Pulse Shaping Algorithm for PAPR Reduction in ZCT Precoded OFDM System

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Abstract

In Orthogonal Frequency Division Multiplexing (OFDM) PAPR (Peak to Power Ratio) is one of the major drawbacks. In this paper we propose a study of PAPR and PAPR reduction technique using ZCT. The Zadoff-Chu Matrix transform (ZCT) is one type of technique used to reduce the PAPR analyzed. In ZCT (Zadoff-Chu Matrix transform) we obtain the ZCT sequences from the ZCT kernel row wise filling which gives rise to the constant Envelope OFDM (CE-OFDM) system with the zero dB and ZCT sequences form column wise gives rise to the nearly 7.8 dB with the 10^{-3}clip rate and the number of sub carriers used in the OFDM is $N = 64$ with QPSK or QAM modulation. In CE-OFDM it is required to use the pulse shaping which helps to meet the required transmission spectrum and to keep the out of band radiation low and the PAPRs are no longer zero dB. So in the present paper we analyze the various PAPR reduction techniques with the Root Raised Cosine (RRC) pulse shaping method. The final simulation results show the ZCT Row wise precoder based OFDM (ZCT-R-OFDM) system has less PAPR than the ZCT Column wise precoder based OFDM (ZCT-C-OFDM), conventional OFDM.

Keywords: Zadoff-Chu Matrix transforms (ZCT) system, ZCT-C-OFDM system, ZCT-R-OFDM system, conventional OFDM, PAPR (Peak to Average Power Ratio).

1. Introduction

Multicarrier transmission is also known as orthogonal Frequency Division multiplexing which is used for wireless and wire line digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and frequency selective fading [9]. The interest in this technique is mainly due to the recent advantages in the Digital Signal Processing technology. International standards will use the OFDM for the high speed wireless communication or being established by the IEEE 802.20 [9]. OFDM is popular modulation technique for the both wireless and wired communication. It will provide the very high data rates over the harsh wireless channels that are characterized by the multi path fading. OFDM widely used in Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Digital subscriber Lines (DSL), Wireless Local Area networks (WLANs), Wireless Metropolitan Area Networks (WMANs), and even it is used beyond the 3G Wide Area Networks (WANs). OFDM provides Inter symbol Interference (ISI) by inserting the Guard Interval (GI) using the cyclic prefix (CP) and it will moderate the frequency selectivity of the Multi path channel with a simple equalizer [8].

One of the major drawbacks of Orthogonal Frequency Division Multiplexing is Peak to average Power Ratio (PAPR). The High PAPR is occurred at the transmitter [10]. The high PAPR makes the OFDM sensitive to the non linear distortion caused by the transmitter power amplifier (PA). Without sufficient power back off the system suffers from the inter modulation distortion, performance degradation, etc [5, 10]. These types of problems are reduced by increasing the back off which results in reduced power amplifier (PA). The high PAPR increases the complexity of Analog to Digital converter in the transmitter section, Digital to Analog converter in the receiver section and also it reduces the efficiency of power amplifier [3]. To reduce the PAPR we propose the different types of techniques like clipping, coding, peak windowing. Clipping is the simple technique which provides serious out of band radiation. Similarly the coding and peak windowing are the similar techniques which mainly used to reduce the PAPR but this type of techniques are not good for coding solutions.
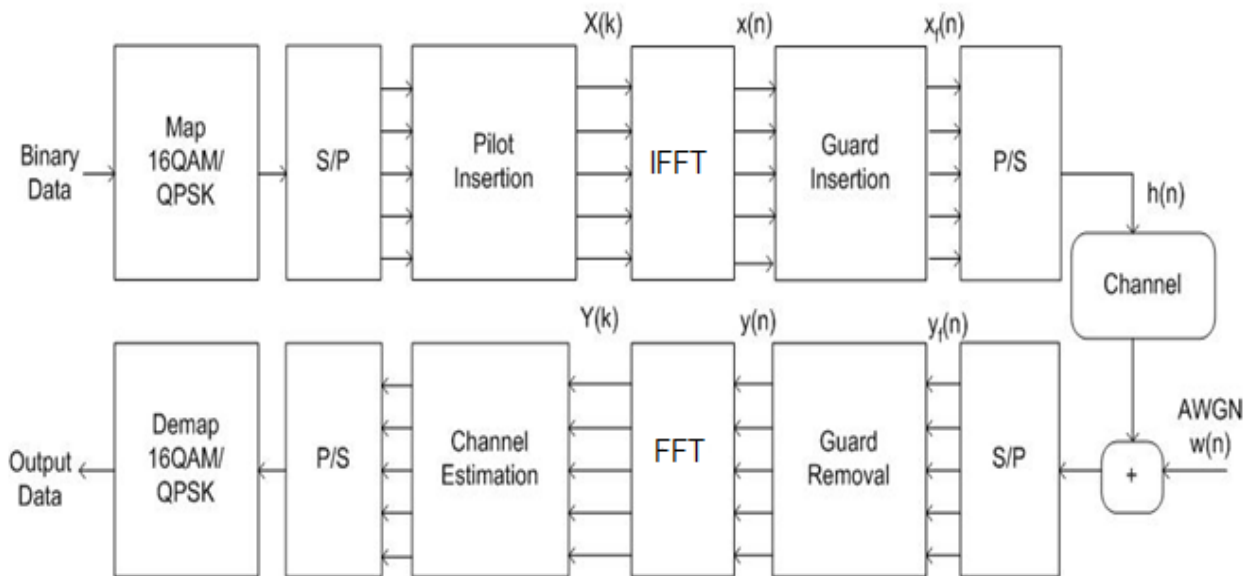
Fig1: Conventional OFDM system

So for zero distortion of the OFDM signal the High Power amplifiers (HPA) need to be used for sufficient back off. Even though large number of PAPR techniques proposed the precoding based technique is effectively used because of it provides the simple linear techniques. If we reducing the PAPR it means that we are reducing the OFDM system cost, reducing the complexity of the Analog to Digital (A/D), Digital to Analog (D/A) conversion, increasing the transmit power for the same range of receiving Signal to Noise Ratio (SNR).

In this paper we analyze the Zadoff-Chu Matrix Transform with Row wise precoder OFDM (ZCT-R-OFDM) system using the Root Raised cosine (RRC) Pulse shaping and the results are compared with the other precoding based OFDM systems [1, 2]. The below Paper is organized as follows section 2 discuss the OFDM block diagram and PAPR calculation, section 3 presents the proposed method (ZCT-R-OFDM), section 4 provides the project simulation results, and final section 5 concludes the project.

2. OFDM Analysis and PAPR

In OFDM initially the binary data is calculated and modulated using the Quadrature Amplitude Modulation or Quadrature Phase shift keying and it is converted to the parallel order which is mainly used to reduce complexity of the signal. After modulation the pilot tones are inserted in between the modulated data for securing the information, the modulated output is of complex vector of order N where N is sub carriers. The parallel converted modulated data $X(k)$ is passed to the Inverse Fourier Transform. The IFFT converts the modulated complex data to the time information $x(n)$ which is mainly used to reduce the complexity of the system and improves the fastness of the Algorithm [9]. The transformed data is passed to the receiver section through the channel. Before the passing through the channel Guard intervals are inserted in between the transformed data $x(n)$ and forms $x1(n)$. The Guard interval is used to reduce the overlapping in between the orthogonal signal.

In the channel the Additive White Gaussian Noise (AWGN) is combined to the transformed data and it passed to the receiver section [5]. In the receiver section the guard interval is removed and demodulated the data after FFT, finally the output data is calculated

2.1 OFDM signal

In OFDM signal we use N sub carriers for the message bit to transfer from transmitter to receiver. After QAM the message bits are of complex vector of size N as [9] $X(k) = [X1(k), X2(k), X3(k) … … … … .]^T$ where X is passed through the IFFT block. The complex base band OFDM signal with N sub carriers is written as [10].

$$x_n = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X(k). e^{-\frac{j2\pi nk}{N}}, \qquad (1)$$

$Where\ n = 0,1,2,3, …. N − 1$

Here $j = \sqrt{-1}$ and x_n is the IFFT block and $X(k)$ is the complex vector.

The PAPR is calculated as

$$PAPR = \frac{\max(x_n)^2}{E[x_n]^2} \qquad (2)$$

Where $E[.]$ denotes the expectation of the signal.

The Complementary Cumulative Distribution Function (CCDF) for OFDM is calculated as

$$P(PAPR > PAPR0) = 1 - (1 - e^{-PAPR0})^N \quad (3)$$

Here $PAPR0$ is the clipping level and it exceeds some clip level $PAPR0$.

3. System model

3.1 Zadoff-Chu (ZC) Sequences and Zadoff-Chu Matrix Transform (ZCT)

Zadoff –Chu (ZC) Sequences has an ideal periodic auto correlation with constant magnitude. The ZC Sequences of length N can be defined as [1]

$$a_n = \begin{cases} e^{\frac{j2\pi r}{N}\left(\frac{k^2}{2} + qk\right)} & for\ N\ is\ even \\ e^{\frac{j2\pi r}{N}\left(\frac{k(K+1)}{2} + qk\right)} & for\ N\ is\ odd \end{cases} \qquad (4)$$

$Where\ k = 0,1,2\ldots.N-1,\ q\ is\ the\ integer,$
$r\ is\ the\ relatively\ prime\ to\ N\ and\ j = \sqrt{-1}$

The ZCT kernel is obtained by reshaping the ZC sequences in the $k = mL + l$

$$A = \begin{pmatrix} a00 \ a01 \ a02 \ a03 \ a04 \ \ldots\ldots\ldots a0(L-1) \\ a10 \ a11 \ a12 \ a13 \ a14\ldots\ldots\ldots\ldots a1(L-1) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a(L-1)0 \ a(L-1)1 \ a(L-1)2 \ \ldots\ldots\ldots a(L-1) \end{pmatrix}$$

The row variables are m and the column variables are l, in other words $N = L^2$ point for long ZC sequences with row wise kernel matrix. The PAPR will reduce to zero dB in this case [1,2].

If the kernel matrix is in column matrix then $k = m + lL$ in this case PAPR will not reduce completely to zero dB but it reduces up to 7.8 dB.

3.2 ZCT Precoded OFDM system

The below shown Figure is the ZCT Precoded OFDM system. In this system the ZCT row wise Precoded matrix A is applied to the modulated constellation symbols before the Inverse Fast Fourier Transform (IFFT) to reduce the PAPR. In ZCT method

the modulated data of complex vector of size L can be written as $X = [X_0, X_1 \ldots\ldots\ldots X_{L-1}]^T$. After modulating the data ZCT precoding is applied to the modulated complex vector and transformed to the Inverse Fast Fourier Transform. The ZCT precoding can be written as [1, 2]

$$Z = AX = [Z_0, Z_1, Z_2, Z_3 \ldots\ldots Z_{L-1}]^T$$

Where A is the ZCT precoder matrix with the size $L * L$ and X is the modulated complex data.
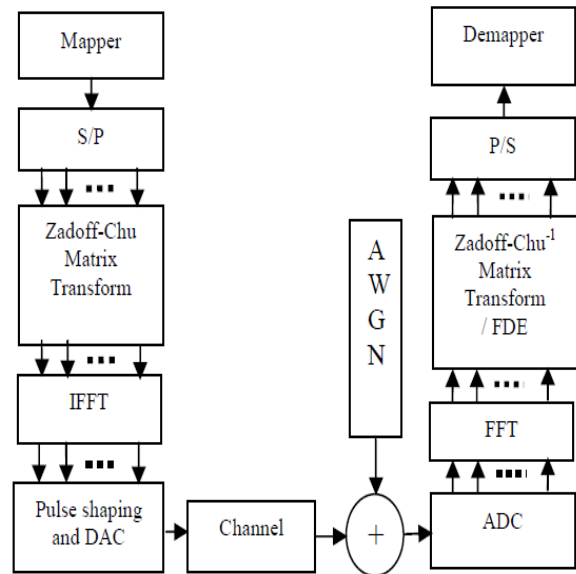

Fig 2: ZCT precoded OFDM system

The ZCT precoder matrix can also be written as

$$Z_m = \sum_{l=0}^{L-1} a_{m,l} . X_l \qquad (5)$$
$$where\ m = 0,1,2,\ldots.L-1$$
$$a_{m,l}\ means\ m^{th}\ row\ and\ l^{th}\ column$$

From the above equation using the row wise shaping $k = mL + l$ and putting the $q = 0, r = 1$ in the equation (3)

$$Z_m = \sum_{l=0}^{L-1}\left(e^{\frac{j\pi(mL+l)^2}{L^2}}\right).X_l \qquad (6)$$

Where $m = 0,1,2 \ldots L-1$. The above equation provides the ZCT precoded constellation symbols.

The ZCT-Row Wise OFDM signal with the L sub carriers after IFFT is written as [1, 2]

$$x_n = \frac{1}{\sqrt{L}}\sum_{m=0}^{L-1} Z_m . e^{\frac{j2\pi nm}{L}}, n = 0,1,2,3\ldots L-1. \quad (7)$$

The pass band transmit signal $x(t)$ of ZCT Row wise OFDM after Root Raised Cosine (RRC) pulse shaping and D/A of x_n can be written as [1]

$$x(t) = e^{jwct}\sum_{n=0}^{L-1} x_n\ .r(t - n\tau) \qquad (8)$$

Here w_c is the carrier frequency, $r(t)$ is the base band pulse, $\tau = \left(\frac{M}{N}\right)$. T is the compressed symbol duration in the seconds after inverse Fast Fourier Transform.

3.3 Root Raised Cosine

The Root Raised Cosine Pulse shaping filter can be defined as [2]

$$r(t) = \frac{\left(\sin\left(\frac{\pi t}{T}(1-\alpha)'\right) + \frac{4\alpha t}{\tau}\cos\left(\frac{\pi t}{\tau}(1+\alpha)\right)\right)}{\frac{\pi t}{\tau}(1-(\frac{16\alpha^2 t^2}{\tau^2}))} \qquad (9)$$

$0 \leq \alpha \leq 1$, where α is the roll off factor.

The PAPR of ZCT-R-OFDM signal with pulse shaping can be written as

$$PAPR = \left(\frac{\max\limits_{0 \leq t \leq N\tau}[x(t)^2]}{\frac{1}{N\tau}\int_0^{N\tau}[x(t)^2]dt}\right) \qquad (10)$$

The PAPR of ZCT-R-OFDM without pulse shaping can be written as [1]

$$PAPR = \left(\frac{\max\limits_{n=0,1,2,\ldots N-1}[x[n]^2]}{\frac{1}{M}\sum_{n=0}^{N-1}[x[n]^2]}\right) \qquad (11)$$

After precoding using the ZCT the orthogonality is maintained between the symbols. Because the precoding matrix is auto orthogonal. The PAPR is better reduced in ZCT-R-OFDM with Pulse Shaping Algorithm than ZCT-R-OFDM without Pulse shaping.

4. Simulation results

The complete analysis of the PAPR reduction is carried out in MATLAB software. It is used to find PAPR of ZCT-C-OFDM, ZCT-R-OFDM with the Root Raised Cosine Pulse Shaping. The PAPR is statically calculated using the Complementary Cumulative Distribution Function (CCDF). The CCDF of PAPR OFDM signal is used to find the probability of exceeding given $PAPR_0$.
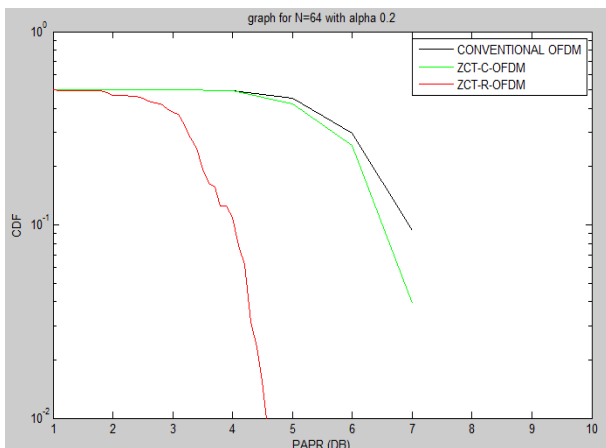
$$Prob(PAPR > PAPR0) = 1 - (1 - e^{-PAPR0})^N.$$

Fig: 3 CCDF comparison of ZCT-R-OFDM with ZCT-C-OFDM and conventional OFDM with Pulse shaping using N=64 and α =0.2.

In the Figure 3 the number of sub carriers N=64 and the roll off factor α=0.2 are used for Pulse shaping and the ZCT Row wise OFDM has PAPR 4.5 dB and ZCT-C-OFDM has 7.5 dB and conventional OFDM has above 8 dB with the clip-rate 10^{-2}.
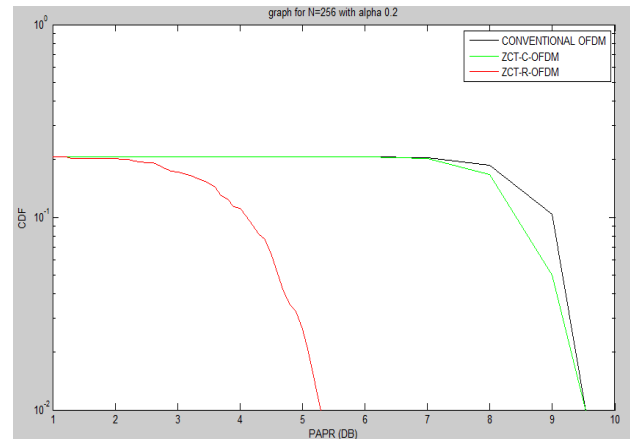

Fig: 4 CCDF comparison of ZCT-R-OFDM with ZCT-C-OFDM and conventional OFDM with Pulse shaping using N=256 and α =0.2.

In the Figure 4 the number of sub carriers N=256 and the roll off factor is α= 0.2 are used for Pulse shaping and the ZCT-R-OFDM system, ZCT-C-OFDM system, Conventional OFDM system PAPR has 5.3 dB, 9.5dB, 9.7dB with clip rate 10^{-2}.

5. Conclusion

In this paper the PAPR analysis using ZCT-R-OFDM system with RRC pulse shaping is presented. The advantage of using the ZCT method is the PAPR reduction with less complexity. In the Figure 3 and Figure 4 it is analyzed that the ZCT-Row Wise-OFDM has lower PAPR than the ZCT- Column Wise-OFDM, conventional OFDM System. So it is concluded that the ZCT-R-OFDM has better PAPR reduction than the conventional OFDM, ZCT-C-OFDM and other types of PAPR reduction techniques. Another advantage of using the ZCT-R-OFDM is pulse shaping the symbol using RRC which does not require for increasing the power, complex optimization.

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