

Procedural use of Hybrid Fuzzy Fault Tree Analysis

Avinash Selot

Student

Department of Mechanical Engineering
Maulana Azad National Institute of Technology
Bhopal, India

Amit Suhane

Assistant Professor

Department of Mechanical Engineering
Maulana Azad National Institute of Technology
Bhopal, India

Abstract— Conventional Fault tree analysis gives us Probability risk analysis (PRA) method used to identify the basic cause of the failure. To conduct the fault tree analysis it is required to know the probability for the basic events and through which we get the requisite probability of the top event. However with the increase in the complexity of the system it has become very difficult to obtain those failure rates well in advance due to insufficient data, environment changing or new components. Overall failure probability may be questionable with the insufficiency of the data. So here we use hybrid approach of the fuzzy numbers which are helpful to solve the conventional problem which are ambiguous. This paper gives us the use of fuzzy logic implementation in the fault tree analysis when the crisp values are not given.

Keywords— fault tree analysis, probability risk analysis, fuzzy number

INTRODUCTION

Fault tree analysis is a diagrammatic and deductive approach by which we get to know the probability of top event. FTA uses the various Boolean logic or the Boolean gates to combine the series of basic events. Fault Tree Analysis (FTA) was originally developed in 1962 at Bell laboratories by H.A. Watson, under a U.S. Air force ballistic system division contract to evaluate the Minuteman Intercontinental Ballistic Missile (ICBM) Launch Control System. Since then it is used in number of system safety assessment and reliability engineering field such as nuclear reactor, chemical industry, manufacturing industries, circuit board, petrochemical industry etc.

Fault tree analysis is a diagnostic technique widely used to know the root cause of the undesired element and get to know the functional relationship between the various components, subsystem etc. Today FTA is used widely in system safety and reliability engineering.

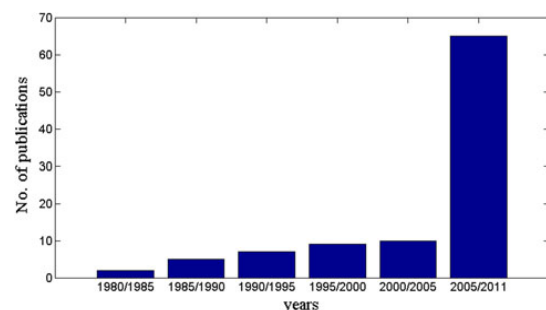
In conventional FTA, the process should have known probabilities of basic events. However in real situation it is very difficult to find failure probabilities of each and every component as data is not sufficient for the basic events. In conventional FTA the failure probabilities are known as crisp or exact one but in real situations it is very difficult to know the exact or crisp value of failure each and every component due to the indefinite terms or the failure probabilities which are not clear. Fuzzy method is the only way to find out the failure probabilities of the components when little

quantitative information is known about it. Fuzzy numbers are used to describe the basic events and with the help of the fuzzy number and expert elicitation the failure probabilities are known. However inadequate data makes it difficult to quantify the probability. So the human linguist variable or the expression such as very high, high, low, very low is used.

The above mentioned limitations are to be taken in account so as to get or overcome over the difficulties faced by the conventional FTA with the increasing of the complexities of the system. Hence a review concept using fuzzy logic with the FTA and reflection of the FFTA and its applications in different area with the algorithm used is provided in the following paper.

Fuzzy set theory had been proposed by many of the researchers, such as Tanaka et al. (1983), Misra and Weber (1989) and Kenarangui (1991) to overcome the limitation of conventional FTA. Fuzzy logic provides a framework whereby basic notions such as similarity, uncertainty and preference can be modeled effectively.

The classification of the numbers of paper published on fuzzy fault tree analysis according to the scopus bibliographic data are given in the fig as follows: [1]



Year's distributions of published papers of FFTA, when putting "Fuzzy fault tree" in Scopus

I. FUZZY LOGIC

Fuzzy Logic is a branch of mathematics that deals with fuzzy statements (linguistic variables). Fuzzy Logic provides a simple way to draw definite conclusions from vague, ambiguous or uncertain information (Zadeh 1978, 1988).

It is being applied to many fields these days especially when there is an uncertainty in the failure probability or where vague data is to be handled. Applications are as

follows control theory, artificial intelligence, chemical industry, nuclear industry etc.

The AND, OR and NOT operators of the Boolean logic exist in fuzzy logic usually defined as minimum, maximum and complement. When they are defined this way they are called Zadeh operators.

For example fuzzy variables x and y:

$$\text{NOT } x = (1-\text{truth}(x))$$

$$x \text{ AND } y = \text{minimum}(\text{truth}(x), \text{truth}(y))$$

$$x \text{ OR } y = \text{maximum}(\text{truth}(x), \text{truth}(y))$$

II. TRADITIONAL FTA

In traditional FTA it is assumed that the basic events in the fault tree are independents and can be represented by probability number. Two cases can be considered:

- a). fault tree without repeated events
- b). Fault tree with repeated events

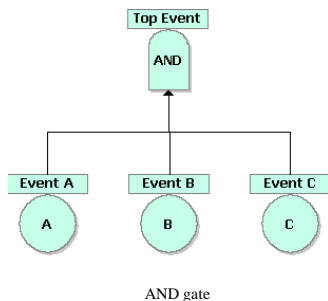
A. Fault tree without repeated events

Here the simple approach is followed by taking up the gates and accordingly the probabilities of failure are worked up through the tree. Calculation starting from the base and moving upward till the top event is calculated so forth.

If AND gate is used

$$P = \sum_{i=1}^n p_i \tag{1}$$

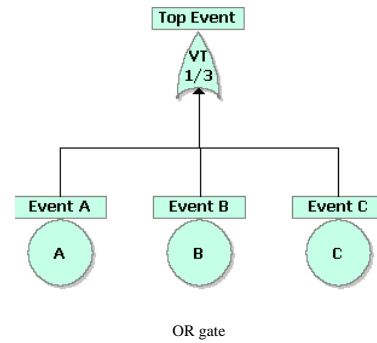
where , P is the probability of top event and p_i is the probability of basic event n is number of basic event gates associated



If OR gate is used

$$P = 1 - \sum_{i=1}^n (1-p_i) \tag{2}$$

where, P is the probability of top event and p_i is the probability of basic event n is number of basic event gates associated



B. Fault tree with repeated events

When the basic events happen more than once the probabilities are found by Minimal Cut Set (MCS) method. If all these events occur, the Top Event is guaranteed to occur; however, if any BE does not occur, the Top Event will not occur. Therefore, if a fault tree has 'n_c' MCSs (MCS_i, i = 1,..., n_c) then the Top Event 'T' exists if atleast one MCS exists[7].

III. FUZZY FAULT TREE ANALYSIS(FFTA)

In many systems exact probability of failure cannot be determined and these vague value leads to the ambiguous selection of the failure probability of the top event. In the absence of the exact data it may be necessary to work with approximate data probabilities and under such conditions it may be not possible to use the traditional approach. Now instead of probability of data it is appropriate to propose its possibility [5]. To overcome the difficulty the fuzzy logic is used up in the fault tree analysis.

Extensive research is being done on fuzzy logic integration with the Fault Tree Analysis. This pioneering work was conducted by Tanaka et al (1983) which treated the probability of basic events as trapezoidal fuzzy numbers and applied the fuzzy extension principle to calculate the probability of top event [3]. Misra and Weber (1989) there analysis was based on possibility distribution associated with the basic events and the fuzzy method to combine all these events [5]. Parallel with this, Singer (1990) analyzed fuzzy reliability by using L-R type fuzzy numbers [2]. In order to facilitate the calculation of Singer's method, Cheng and Mon (1993) proposed revised methods to analyze fault trees by specifically considering the failure FPs of BEs as triangular fuzzy numbers. Onisawa (1988) proposed a method of using error possibility to analyse human reliability in a fault tree [6]. Liang and Wang(1993) used ranking values to evaluate fuzzy importance index. Misra and Soman (1995) provided a simple method for FFTA based on the alpha-cut method, also known as resolution identity. This method was then extended to deal with multi- state FTA. Lin and Wang (1997) combined fuzzy set theories with expert elicitation to the failure probability of basic events of a robot drilling system, based on triangular fuzzy numbers.

In evaluation of algorithm, hardware failure probability or rate can be taken up from the reliability handbook such as NPRD (nano electronics part reliability data). But the human

error to be handled with the experts subjective assessment and then converting it into the failure possibility score or crisp failure possibility. Next stage would be converting it into the failure probabilities.

Algorithm:

The stepwise procedure to be followed to get to know the failure probability of the subjective assessment and expert elicitation:

1. Basic events with known probabilities are separated from that of the vague or subjective linguistic evaluation rates
2. Obtain the failure rates of the basic events of known probabilities
3. Conducting the linguistic assessment by expert judgment for the vague events and transforming the same to fuzzy numbers
4. Aggregating the experts opinion for the basic events with vague failure
5. Defuzzification process is then used to transform the fuzzy numbers to the failure possibility score(FPS)
6. Converting the failure possibility score into the failure probabilities
7. Synthesize the probability of failure of top event by integrating the fuzzy failure probabilities with the known probabilities of the basic event also taking into account the minimal cut sets(MCSs)
8. Analyze the result and produce the corrective action

Each step is now given in detail and how the fuzzy number helps us to get the outcome of probability is shown below:

Step 1: Separating hazards

Basic events with known failure rates are separated from that of the vague or subjective linguistic evaluation rates this can be done with the help of the reliability data handbook.

Step 2: Obtaining failure rates of known basic events

This can be obtained from the reliability data handbook such as that of NPRD or OREDA etc

Step 3: Rating stage

In this stage the expert judgment i.e. experts from different fields make judgment about the probability of the event. As for the vague events it is not possible to get to know the exact probability so an educated guess is made by the personnel's who have worked up in the following field. Evaluation committee is formed consisting of reliability analyst, maintenance personnel, factory inspector, apprentice, supervisor etc. These experts cannot directly give the

probabilities so they apply natural linguistic expression such as 'very low', 'low', 'medium', 'high', 'very high' to give the probabilities.

Constitution	Classification	score
Professional Position (PP)	Senior academic	5
	Junior academic	4
	Engineer	3
	Technician	2
	Worker	1
Service Time (ST)	>=30 years	5
	20 – 29	4
	10 – 19	3
	6 – 9	2
	=<5	1
Education Level (EL)	PhD	5
	Master	4
	Bachelor	3
	HND	2
	School level	1

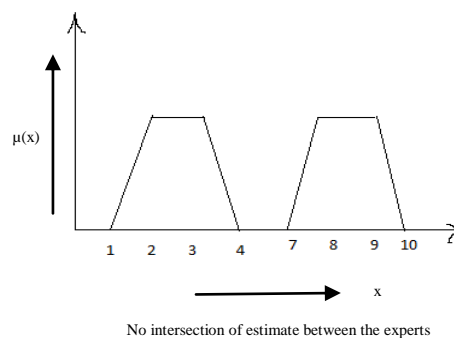
Table 1: Weighting scores of different experts[8]

Step 4: Aggregating stage[10]

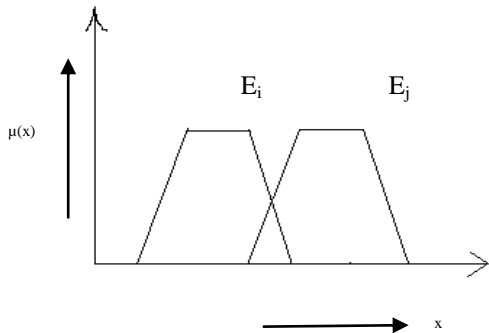
As we all know different experts have different opinions as to come up to a common consensus it aggregation expertise opinion has to be devised. Now for this aggregation the Hsu and Chen (1994)[10] presented an algorithm to aggregate the linguistic opinion of the group of experts.

Here we assume that the opinion or the estimate R_i of each expert E_i ($i=1,2,3,\dots,n$) have a common intersection at a level α -level cut $\alpha \in (0,1]$. The below example will provide why are we taking the assumption of α -level cut.

Suppose the expert A and expert B constructs their estimates as $R_A = (1,2,3,4)$ and $R_B = (7,8,9,10)$ respectively. In the estimates provided there are no common points of intersection. If the aggregation results fall between $[4,7]$ the result is not accepted by the two experts. In such case the aggregation result is unreasonable and the experts should resume discussion and must get the new information to adjust their estimate. In other words assumption $R_a \cap R_b \neq \emptyset$



If R_i and R_j are two experts estimate with degree of agreement as $S(R_i, R_j)$ between experts E_i, E_j can be determined by proportion of the consistent area (i.e. $\int_x \min \{ \mu_{R_i}(x), \mu_{R_j}(x) \} dx$) to the total area (i.e. $\int_x \max \{ \mu_{R_i}(x), \mu_{R_j}(x) \} dx$).



Overlap of two expert opinion

$$S(R_i, R_j) = \frac{\int_x \min \{ \mu_{R_i}(x), \mu_{R_j}(x) \} dx}{\int_x \max \{ \mu_{R_i}(x), \mu_{R_j}(x) \} dx}$$

Where $S(R_i, R_j)$ is also called as similarity measure function by Zwick et al. [11]

If the estimate of the two experts is same i.e. $R_i=R_j$ then the degree of agreement between them is one and if the estimate of the two experts are entirely different then the degree of agreement is zero. Therefore $S(R_i, R_j) \in [0, 1]$

Now we calculate the Average Agreement degree $A(E_u)$ of the experts $E_i (i=1, 2, 3, \dots, n)$ and the equation is given by

$$A(E_i) = \left(\sum_{j=1}^n S_{ij} \right) / (n-1)$$

Where n- are no. of experts

Now calculate the Relative Agreement (RA) degree of the experts $E_i (i=1, 2, 3, \dots, n)$

$$RA(E_i) = \frac{A(E_i)}{\sum_{i=1}^n A(E_i)}$$

In some cases the relative importances of experts are widely different for example the personnel in the company may be more experienced than the other. So here comes the importance of the weights assigned for each expert. First we select the most important of the person and assign them the weight equal to one i.e. $r_i=1$. then we compare the other j^{th} expert with the most important person and the relative weight of the j^{th} expert $r_j, j=1, 2, 3, \dots, n$ is known

Finally we define the degree of importance w_i as follows

$$w_i = (r_i) / \sum_{i=1}^n r_i$$

Now next we will compute the Consensus Coefficient (CC) degree of the expert $E_i (i=1, 2, 3, \dots, n)$

$$CC(E_u) = \beta \cdot w(E_i) + (1-\beta) \cdot RA(E_i)$$

Where $0 \leq \beta \leq 1$ is a relaxation factor it shows the importance of weight of expert over the Relative Agreement degree. When $\beta=0$ no importance has been given to the weight of the expert hence homogeneous group of experts is used. When $\beta=1$, the Consensus Coefficient degree of an expert is same as its importance weight.

Let 'R' be the overall fuzzy number of the combining experts opinion. Finally the aggregated result of the experts' judgment can be obtained as

$$R = CC(E_1) \cdot R_1 + CC(E_2) \cdot R_2 + CC(E_3) \cdot R_3 + \dots + CC(E_i) \cdot R_i$$

$$R = \sum_{i=1}^n (CC_i \cdot R_i)$$

where (\cdot) is a fuzzy multiplication operator [12]

Step 5: Defuzzification process

Defuzzification basically means conversion from fuzzy to crisp value. It produces a quantifiable result in the fuzzy logic. Fuzzy number defuzzification is an important procedure for making decision in the fuzzy environment. There are many methods which are used to defuzzify the fuzzy number, but here we will choose the center of area defuzzification method. This technique was developed by Sugeno in 1985 [13] this is the most widely used technique and is accurate too. The expression for the method is given by:

$$z^* = \frac{\int \mu_C(z) \cdot z dz}{\int \mu_C(z) dz}$$

where \int denotes an algebraic integration.

Where

z^* is the defuzzified output $\mu_c(z)$ is aggregated membership function, x is the output variable.

Step 6: Converting the failure possibility score into the failure probabilities

As we all have mentioned earlier that data for failure rate of some events are known whereas for the others it is vague. This inconsistency can be sorted out by transforming the failure possibility to failure probability. Onisawa [6] has proposed that a function can be used for converting the failure possibility to failure probability. This function is derived by addressing some properties such as human sensation to the logarithmic values of a physical quantity. Fuzzy failure probability (FFP) was defined by Onisawa, [6][9] as:

$$FFP = 1/10^k, \text{ CFP} \neq 0$$

$$FFP = 0, \text{ CFP} = 0$$

Where,

$$K = [(1-\text{CFP})/\text{CFP}]^{1/3} \times 2.301$$

CFP- Crisp Failure Possibility

Step 7:

One of the most important outputs of an FTA is the set of importance measures that are calculated for the TE. Both the intermediate events as well as MCSs can prioritize according to their importance. Importance measure also used to optimize the system design and its reliability it also provides information to improve fault diagnosis of complicated system and provide reasonable guidance for maintenance. This can be used to list the examination table and instruct for operation and maintenance [15].

The importance value is measured by using the probabilities of basic events as well as that of the top event [16]. However, these probability-based definitions may be insignificant for a case in which the occurrence of a basic event can be imprecisely specified. This means that if the state of a basic event is expressed by a fuzzy event, it may not be possible to identify the importance of each event on the basis of only probability information.

Various importance measures are available in probabilistic approach like Risk assessment worth, Birnbanm importance , Fussel-vesely importance etc.

In fuzzy methodology, different importance measures are introduced such as fuzzy importance measure, fuzzy uncertainty importance measure.

Suresh et al [17] introduced an approach to rank the components depending on the contribution of their top event probability and their uncertainty contribution to the uncertainty of the top event. They introduced fuzzy importance measure (FIM), which is defined as

$$FIM_i = ED[Q_{qi=1}, Q_{qi=0}]$$

Where, ED[A,B] is the Euclidean distance between two fuzzy sets A and B and is defined as

$$ED[A,B] = \sum_{X=1,2,\dots,N} ((a^1 - b^1)^2 + (a^u - b^u)^2)^{0.5}$$

Where a^1 and a^u are lower and upper values of fuzzy set A.

IV. CONCLUSION

In conventional FTA probability of failure of basic events must be known in advance. These are in general, obtained from the international database which may not be exactly applicable to Indian conditions. Therefore, the failure probability values obtained here are different from the available values. The differences in the operating procedures as well as climatic factors contribute to the variations. Now as we seen from the paper so forth that the FFTA method is more helpful in finding out the relational parameters of the various uncertainties prevailing in the system and which are not encountered previously, can be found with the help of expert elicitations. Furthermore FTA isn't suitable for the where available data are insufficient for statistical inference, fuzzy methodology is the only way out when little information is available. The review uncovers the

effectiveness of FFTA in comparison to the conventional FTA when there is inadequate amount of accurate reliability information. The proposed method increases up the improvement level of the system and greatly increases the correction of fault component decision.

This approach can help the decision maker to redesign or change the critical parts accordingly to improve the reliability and safety of the system. Here the independency of the basic events is taken up but for future work, method is to be developed so that the dependencies of various hazards of the basic events are to be taken into account

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