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Properties Of The Bilinear Concomitant Of A Matrix Differential Equation

1. **Dr. Brijendra Kumar Singh**, *Department of Mathematics, J.P. University, Chapra,India*
2. **Himanshu Shekhar**, *Research Scholar*

ABSTRACT: The problem considered in this article is an eigenvalue problem associated with a matrix differential operator. In a previous article the problem has been defined, existence theorem has been proved and the expression for bilinear concomitant has been obtained. In this article the properties of bilinear concomitant are proved which are useful in obtaining further results concerned with the eigenvalue expansion associated with the problem.

1. We have considered the Matrix Differential Equation

$$L\phi = -F\phi \qquad \text{......................... (1.1)}$$

where

$$L = \begin{pmatrix} \frac{-d}{dx}\left(P_O \frac{d}{dx}\right) + p & r \\ r & i\frac{d}{dx} + q \end{pmatrix} \qquad \text{....... (1.2)}$$

$$F = \begin{pmatrix} F_{11}, F_{12} \\ F_{21}, F_{22} \end{pmatrix} \qquad \text{................(1.3)}$$

$$\text{and } \phi = \begin{pmatrix} u \\ v \end{pmatrix} \qquad \text{......................(1.4)}$$

with proper conditions as given in the previous paper.

The Bilinear Concomitant has been defined to be

$$\left[\phi_j \, \phi_k\right] = P_0\left(u_j \overline{u_k'} - u_j \bar{u}_k\right) + iv_j v_k \qquad \text{..... (1.5)}$$

Properties of the Bilinear Concomitant:

Theorem – 1:

If ϕ_j (x, λ) and ϕ_k (x, λ) are two solutions of (1.2) for the same value of λ, then $[\phi_j\,(x, \lambda)\ \phi_k\,(x, \bar{\lambda})]$ in independent of x.

Proof:

Using Green's formula we have

$$\int_{x_1}^{x_2} \left(\phi_k^{-T}\,(x, \bar{\lambda})\, L\phi_j\,(x, \lambda) - \phi_j^T\,(x, \lambda)\,\overline{L\phi_k\,(x, \bar{\lambda})}\right) dx$$

$$= \left[\phi_j(x, \lambda)\,\phi_k(x, \bar{\lambda})\right](x_2) - \left[\phi_j(x, \lambda)\,\phi_k(x, \bar{\lambda})\right](x_1)$$

Where a $\le x_1$, $x_2 \le$ b, which with the help of (1.2) reduces to the form.

$$\int_{x_1}^{x_2} \left(-\phi_k^{-T}\,(x, \bar{\lambda})\, F\phi_j\,(x, \lambda) + \phi_j^T\,(x, \lambda)\, F\phi_k\,(x, \bar{\lambda})\right) dx$$

$$= \left[\phi_j(x, \lambda)\,\phi_k(x, \bar{\lambda})\right](x_2) - \left[\phi_j(x, \lambda)\,\phi_k(x, \bar{\lambda})\right](x_1) \qquad \text{...... (1.6)}$$

Evidently, the relation $\phi_j^T F\phi_k = \bar{\phi}_k^T F\phi_j$ reduces (1.6) to

$$\left[\phi_j(x,\lambda)\,\phi_k(x,\bar{\lambda})\right](x_2) = \left[\phi_j(x,\lambda)\,\phi_k(x,\bar{\lambda})\right](x_1) \qquad \text{...... (1.7)}$$

But x_1, x_2 are any two points in [a, b] such that (1.7) holds. This establishes the proof of the statement.

Remarks:

(i) For brevity $\left[\phi_j(x,\lambda)\,\phi_k(x,\bar{\lambda})\right]$ will be denoted by $\left[\phi_j\,\phi_k\right](\lambda) = \left[\phi_j\,\phi_k\right]$.

(ii) If ϕ_j (x, λ) and ϕ_k (x, λ) satisfy the conditions of theorem (1), then by the representation $\bar{\phi}_k^T L\phi_j - \phi_j^T L\bar{\phi}_k = \frac{d}{dx}\phi_j\,\phi_k$, $\phi_j(x,\lambda)\,\phi_k(x,\bar{\lambda})$ is an integral function of independent of x where λ is real but $\left[\phi_j\,\phi_k\right]$ is not necessarily real.

(iii) With the help of (1.6), (1.7) and the Green's formula, we have

$$\left(\bar{\lambda}-\lambda\right)\int_a^b \phi_j\,(x,\lambda)\,F\bar{\phi}_k^T\,(x,\lambda)$$

$$= \quad \left[\phi_j\,(x,\lambda)\,\phi_k\,(x,\lambda)\right](b) - \left[\phi_j\,(x,\lambda)\,\phi_k\,(x,\lambda)\right](a) \qquad \text{...... (1.8)}$$

Theorem – 2:

$$\phi_j(x,\lambda)\,\phi_k(x,\bar{\lambda}) = \overline{-\phi_k(x,\lambda)\,\phi_j(x,\lambda)}$$

Bar as usual denotes the complex conjugates. ...... (1.9)

Proof:

The expression for bilinear concomitant and its conjugates, from (1.5) yields the result (1.9).

Corollary:

$\left[\phi_j(x,\lambda)\,\phi_k(x,\lambda)\right]$ is purely imaginary .... (1.10)

This is evident from (1.9) by putting $\phi_j = \phi_k$.

Theorem – 3:

$\left[\phi_j(x,\lambda)\,\phi_k(x,\lambda)\right]$ is a semi bilinear form which reads

$$\left[\left(\phi_j(x,\lambda)+\phi_k(x,\lambda)\right)\phi_l(x,\lambda)\right]$$

$$= \left[\phi_j(x,\lambda)\,\phi_l(x,\lambda)\right]+\left[\phi_k(x,\lambda)\,\phi_l(x,\lambda)\right] \qquad \text{...... (1.11)}$$

$$\left[\phi_j(x,\lambda)\,(\phi_k(x,\lambda)+\phi_l(x,\lambda))\right]$$

$$= \quad \left[\phi_j(x,\lambda)\,\phi_k(x,\lambda)\right] + \left[\phi_j(x,\lambda)\,\phi_l(x,\lambda)\right] \qquad \text{...... (1.12)}$$

and $$\left[\alpha\phi_j, \beta\phi_k\right] = \left[\alpha\bar{\beta}\,\phi_j\phi_k\right] \qquad \text{...... (1.13)}$$

where α and β are constants real or complex.

The proofs are very simple.

References:

1. Singh, S.N., Chandra, Subhas,
On Existence – Differential Equations, Proc. Math. Soc. B.H.U. Vol. 5, P. 194, Varanasi.

2. Ramadhar Prasad and Devnandan Sharma,
Existence and Uniqueness of Greens Matrix associated with a n x n matrix differential operator: Bulletine of Pure and Applied sciences. Vol. 16E (No. 1) 1997; P. 55-59.

3. Lallu Prasad,
Fourier coefficients in a partial differential equation of fourth order, Ideal Research review Vol. 22, No. I, 11 June 2009.

4. Himanshu Shekhar and Brijendra Kumar Singh
Bilinear Concomitant and Green's Formula associated with a Matrix Differential Operator, To be published shortly.