested. For each order, the smoothed codes are fitted (using the available 101 smoothed codes) and the corresponding polynomial coefficients are estimated using least squares principles. Then, the fitted smoothed codes are computed at different epochs. Finally, the smoothness indicator (I_{sm}) is computed for the ten tested polynomials. All the involved computations were performed using C^{++}. Results are summarized in table (1).

By observing the obtained results in table (1) it can be stated that, increasing the order of the used polynomial enhances the degree of fitness of the smoothed codes. Such enhancement is reflected in the significant decrease in the resulted smoothness indicator. This is true up to the 5^{th} order. Beyond the 5^{th} order, the resulted smoothness indicator fluctuates randomly up and down. Throughout its fluctuations, no values less than the one obtained using the 5^{th} order were observed. So, in this case, the 5^{th} order polynomial is considered the best fitting one with a smoothness indicator of 17.09m.

VII. Effect of the Smoothing Window Size on the Internal and External Consistencies of the Smoothed Codes

It is a matter of fact that the elongation of the used smoothing window yields smoother codes. However, such smoother codes will be contaminated by larger biases [6]. In other words, increasing the smoothness (internal consistency) of the smoothed codes will certainly reduce the reliability (external consistency) of such codes and vice versa. The question arises now is "*To what extent the elongation of the used smoothing window degrades the external consistency of the smoothed codes and enhances its internal consistency?*"

TABLE I. RESULTED SMOOTHNESS INDICATOR FOR DIFFERENT TESTED POLYNOMIAL ORDERS (APPLYING TRADITIONAL HATCH SMOOTHING PARAMETERS)

Used Polynomial Order	Resulted I_{sm} (m)	Tendency of I_{sm}
1^{st} (Linear)	24611.37	Decreases ↓
2^{nd}	579.86	
3^{rd}	175.15	
4^{th}	25.37	
5^{th}	17.09	
6^{th}	23.44	Randomly Fluctuates ↕
7^{th}	22.87	
8^{th}	21.09	
9^{th}	24.78	
10^{th}	22.67	

To answer the above mentioned question, several smoothing window sizes will be tested. Of course, all the tested smoothing windows should be located within the same previously used one hour of observations. This is to unify all the factors that are affecting the consistency of the resulted smoothed codes like the ionospheric conditions, satellite elevation angle, tropospheric conditions…etc. Here, twelve different smoothing window sizes will be tested. Such windows start with a relatively small size (only 5 minutes) and increased by a rate of 5 minutes till it reaches the considered one complete hour. For each window size, the number of smoothing epochs will be different. This will result in a different rate of change of the implied phase weight factor. Statistical information of the twelve tested smoothing windows are summarized in table (2).

At the beginning, the reliability indicator (I_{rel}) is computed for the twelve tested smoothing windows. For each smoothing window, the summation of the smoothing discrepancies is performed and averaged within the adopted smoothing epochs. All computations are performed concerning the same previously used satellite (SN 13). The resulted reliability indicators are listed in table (3). Also, the enhancement and degradation in the external consistency of the smoothed codes are listed in the same table as compared to a reference value. Such reference value is taken as the resulted reliability indicator when applying the classical Hatch smoothing parameters.

TABLE II. CHARACTERISTICS OF THE DIFFERENT TESTED SMOOTHING WINDOWS

Duration of Smoothing Window (min.)	Number of Smoothing epochs	Reduction in weight factor between consecutive epochs
5	21	5.00 %
10	41	2.50 %
15	61	1.67 %
20	81	1.25 %
25	101	1.00 %
30	121	0.83 %
35	141	0.71 %
40	161	0.63 %
45	181	0.56 %
50	201	0.50 %
55	221	0.45 %
60	241	0.42 %

TABLE III. RELIABILITY INDICATORS FOR SMOOTHED CODES USING DIFFERENT SMOOTHING WINDOWS

Smoothing Window Duration (min.)	Reliability Indicator (m)	Enhancement / Degradation * in External Consistency (%)
5	0.20	89.9
10	1.10	44.7
15	1.55	22.1
20	1.81	9.0
25	1.99	Zero (Reference Reliability)
30	2.11	-6.0
35	2.19	-10.6
40	2.23	-12.9
45	2.27	-15.1
50	2.31	-17.0
55	2.35	-18.1
60	2.38	-19.6

*
- *Positive values indicate enhancement in the reliability.*
- *Negative values indicate degradation in the reliability.*

After the computation of the reliability indicators for different smoothing window sizes, the smoothness indicator (I_{sm}) will now be computed for the same applied smoothing windows. In spite of the fact that the 5^{th} degree polynomial was proved to be the best fitting one for the smoothed codes when using the window size of 25 minutes, other higher (or lower) order polynomials can fit the smoothed codes, resulted from other window sizes, better. So, for all the 12 tested smoothing windows, the same pre-tested ten polynomials are applied for all the used smoothing windows. Also, the enhancements / degradations in the smoothness of the resulted codes are computed by considering a reference smoothness indicator of 17.09 m (computed using the classical smoothing parameters). Results are summarized in table (4).

To better visualize the behavior of both the reliability and smoothness indicators when changing the size of the adopted smoothing windows, the obtained values of the two types of indicators are drafted against the different tested sizes of smoothing windows in Fig. 5. By noticing the behavior of the computed two indicators in Fig. 5 it is very evident that, by increasing the size of the used smoothing window, the reliability of the computed smoothed codes decreases (solid curve), whereas the smoothness of the resulted codes is increased (dashed curve). So, it can be stated here that, reliability and smoothness exhibited a very significant inverse correlation.

TABLE IV. SMOOTHNESS INDICATORS FOR RESULTED USING DIFFERENT SMOOTHING WINDOWS

Smoothing Window Duration (min.)	Best Fitting Polynomial Order	I_{sm} (m)	Enhancement / Degradation * in Internal Consistency (%)
5	4^{th}	37.12	-117.2
10	5^{th}	25.06	-46.6
15	4^{th}	19.95	-16.7
20	5^{th}	18.12	-6.0
25	5^{th}	17.09	Zero (Reference Smoothness)
30	6^{th}	16.11	5.7
35	6^{th}	15.02	12.1
40	5^{th}	13.99	18.1
45	5^{th}	13.02	23.8
50	6^{th}	12.11	29.1
55	6^{th}	10.97	35.8
60	5^{th}	9.88	42.2

*
- *Positive values indicate enhancement in the smoothness.*
- *Negative values indicate degradation in the smoothness.*

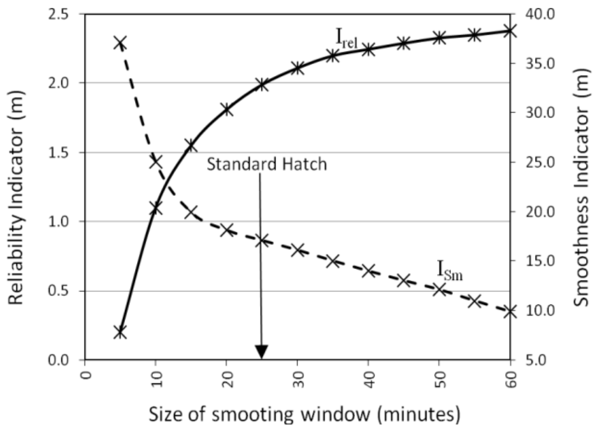

Fig. 5. Behavior of reliability and smoothness indicators for different smoothing window sizes

VIII. EFFECT OF THE DATA ACQUISITION TIME ON THE EFFICIENCY OF THE SMOOTHING PROCESS

To study the effect of the data acquisition time on the efficiency of the smoothing process all over the day, a continuous 24 hours of GPS data should be available. Of course, such data will never be available for the same satellite. To overcome this problem, different satellites will be studied along the same day, in different time intervals, with a total coverage of 24 hours. It should be noted here that the different selected satellites should have almost the same mask angles at the borders of the assigned intervals. This means that the mask angle of any new satellite at the beginning of its assignment should be as near as possible to the mask angle of the previous satellite at the end of the preceding interval. This is to unify all the factors that can affect the efficiency of the smoothing process. These criteria were achieved using four intervals. The properties of these four intervals and the used satellites are summarized in table (5).

After assigning the four listed intervals, each of them is divided into some sub-intervals with a duration of one hour. So, 24 different one hour data sets are resulted covering the whole day. For each one hour data set, the smoothing algorithm (1) is applied using the traditional parameters (after checking the existence of cycle slips). Then, both the reliability and smoothness indicators are computed for each one hour of observations. Also, the degree of the best fitting polynomial of the obtained smoothed codes is determined for each data set. Variations in the obtained reliability indicators along the day are drafted in Fig. 6. In addition, Fig. 7 shows the resulted smoothness indicators for each hour along the day as well as the degree of the best fitting polynomial for the obtained smoothed codes. In Fig 7, smoothness indicators are represented by the narrow hatched bars and scaled to the left axis, whereas the degrees of the best fitting polynomials are represented by the wide hollow bars and scaled to the left axis.

TABLE V. CHARACTERISTICS OF THE CONSIDERED INTERVALS AND SATELLITES

Interval no.	Duration		SN	Mask angle of the considered satellite	
	From	To		Start	End
1	0:00	7:00	14	4.35°	3.94°
2	7:00	14:00	13	4.01°	5.22°
3	14:00	18:00	26	6.01°	38.17°
4	18:00	24:00	30	37.94°	10.09°

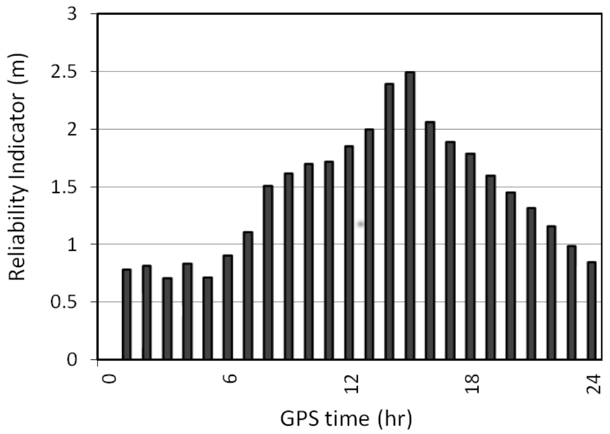

Fig. 6. Behavior of the Smoothed Codes Reliability Along one day

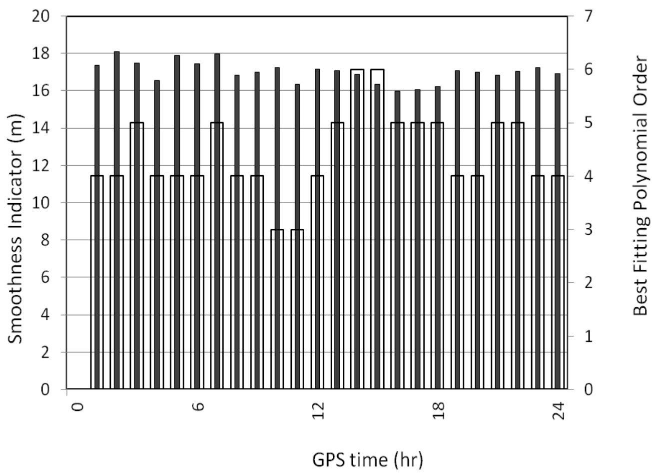

Fig. 7. Behavior of the Smoothness Indicator and the degree of the best fitting polynomial along one day

From Fig. 6 it can be seen that, the obtained reliability at night hours is significantly better than that obtained at day hours. This can be interpreted by the great change in ionospheric conditions between day and night hours along the same day. The obtained reliability at night hours is about three times better than that obtained at day hours. Also, at the beginning of the day (during night hours), slight random fluctuations in reliability are observed from hour to another. On the other hand, the reliability exhibited a tendency to degradation (from GPS time 5:00 to GPS time 15:00). Then, it begins to increase from GPS time 15:00 till the end of the day.

Finally, Fig. 7 indicates that the obtained smoothness is not affected by the time of the smoothing process. This is reflected by the continuous random fluctuations in the obtained smoothness indicators along the whole day (narrow hatched bars). The same fact is observed concerning the degree of the best fitting polynomial of the smoothed codes (wide hollow bars) which also exhibited the same random fluctuations. So, the time of the smoothing process within the day does not affect neither the smoothness indicator nor the degree of the best fitting polynomial. This is due to the fact that changing the time of the data acquisition process affects mainly the error budget of all the collected GPS observables. So, both the degree of the smoothness of such observables and its tendency to be fitted could not be affected.

IX. Conclusions

Based on the performed tests, and based on the above drafted obtained results, many important conclusions can be extracted concerning the process of smoothing GPS code data using phase data. Such conclusions can be summarized in the following points:

- Any group of phase data should be tested against the existence of cycle slips before using such data in smoothing the code data. Otherwise, any smoothing algorithm may be subjected to serious problems concerning both the reliability of the smoothed codes or its ability to be accepted by any GPS processing software.
- Accuracy of the obtained smoothed codes is inversely proportional to the value of the reliability indicator.
- Precision of the smoothed codes is inversely proportional to the value of the smoothness indicator.
- The 5th degree polynomial is the best one that fits the smoothed codes using traditional smoothing parameters.
- Reliability of the smoothed codes degrades very rapidly by increasing the duration of the smoothing process.
- Elongation of the duration of the smoothing process enhances significantly the smoothness of the resulted codes which will be certainly result in a larger number of accepted epochs in further processing of such smoothed codes.
- Changing the duration of the smoothing process does not affect the order of the best fitting polynomial of the resulted smoothed codes.
- During night hours, reliability of the smoothed codes is about three times better than its corresponding value at day hours.
- The time of the smoothing process within the day does not affect neither the smoothness of the resulted smoothed codes nor the order of its best fitting polynomial.

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