Proposed Modification of Holt's Method for Short Term Forecasting

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***Abstract*— Forecasting has long been our part of life. It was centered to weather forecasting only till 19th century but in 20th century it gets new dimension in the business planning. Since 1950's a lot of research works has been carried out on business forecasting and is continuing today to improve the existing forecasting methods and develop a new method or model. This article deals with such an existing method namely Holt's method (or sometimes called Holt-Winter's method) to forecast the time series data containing trends or linear trends but no seasonality. It is noted that this method used only the observed (real) data to predict data for all the next periods ahead (3 to 5) but it does not take into consideration the most recent inter trends relation. We know that recent (last few periods) data have more significant effect rather than far old data on forecast. Exploiting this idea in this research works a modification is proposed to estimate future data. In the proposed modified approach, we take into account the recent available data (may be real or predicted) as weight parameter along with previous trend to forecast the next period outcome. We expect that our modified forecasts can be a better approximation or give the best upper or lower limit of the forecast depending on the nature of last few data.**

Keywords*— *Forecasting, Trends, seasonality, Holt's method

I. INTRODUCTION

We all make and use forecasts every now and then, both in our jobs and everyday life. Armstrong [1] defined forecasting as the prediction of an actual value in a future time period. Makridakis et al. [2] stated that forecasting supplies information of what may occur in the future. And therefore, it is used to estimate when an event is likely to happen so that we can take necessary actions.

In business, forecasting is the basis for budgeting, planning capacity, sales, production and inventory, personnel, purchasing etc. which affects decisions and activities throughout an organization [3]. Business forecasting is used not only in predicting demand but also that of profits, revenues, costs, productivity changes, raw materials, interest rates, movement of key economic indicators(e.g., GDP, inflation, government borrowing) and prices of stocks and bonds. Though computers and sophisticated mathematical models are used in forecasting they are not exact science rather successful forecasting requires a proper blending of art and science. So in this modern age of business competitiveness is everywhere and to survive in such competitive world market business organization needs to predict the business involved future events as precisely as possible. To serve this purpose they have to use some mathematical model to predict the future outcomes based on the historical data available to them. The sequence of historical data collected at uniform time intervals is called time series [4]. The time intervals may be in hour(s), day(s), week(s), month(s), quarter year or year(s).

Holt's (linear exponential smoothing [5]) method performs well for the time series where only trends [6] exist but no seasonality. Its extended version called Holt-Winters' method which is also a univariate method is used for the time series where trends and seasonality both exists [7]. Holt's method is easy than some other method such as ARIMA [8].

II. EXISTING HOLT'S METHOD

Exponential or single exponential [5] method does not work well if the time series data contains trends or seasonality. To overcome the In that case several methods were developed to overcome the difficulties involving errors in forecasting and usually they are referred to "double exponential smoothing method". One of the methods is named "Holt-Winters double exponential smoothing" or only "Holt's Method". This method works as follows:

We suppose that the raw data sequence of observations is represented by $\{X_t\}$, beginning at time $t = 0$. We use $\{S_t\}$ to represent the smoothed value for time t, and $\{B_t\}$ is our best estimate of the trend at time t. The output of the algorithm is now written as F_{t+m}, an estimate of the value of X_t at time $t+m$, $m>0$ based on the raw data up to time t. Double exponential smoothing is given by the formulas:

$$S_t = \alpha X_t + (1-\alpha)(S_{t-1}+B_{t-1}) \quad (1)$$

$$B_t = \beta (S_t - S_{t-1}) + (1-\beta) B_{t-1} \quad (2)$$

Where α and β are smoothing constants such that $0 < \alpha, \beta < 1$; X_t denotes observed data whereas B_t indicates tend value at time t and S_t be smoothed value at time t. Now we need the initial value of S_t, B_t and for $t > 1$ they have the following form:

$$S_0 = X_0 \text{ and } B_0 = (X_n - X_0)/n \quad (3)$$

And the h-step forecast by this method is given by the following equation:

$$F_{t,h} = S_t + h \cdot B_t \quad (4)$$

Here $F_{t,h}$ denotes the forecasting value determined for t+h period at period t based on available data for the first t period data. The following section numerically illustrates how this method functions.

III. NUMERICAL EXAMPLE

Let us consider the following time series data. We are to forecast the value for the next five periods.

TABLE I. TIME SERIES DATA

Period	Value	Period	Value	Period	Value
1991	591	1999	1301	2007	2146
1992	620	2000	1440	2008	2430
1993	699	2001	1661	2009	2746
1994	781	2002	1770	2010	3069
1995	891	2003	1851	2011	3649
1996	993	2004	1954	2012	4159
1997	1111	2005	2023	2013	4686
1998	1149	2006	2079		

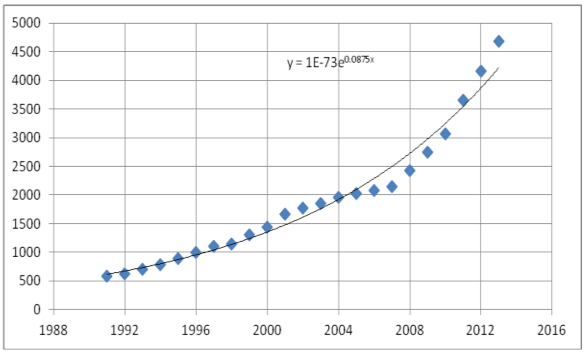

Fig. 1. Graph of observed data with trend line

At first, we plot the data to see if the trend exists but no seasonality. From the Fig. 1, we observe that there exist trends only but no seasonality. So we can apply Holt's method to forecast. It is noted that a trend line plotted on the same axis is fitted well to the observed data.

According to the Halt's for the initialization we set $S_0 = X_0 = 591$ which is the first observed data and

$$B_0 = \frac{X_2 - X_0}{2} = \frac{699 - 591}{2} = 54$$

After testing several combination of different values of these two parameters to find very closer smoothed or fitted values, we set $\alpha = 0.7$ and $\beta = 0.7$ for this numerical instance.

Now using the formulas of Holt's approach we have obtained the trends values. The details numerical results are shown in the Table II.

TABLE II. SMOOTHING DATA

T	Year	X_t	S_t	B_t	$F_{t-1,1}$
0	1991	591	591.00	54.00	
1	1992	620	627.50	41.75	645.00
2	1993	699	690.08	56.33	669.25
3	1994	781	770.62	73.28	746.40
4	1995	891	876.87	96.36	843.90
5	1996	993	987.07	106.05	973.23
6	1997	1111	1105.63	114.81	1093.12
7	1998	1149	1170.43	79.80	1220.44
8	1999	1301	1285.77	104.68	1250.24
9	2000	1440	1425.13	128.96	1390.45
10	2001	1661	1628.93	181.34	1554.09
11	2002	1770	1782.08	161.61	1810.27
12	2003	1851	1878.81	116.19	1943.69
13	2004	1954	1966.30	96.10	1995.00
14	2005	2023	2034.82	76.80	2062.40
15	2006	2079	2088.78	60.81	2111.62
16	2007	2146	2147.08	59.05	2149.60
17	2008	2430	2362.84	168.75	2206.13
18	2009	2746	2681.68	273.81	2531.59
19	2010	3069	3034.95	329.43	2955.49
20	2011	3649	3563.61	468.90	3364.38
21	2012	4159	4121.05	530.88	4032.51
22	2013	4686	4675.78	547.57	4651.93

It is observed that the observed values and estimated values obtained by the method are almost identical. So the values $\alpha = 0.7$ and $\beta = 0.7$ are perfect enough for the instance considered.

Now the goal that is to forecast for the next five future years (namely year 2014, 2015, 2016, 2017 and 2018), with period T = 23, 24, 25, 26 and 27 respectively from the last period 22 (i.e. year 2013). Then the forecast for h =1, 2, 3, 4 and 5 based on 22^{nd} period smoothed and trend value by (4) which are accomplished below:

$F_{23}=F_{22,1}=S_{22}+1*B_{22}=4675.78+1*547.57=5223.35$

$F_{24}=F_{22,2}=S_{22}+2*B_{22}=4675.78+2*547.57=5770.92$

$F_{25}=F_{22,3}=S_{22}+3*B_{22}=4675.78+3*547.57=6318.49$

$F_{26}=F_{22,4}=S_{22}+4*B_{22}=4675.78+4*547.57=6866.06$

$F_{27}=F_{22,1}=S_{22}+5*B_{22}=4675.78+5*547.57=7413.63$

The forecasted values are displayed in the Table III. It is observed that for the predicted values of the years 2014, 2015, 2016, 2017, and 2018, the smooth value and trends value of based year remain constant for all the cases.

TABLE III. FORECAST VALUE OBTAINED BY THE HOLT'S METHOD

Year	Forecasted Value
2014	5223.35
2015	5770.92
2016	6318.49
2017	6866.06
2018	7413.63

IV. OUR PROPOSED MODIFIED METHOD

It is observed in Holt's method that to obtain the smoothed values they used available immediate observed values and immediate previous trends. But for forecasting any far years they only used just last smoothing observed value (in the above example that was 22^{nd}) as their base and added the consecutive multiple of the corresponding trend of the base period (here 22^{nd}).

It might be assumed that the recent available data have more significant effect on future prediction rather than far old data. By exploiting this idea we want to develop a forecast model based on Holt's approach. However before formulation the proposed modification, it is better to analyze the estimated values obtained by the Holt's method of the given instance.

At first we have compared the actual values and estimated value for recent periods namely year 2008 – 2013. The comparison is shown in the Fig. 2. It is observed that the actual values are always greater than estimated values for all the period (recent) considered.

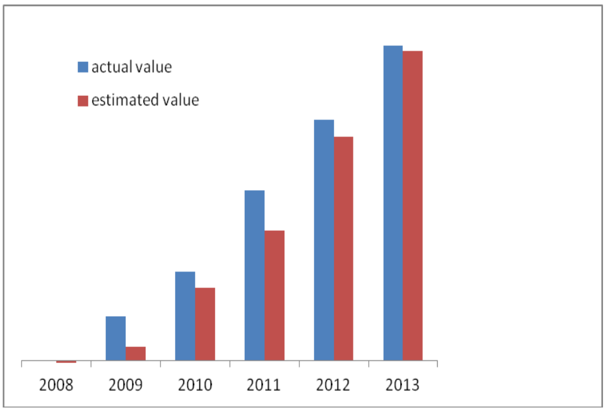

Fig. 2. Comparison between actual and estimated values

In order to measure the relative error between actual value and estimated value obtained by Holt's method for these recent periods, we set the following formula

Relative error = (actual value – estimated value)/ actual value ×100.

The relative errors are plotted against the years (periods) which given in the Fig. 3. It is noticed in the figure that there are significant errors.

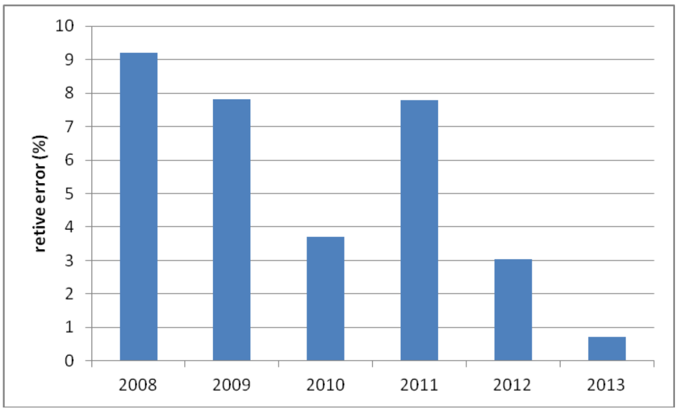

Fig. 3. Relative percentage error in 1-step estimated values of Holt's method

Moreover, we have noticed that there is a gradual change in the trend which is the increment compared to the immediate last trend value. The numerical example in Section III has the following trend values for the last four periods:

$B_{19} = 329.43$; $B_{20} = 468.9$; $B_{21} = 530.88$; $B_{22} = 547.57$

Now we want to observe the inter trend relation of the existing method which is shown in Table IV.

TABLE IV. MOST RECENT INTER-TREND RELATIONS

Trend Values	Difference $D_t = (B_t - B_{t-1})$	Remarks
B_{19}=329.43	---------	--------
B_{20} = 468.9	D_{20}=139.47	$B_{19}+D_{20}$
B_{21}=530.88	D_{21}=61.98	$B_{20}+D_{21}$
B_{22}=547.57	D_{22}=16.69	$B_{21}+D_{22}$

Thus, when we make forecast according to (4) based on the current period smoothed data we incorporate here only the current trend but it is obviously true that we do not take the changes in the trend into consideration. It is seen from Table IV that trends do not remain fixed rather it changes from the previous trend value by some amount and so it is convincible that in future this gradual change also remain in the time series data and to forecast adding this change in the model (4) is reasonable.

From this analysis it may be concluded that the forecasting value by Holt's method based on the last smoothed value (corresponding to the observed value) contain a significant error. To reduce this error we want to modify (4). The proposed modified equation of (4) is given as follows:

$$F_{t,h} = S_t + h \cdot (B_t + D_{t+h}) \qquad (5)$$

where,

$$D_t = (B_t - B_{t-1}) \qquad (6)$$

$$D_{t+h} = \text{Harmonic Mean of } (D_{t+h-1}, D_{t+h-2}, D_{t+h-3}) \qquad (7)$$

Here (7) is formulated by using recent trend values. So (5) provides the forecasts where trend is updated by adding the additional parameter by (7) in each and every time of forecasting. Thus it adds more weight to the most recent trend than the far old data.

TABLE V. UPDATED DIFFERENCE IN FUTURE TRENDS

Updated Difference in Trends	Value
D_{23}	HM (D_{20} ,D_{21} ,D_{22}) =36.05
D_{24}	HM (D_{21} ,D_{22} ,D_{23}) =28.90
D_{25}	HM (D_{22} ,D_{23} ,D_{24}) =24.54
D_{26}	HM (D_{23} ,D_{24} ,D_{25}) =29.10
D_{27}	HM (D_{24} ,D_{25} ,D_{26}) =27.34

*HM means Harmonic mean

TABLE VI. FORECASTS COMPARISON TABLE

Year/ Period	Old	Modified
2014=F_{23}=$F_{22,1}$	5223.35	5259.4
2015=F_{24}=$F_{22,2}$	5770.92	5828.72
2016=F_{25}=$F_{22,3}$	6318.49	6392.11
2017=F_{26}=$F_{22,4}$	6866.06	6982.46
2018=F_{27}=$F_{22,5}$	7413.63	7550.33

Now we have implemented this modified formula to the numerical example given in section III. So modified forecasting values obtained by our proposed modification (5) which is displayed in the Table VI. Now we have compared the modified forecasting with Holt's forecasting given in the Table VI.

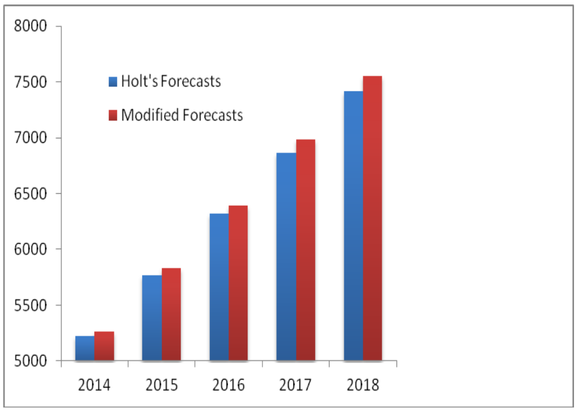

Fig. 3. Comparison between Holt's and Modified Forecasts

It is observed that our modified forecasts have a bit greater value than the Holt's forecasts. To observe the relatve increase in forecasts by our proposed method relative to Holt's method we use the following formula:

Relative increase = (Modified value – Holt's value)/ Modified value ×100.

Relative percentage increase of the proposed method over Holt's method is shown in the Fig. 4. It is observed in the figure that the increase of predicted values obtained by our proposed method is not much larger. So this increment of forecasts by (5) is very much resonable and it should be much closer to the real value than the the forecasts by Holt's method.

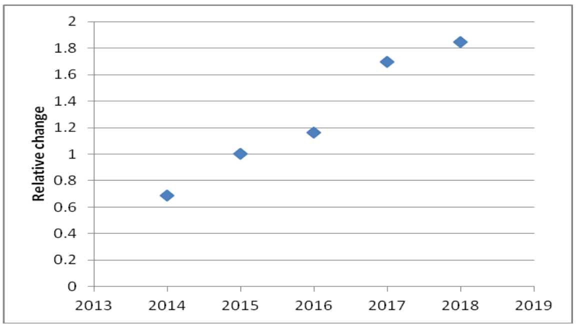

Fig. 4. Relative percentage changes in new forecasting

V. CONCLUSION

To forecasts for any periods, Holt's method uses the last smoothed observed value. On the other hand, our proposed modified method uses the last smoothed observed value along with most recent estimated trend values to weight the most recent estimated trend values over far old data to forecast. Numerical experiments suggest that the estimated values obtained by the proposed method are much closer to the real values than that of Holt's method. It is also expected that the forecasts values obtained by modified method will be closer to the real values.

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