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Probabilistic Methodology to Determine The Shaft's Diameter and Designed Reliability

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***Abstract*— In the proposed probabilistic shaft's methodology the stress average and the standard deviation are both determined based on the binary synthesis approach. The efficiency of the proposed methodology is compared with the static and fatigue approaches. The application is performed by using a speed reducer. Then the estimated stress average and standard deviation are both used in the stress/strength methodology to determine the reliability of the designed shaft. Additionally, since by applying the probabilistic method, the mean and the standard deviation of the alternating S_a and fatigue S_e values are both always determined, then the proposed method is generalized to determine the shaft's reliability by considering that both S_a and S_e follow either a normal, Weibull or lognormal distribution. Finally, the guidelines to select which distribution we should use in the stress-strength analysis are also given.**

Keywords*— Probabilistic shaft design, Fatigue; Weibull distribution; Binary synthesis; Stress/strength analysis; Torsional rigidity.

I. INTRODUCTION

In machines and equipment a shaft is used to let movement and transmit power [1], [2]. Therefore, its design is based on the applied alternating (S_a) and the midrange stress (S_m) values. However, when a shaft fails due to the S_m value, its failure occurs at the first cycle (or after a few cycles), meaning that the instantaneous applied S_m value was higher than the shaft's strength. However, because in the design shaft's phase, a security factor is used, then the material's strength (S_y) value is higher enough than the S_m value (say S_y>>S_m), implying no first cycle failure occurs. Consequently, the shaft fails by fatigue. Fatigue is a failure generated by the cumulated damage, generated by the cyclical application of the stress. Therefore, in the analysis the shaft's failure is generated by the alternating S_a stress value. Consequently, the shaft's reliability is also based on the S_a value. Thus, in this paper the fatigue shaft's design as well as the shaft's reliability are both determined by using both the nominal S_a value and its corresponding standard deviation. The S_a standard deviation is determined based on the fatigue and binary synthesis methodologies. The efficiency of the proposed method is shown by designing an intermediate shaft of a fan speed reducer, and by comparing the designed shaft's diameter with those given by the static and dynamic (fatigue) methodologies.

In the static approach, the speed reducer shaft's design is performed based on the bending and torsional stresses that are acting at the critical point of the shaft, in the yield strength S_y and ultimate strength Sut material's values [3]. And to determine if the designed shaft is whether safe or not, the Von Mises and the distortion energy (DE) criteria, with a safety factor of two [4-5] are used. The designed static shaft's diameter was d_s=0.0235m (0.928 in).

In similar form, in the dynamic fatigue approach [4], the shaft's design analysis was based on both the alternating S_a and midrange S_m stresses values at which the shaft is subjected. The applied S_a value was determined based on both the bending loads generated by the gears, and on the radial forces generated by the applied torque. And the S_m value was determined through the Soderberg's fatigue method, were the modified endurance limit (S_e) value was determined by using the corresponding endurance modification factors. The designed fatigue shaft diameter was d_f=0.0375m (1.48 in).

On the other hand, in the binary synthesis approach [5], the shaft's design analysis was performed by considering that all the endurance limit modification factors of the above fatigue approach are random [6] [7], and that they can be modeled by a normal distribution [8]. Therefore, after all the modifier factor were synthetized by using the binary synthesis approach. Then from the synthetized average values, the corresponding modified endurance limit (S_e) value was determined. And by using the Se value with the corresponding Soderberg line and the stress ratio ($r=S_a/S_m$) in the Soderberg's diagram, the maximum allowed S'_a value was determined. Finally, by using the S_a value of the fatigue analysis, and the addressed binary S'_a value and a safety factor of 2, the designed binary shaft diameter was d_b=0.0381m (1.50 in).

However, here it is important to observe that 1) although in the static, fatigue and binary approaches we conclude the designed shaft is considered safe, and we use a safety factor of two, from neither of these analyses it is possible to determine the reliability that the designed shaft presents. And 2) because $d_b>d_f>d_s$ (0.0235m>0.0375m>0.0381m), then we have that the robust design is the one given by the binary method with d_b=0.0381m. 3) because in the binary method, the estimated S_a value is higher than the minimum expected strength S_{emin} value ($S_a>S_{emin}$), then although we design the shaft with d_b=0.0381m, failures are expected. Hence in order to avoid failures, in

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section VI a probabilistic method to avoid selecting a S_a value higher than the minimum S_{emin} value, is presented. The proposed method is based on the fatigue and binary approaches. And its main contribution consists of 1) given the minimum expected modified endurance limit S_{emin} value, to create a new Soderberg line, and from it to derive the mean and the standard deviation of S_a in such a way that the maximum expected S_a value never will be higher than the minimum expected S_{emin} value ($S_a<S_{emin}$). And 2) by using the addressed normal distributions families of S_a and S_e in the corresponding stress/strength function, the reliability of the designed shaft is determined. Therefore, by applying the proposed method, the designed probabilistic shaft diameter was d=0.0418m (1.647 in), with a designed reliability of R(t)=0.9950. However, at this point, it is important to mention that because the torsional rigidity method requires a minimum shaft diameter of 0.0419m (1.65in), then finally the recommended diameter for shaft 2 was 0.0419m. Finally, since by applying the probabilistic method, the mean and the standard deviation of S_a and S_e are both always determined, then the stress-strength analysis is generalized to determine the shaft's reliability, for any combination among the normal, Weibull and lognormal distribution, and the guidelines to select the right distribution are also given.

II. DATA OF THE ANALIZHED CASE

In order to perform the comparison between the static, dynamic and binary synthesis design approaches, as well as to formulate the probabilistic proposed method, the intermediate shaft of a speed reducer used for a grain drying process is designed. In the application, it is required to move the fan at 450 rpm with a power of 12 hp. The shaft design's material is an AISI 1020 steel normalized at 925 ° C (1,700 ° F) air cooled, 50 mm (2 in) round with ultimate tensile strength of S_{ut} = 438 MPa (63,500 psi), tensile yield strength of S_{yt} = 319 MPa (46,300 psi), and a shear modulus of G = 72 GPa (10,400 ksi) [9]. Therefore, because the selected motor is a 12 hp motor and a turning speed of 1,800 rpm, then the speed reducer must be designed to decrease the speed from 1,800 rpm to 450 rpm, and this has to be made by conserving the initial motor's power of 12 hp.

The scheme of the reducer, coupled to the motor, the position of the spur gears in the shafts, and the shaft's length are all presented in Figure 1. This system can transfer motion and torque via two accurately stages of configuration and efficiently [10]. The selected motor has a shaft diameter of 0.035m(1 3/8 in) [11] [12]. The motor is connected to the shaft 1 of the reducer by using a flexible coupling. The spur gear A is mounted on shaft 1, which engages with spur gear B of shaft 2. A spur gear C is mounted on shaft 2, which is connected to spur gear D that is mounted on the shaft 3. And according to [12], the main characteristics of the 4 spur gears to reduce the engine speed are shown in Table 1. From Table 1, we have that due to the relationship between spur gears A and B, the initial angular velocity of shaft 1 of 1,800 rpm is reduced in shaft 2 to 900 rpm. Similarly, note due to the relation between spur gears C and D, this angular velocity in shaft 3 is finally reduced to the required 450 rpm.

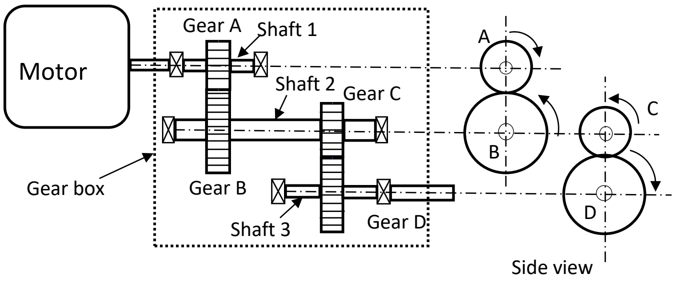

Fig. 1. Electric motor connected to speed reducer.

On the other hand, due to spur gears B and C, shaft 2 is subjected to bending stresses and constant torsional forces generated by the transmission of power, then in order to avoid plastic deformation, its design must comply with a minimum torsional rigidity of (0.25 °/ m) [14] [15].

Similarly, due to the bending and torsional stresses at which shaft 2 is subjected (see Fig. 2), the expected failure mode of the shaft is due to fatigue. This implies that for the dynamic design, the modified endurance limit S_e has to be calculated [3][4]. And according to [16], the endurance limit modification factors that affect S_e are surface factor k_a = 0.8, size factor k_b, load factor k_c = 1 and temperature factor k_d = 1. Also, due to the concentration of stresses generated by the loads at which the shaft is subjected, the holes for the assembled gears and the type of material, the fatigue stress concentration factors for bending K_{FF} and for torsion factor K_{FT} must be considered also. However, since in this stage neither the dimensions of the geometry nor the shaft's diameter are known, then following [15] K_{FF} = 2 and K_{FT} = 1.6 were used.

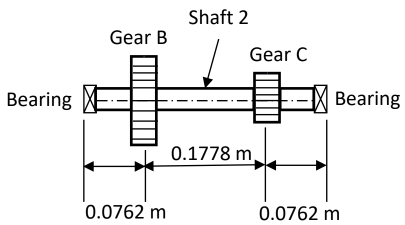

Fig. 2 Top view of shaft 2.

Table 1. Spur gears characteristics of the speed reducer

Spur gear	Diametral Pitch (in-1)	Pitch diameter (in)	Number of teeth	Face width (1.5)
A	8	2.5	20	1.5
B	8	5	40	1.5
C	8	3	24	1.5
D	8	6	48	1.5

Finally, before to present the design analysis of shaft 2 through the static, dynamic and binary synthesis approaches, it is necessary to determine the critical point where the shaft could fail. And because the selection of the critical point depends on 1) the applied torque generated by the torsional forces of the gears B and C and on 2) the maximum applied bending moment generated by both the radial forces and their corresponding reactions, then let first present the torque analysis.

A. *Torque Calculations for Shaft 2*

The torque generated on shaft 2 depends on the angular velocity (ω) at which the shaft will rotate, and on the power (P) that the shaft has to transmit. Hence, the generated torque is given by:

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$$T = \frac{63000P}{\omega} \quad (1)$$

In (1), P is the power measured in horse power hp units, 63,000 is a conversion factor from hp to pounds, ω is given in revolutions per minute (rpm), and T is measured in pounds per inch (lb in). Therefore, under the assumption that there is no loss of power, P=12 hp, and due to the ratio of gears A and B (see Fig.1), ω=900 rpm, then the generated torque in shaft 2 is

$$T_B = \frac{63{,}000\ (12\ hp)}{(900\ rpm)} = 840\ lb\ in = 94.9Nm$$

And because the analyzed system is in equilibrium, then as it is shown in Figure 3 and in Figure 4, the torque in gear B of T_B = 94.9Nm is equal to that in gear C, but in the opposite direction, Tc = -94.9Nm. It is to say; the generated torque is constant through the shaft at T=94.9Nm.

B. *Maximum Bending Moment for Shaft 2*

Since the radial and tangential forces generated by the spur gears generate bending moments in the shaft, and because these generated moments by themselves generate bending stress on the shaft, then the maximum bending moment at which the shaft is subject is determined from the bending moments diagram [17][18]. However, because the bending moment diagram is based on the shear forces diagram, which is built based on the radial and tangential forces, and on the corresponding reactions forces, then let us first calculate the radial, tangential and reaction forces. The analysis is as follows.

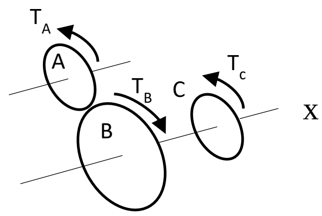

Fig. 3. Gear torque direction

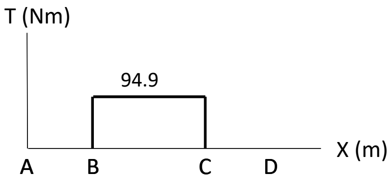

Fig. 4. Shaft 2 torque diagram.

C. *Radial and tangential forces calculation*

The calculation of the radial and tangential forces, and reactions (see Fig. 5) that are acting on the teeth of gears B and C depends on their corresponding pressure angle (Φ) (see Fig. 6). In this case the pressure angle of spur gears B and C is Φ = 20 °. Thus, the generated tangential force, which is in function of the torque and the spur gear radius is given by [3][19]

$$F_t = \frac{T}{r} \quad (2)$$

Hence, in function of Φ and F_t, the radial force [3][20], is given by

$$F_r = F_t\ (\tan \emptyset) \quad (3)$$

Numerically, by using from Table 1 r_B=0.0635m (2.5 in), for gear B the tangential and the radial forces are

$$F_{tB} = T/r_B = 94.9Nm/0.0635m = 1494.48N$$

$$F_{rB} = F_{tB}(\tan\theta) = 1494.48\ (\tan 20°) = 543.94N$$

Similarly, since from Table 1 r_C=0.0381m (1.5 in), then for gear C the tangential and the radial forces are

$$F_{tC} = T/r_C = 94.9Nm/0.0381m = 2490.81N$$

$$F_{rC} = F_{tC}(\tan\theta) = 2490.81\ (\tan 20°) = 906.58N$$

Now let present the reactions calculations.

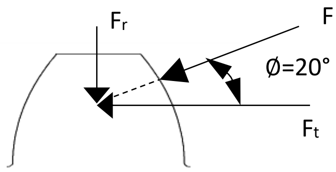

Fig. 5. Forces and reactions acting on the bearings and gears

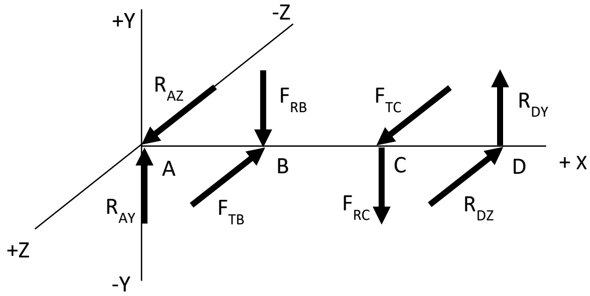

Fig. 6. Generated forces on the gear teeth

D. *Bearing reactions calculations*

The calculation of the reactions, generated in the bearings by the radial and tangential forces, depends on the axis of symmetry in which these forces are being generated. In our case, since these forces are in different axis of symmetry, then in the analysis we have to consider that the forces are acting in two different planes; the x-y and the x-z planes (see Fig. 6). Thus, the analysis is based in the force and reaction diagrams given in Figure 7 and Figure 8.

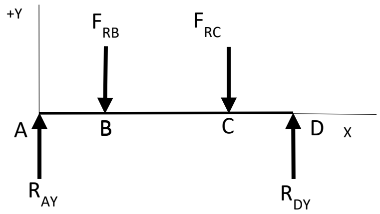

Figure 7. Force and reaction in x-y plane

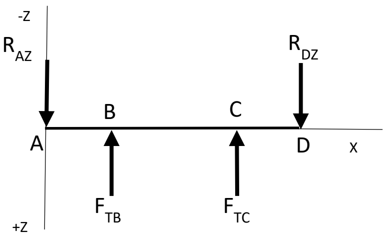

Figure 8. Force and reaction in x-z plane

As it is shown in Fig. 7, the forces and reactions which are acting in the x-y plane are.

$$+\circlearrowleft \sum M_D = 0 = (543.94\,N)(0.254m) + (906.58\,N)(0.0762m) - (0.3302m)(R_{Ay}); \qquad R_{Ay} = 627.62\,N$$

$$\sum F_y = 0 = 627.62\,N - 543.94\,N - 906.58\,N + R_{Dy};$$
$$R_{Dy} = 822.9\,N$$

Similarly, as it is shown in Fig. 8, the forces and reactions which are acting in the x-z plane are.

$$+\circlearrowleft \sum M_D = 0 = (R_{Az})(0.3302\,m) - (1494.48\,N)(0.254m) - (2490.81\,N)(0.0762\,m);$$

$$R_{Az} = 1724.40\,N$$

$$\sum F_z = 0 = -1724.40\,N + 1494.48\,N + 2490.81\,N - R_{Dz};$$

$$R_{Dz} = 2260.89\,N$$

Now based on these forces and reactions let present the shear forces and maximum flexion moment calculations.

E. Shear forces and maximum flexion moment diagram

Using the reaction forces and the estimated radial loads, the shear force diagram for the x-y plane is given in Figure 9. Therefore, based in these forces and on the distances of the shear force diagram, the corresponding bending moments diagram for plane x-y is given in Figure 10.

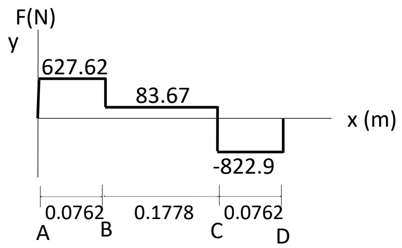

Figure 9. Shear force diagram for x-y plane.

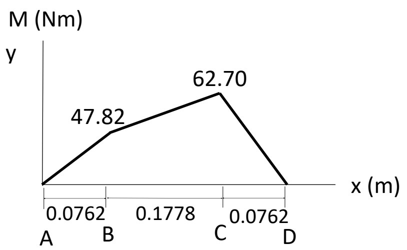

Fig. 10. Bending moments diagram for x-y plane

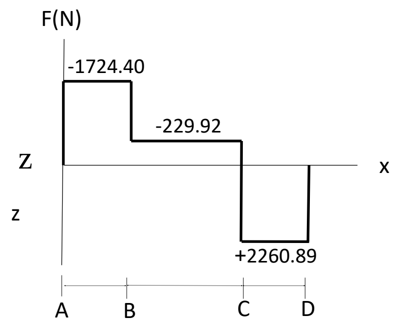

Figure 11. Shear force diagram for x-z plane.

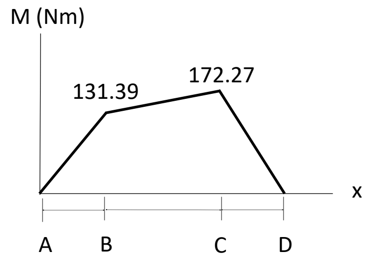

Fig. 12. Flexing moment diagram for x-z plane

Similarly, for the x-z plane, the shear force diagram and the bending moment diagram are given in Figure 11 and Figure 12 respectively. Thus, based on the bending moments diagrams of Figures 10 and 12, the maximum bending moment to be used in the design of shaft 2 is

$$M_{MAX} = \sqrt{M_{x-y}^2 + M_{x-z}^2}\,,$$
$$M_{MAX} = \sqrt{(62.70\,Nm)^2 + (172.27Nm)^2},$$
$$M_{MAX} = 183.32Nm$$

Finally, from the torque diagram given in Fig. 4 and from the flexion moment diagrams given in Figs. 10 and 12, we have that the critical point for the analysis in the design of shaft 2, is the point C. In the same way, we have that to avoid plastic deformation by torsion, then if the size of the designed diameter from either of the three design approaches is smaller than the diameter given by the torsional rigidity approach, then the recommended diameter size must be the one given by the torsional rigidity approach.

F. Torsional Rigidity Angle Approach for Shaft 2

To avoid plastic deformation, the material's strength used in the shaft design has to withstand the applied torsional forces. This is determined by measuring the torsion angle caused by the applied torque along the entire length of the shaft. Thus, to avoid plastic deformation the shaft's diameter given by the rigidity approach has to be used as the minimum allowed diameter in the shaft design [4]. And because the minimum torsional rigidity value to be used in the shaft design depends on the precision of the analysis that the application requires, then in this paper the used value was selected from the classification given in [14][15].

On the other hand, although in practice the procedure to design a shaft, consists of first to determine the shaft diameter, and then based on its value, to determine the torsion angle. And finally, by comparing the torsion angle with the rigidity angle of the used material, we determine if the selected shaft diameter

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is whether safe or not to fail by plastic deformation. Therefore, because using the torsion angle is equivalent to using the shaft's diameter [4], then in this article the conformance of the material with the torsional rigidity, is made through the comparison of the designed diameter with the diameter given by the torsional rigidity approach, which is calculated as

$$d_r = \left[\frac{32T}{\pi G\left(\frac{\theta}{L}\right)}\right]^{\frac{1}{4}} \quad (4)$$

Based on Table 4 in [14], in this case of study a torsion angle of 0.0001108 radians in each inch, is recommended. Hence, by replacing this value in (4), the shaft's diameter of the torsional rigidity approach is

$$d_r = \left[\frac{32(94.9Nm)}{\pi(72x10^9\ N/m^2)(0.00463\ rad/m)}\right]^{\frac{1}{4}} = 0.041m$$

Thus, to avoid plastic deformation d_r = 0.041m (1.65in) represents the minimum allowed diameter value for shaft 2. And therefore, the shaft diameters given by the static, dynamic and probabilistic design approaches have to be compared with it. And in all cases if the designed diameter is lower than 0.041m, then Shaft 2 must be designed at 0.041m.

Here notice that although this torsional rigidity diameter prevents us to have failures by plastic deformation, it does not prevent us of failures generated by fatigue, and neither let us know the reliability that the designed shaft presents. However, before present the method to determine the shaft's reliability, let first present the static shaft's design approach.

III. Static Design Approach

The static shaft's design method offers the initial steps to design a shaft. In this static method, the shaft's diameter is determined based on the principal stress values generated by the forces that are acting on the shaft. And the determination if the designed shaft is whether safe or not, it is performed by using the Von Misses [distortion energy (DET)] and Tresca [Maximum stress shear (MSST)] theories [5] [3] with a selected safety factor value. The static design analysis is as follows.

A. *Static Shaft Design*

Based on the Von Mises criteria, the determination of the designed shaft diameter through the static method is given by the formula

$$d_s = \left(\frac{32SF}{\pi S_y}\sqrt{M^2 + {}^3/_4 T^2}\right)^{\frac{1}{3}} \quad (5)$$

In (5), SF is the safety factor which from [4] is recommended to be SF=2. S_y is the tensile yield strength of the used material, which from sec. II is S_y=319 MPa (46,000 lb/in^2). M is the maximum bending moment estimated in sec. IIE as M=183.32Nm. And T is the applied torque estimated in sec. IIA as T=94.9Nm. Thus, by substituting these values in (5), the estimated static shaft diameter is

$$d_s = \left[\frac{32(2)}{\pi(319x10^6\ N/m^2)}\sqrt{(183.32Nm)^2 + \frac{3}{4}(94.9Nm)^2}\right]^{\frac{1}{3}} = 2.35x10^{-2}m$$

Thus, because the designed static shaft's diameter of d_s=0.0235m (0,928in) in is lower than the minimum acceptable diameter of the torsional rigidity of sec. IIF of d_r=0.041m (1.65in), then the recommended diameter for shaft 2 is the one given by the torsional rigidity of 0.041m.

Here it is important to notice that although (5) was derived based on the Von Mises criteria, and we used a SF=2, as in the rigidity approach, it is not possible to determine the reliability that the designed shaft presents. However before present the method to determine the reliability of the designed shaft, let present the shaft's design through the fatigue method.

IV. Fatigue Design Approach

The fatigue in a mechanical component is caused by the load's repetitiveness. These loads being of lesser magnitude than those that can cause plastic deformation in ductile materials or rupture in fragile materials. Thus, the design of a shaft by the fatigue approach is based on the alternating stress (S_a) and on the mean stress (S_m) values generated by the bending and torsional loads that are acting on the shaft. And while S_a is variant because it is generated by the bending stresses, S_m is constant because it is generated by the applied constant torque (see Fig. 13). However, because initially the shaft's diameter is unknown then at this stage S_a and S_m are both unknown, and therefore they are determined by using a fatigue failure theory. The most common used theories are the Soderberg, Goodman, Gerber and ASME theories. However, because in the shaft 2 design, an AISI-1020 steel is used, which is a ductile material and plastic deformation is not allowed, then in the analysis S_a and S_m are both determined based on the Soderberg theory [3][4]

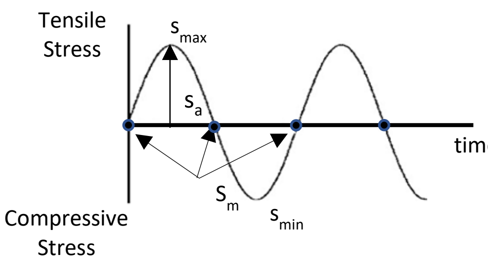

Fig. 13. Alternating and mean stresses.

Similarly, based on the Whöler S-N curve of the used material, the endurance limit S'_e to be used in the fatigue analysis is determined. And then, through the application of the endurance limit modification factors, the S'_e value is lowered to the called modified endurance limit S_e, which in the fatigue analysis is taken as the real fatigue limit of the analyzed case. In our case, the used modifier endurance limit factors are surface, size, type of load, temperature, and fatigue. And because axial forces are not acting on shaft 2, then the designed shaft's diameter is determined based on the Soderberg and Von Mises' theories. Hence, due to the lack of axial forces, the shaft's diameter is only in function of the bending and torque moments [4], and its diameter is given by

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$$d_f = \left[\frac{32FS}{\pi S_y}\sqrt{\left(M_m + \frac{S_y}{S_{ef}}M_a\right)^2 + \frac{3}{4}\left(T_m + \frac{S_{ys}}{S_{et}}T_a\right)^2}\right]^{\frac{1}{3}} \quad (6)$$

In (6), M_m is the mean bending moment, M_a is the mean alternating bending moment, T_m is the mean torque, T_a is the alternating torque, S_y is the yield strength for bending loads, S_{ys} is the shear yield strength for torsional loads, S_{ef} is the modified endurance limit for bending and S_{et} is the shear modified endurance limit for torsional loads. Numerically the shaft's 2 diameter is as follows.

A. *Fatigue Shaft Design*

For the determination of the diameter of the shaft 2, notice first from Figure 13 that when the shaft rotates, the bending moment Mm, which is generated due to radial and tangential loads of the spur gears (see Fig. 5), produces the alternating stresses shown in Fig.13. Then the alternating stress S_a at which the shaft is subjected is equivalent to the maximum stress S_{max}. Also notice that this implies that because in (6) the force moments will be used to determine the shaft's diameter, then the alternating moment M_a is equivalent to the maximum moment of section IIE (M_a=M_{max}=183.32Nm). In the same way, notice from Fig.13 that because the mean stress S_m is zero, then the mean moment M_m is equal to zero (M_m = 0) also. And since the torque on the shaft is constant because the power and angular velocity remain constant, then the mean torque T_m is equivalent to the torque at the critical point C (T_c). That is, T_m = T_c =94.9 Nm. This implies that because there are no variations in the torque, then in (6) the alternating torque T_a element is also zero. And as a consequence, that both M_m and T_a are zero (null), then the terms S_{ys} and S_{se} can also be omitted from (6). Therefore, the final formula to determine the fatigue diameter is given by

$$d = \left[\frac{32FS}{\pi S_y}\sqrt{\left(\frac{S_y}{S_{ef}}M_a\right)^2 + \frac{3}{4}(T_m)^2}\right]^{\frac{1}{3}} \quad (7)$$

Where the bending modified endurance limit S_{ef} is given by

$$S_{ef} = k_a k_b k_c k_d \frac{S'_{ef}}{K_{FF}} \quad (8)$$

In (8), the values of k_a=0.8, k_c=1 k_d=1 and K_{FF}=2 were given in II. On the other hand, the value of the size factor k_b was not provided because the shaft diameter is unknown. Therefore, the k_b value is selected by using as a reference, the static shaft's diameter determined in sec. IIIA of d_s = 0.023m. From [3] it is determined as

$$k_b = 0.85\ if\ (0.0127m < d < 0.0508m)$$

or

$$k_b = 0.70\ if\ (d > 0.0508m)$$

Hence, the selected value is k_b = 0.85. And because the bending endurance limit (S'_{ef}) is given as

$$S'_{ef} = 0.5S_{ut} \quad (9)$$

Then by substituting it in (9), with the S_{ut} value given in sec. II, the S'_{ef} value to be used to determine the shaft diameter is

$$S'_{ef} = 0.5(438\text{ MPa}) = 219\ MPa$$

In the same way, by using the modifier factors, the bending modified endurance limit is

$$S_{ef} = \frac{[(0.8)(0.85)(1)(1)(219\ MPa)]}{2} = 74.46MPa$$

Finally, by substituting these values in (7) the estimated fatigue shaft diameter is

$$d_f = \left\{\frac{32(2)}{\pi(319x10^6\ N/m^2)}\sqrt{\left[\frac{(319x10^6\ N/m^2)(183.38Nm)}{74.46x10^6\ N/m^2}\right]^2 + \frac{3}{4}(94.9Nm)^2}\right\}^{\frac{1}{3}}$$

$$d_f = 0.036m$$

Thus, because the designed fatigue shaft's diameter of d_f=0.036m(1.45in), is lower than the minimum acceptable diameter of the torsional rigidity value of sec. IIF of d_r=0.0419m(1.65in), then as in the static approach, the recommended diameter for shaft 2 is 0.0419m. Here it is important to notice that although (7) was derived based on the Von Mises criteria, and we used a SF=2, then as in the static and rigidity approach, in this fatigue analysis it is not possible to determine the reliability that the designed shaft presents. Now let present the shaft design through the binary synthesis method.

V. BINARY SYNTHESIS DESIGN APPROACH

In the binary synthesis method, the shaft's diameter is determined in function of the maximum applied alternating S_a value determined from the maximum moment M_a value given by the static analysis, the average S_e value, the yield strength of the used material S_y and the S_m values caused by the torque. And because in this approach, S_e is considered to be random, then not only its mean μS_e, but also its standard deviation σS_e are both determined by applying the synthesis method to each one of the modifier factors. And once the S_e and S_y values are known, by using them in the Soderberg diagram with the corresponding stress ratio r (see Fig. 14), the S_a and S_m values are both determined. Here, the SolidWorks routine was used (see Fig. 14).

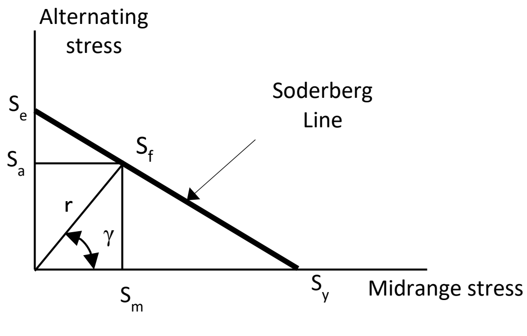

Fig. 14. Soderberg diagram

Now let present the functional relationships to perform the binary synthesis analysis.

A. *Alternating and endurance binary synthesis analysis*

The input variables to determine the shaft's diameter through the binary synthesis method and the Soderberg diagram, are the S_a, S_m, S_y, and S_e values. Where the S_a and S_m values are determined from their ratio (r), their angle γ and from their real endurance limit S_e values, then let first, calculate the

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functional relationship between S_a and S_m (r and γ). And then to present the binary synthesis analysis to determine the average and standard deviation of S_e. And finally, to determine the diameter of the designed shaft. The functional relationships that determine the S_a value are as follows.

B. Alternating stress estimation

The functional relationship for S_a is given as S_a=(Mc)/I where M is the maximum bending moment (M_a), c is the shaft's radius and I is the inertia moment given by I=πd⁴/64. Hence S_a in terms of M and d is given as

$$S_a = \frac{Mc}{I} = \frac{M\frac{d}{2}}{\frac{\pi d^4}{64}} = \frac{64M}{2\pi d^3} = \frac{32M}{\pi d^3} = \frac{10.18M}{d^3} \quad (10)$$

Finally, by substituting the maximum moment M =183.38Nm, addressed in sec. IIE in (10), the equation of S_a in function of the unknown shaft's diameter is given as

$$S_a = \frac{10.18M}{d^3} = \frac{10.18(183.38Nm)}{d^3} = \frac{1866.8Nm}{d^3} \quad (11)$$

Similarly, the equation to determine the mean stress S_m, in terms of the shear stress τ is given by

$$S_m = \sqrt{3}\,\tau \quad (12)$$

Where τ=Tc/J, with T representing the torque estimated in sec.2.1 of T=94.9Nm and J is the inertia moment given as

J=1/2 πc⁴. Thus, by using T=94.9Nm the shear stress τ function it is given as

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{1}{2}\pi c^4} = \frac{16T}{\pi d^3} = \frac{5.093(94.9Nm)}{d^3} = \frac{483.32Nm}{d^3} \quad (13)$$

Therefore, the equation to estimate S_m in function of the unknown shaft's diameter is given by

$$S_m = \left(\sqrt{3}\right)\left(\frac{483.32Nm}{d^3}\right) = \frac{837.13Nm}{d^3} \quad (14)$$

And since both S_m and S_a are in function of the unknown diameter, then in the analysis its ratio is used. It is given as

$$r = \frac{S_a}{S_m} = \frac{\frac{1866.8Nm}{d^3}}{\frac{837.13Nm}{d^3}} = 2.22 \quad (15)$$

Thus, the angle γ to be used in the Soderberg diagram to determine the maximum allowed S_a and S_m values is

$$\gamma = \tan^{-1}\frac{S_a}{S_m} = \tan^{-1}(2.22) = 65.75°$$

Now to estimate the S_a and S_m values let present the determination of the endurance limit S_e value to be used in the estimation process.

C. Endurance fatigue limit estimation

The S_e value used in the Soderberg diagram to estimate the S_a and S_m values, is determined by applying the binary synthesis method as follows. All the modifier factors are considered to follow a normal distribution. Therefore, their standard deviation is estimated as the 10% of their average value [5]. After that, two factors are synthesized by using their corresponding mean and standard deviations, a new binary factor with its own synthesized mean and standard deviation is determined. Then another factor is synthesized with the one obtained previously. The process is repeated until all factors are being already synthesized. Finally, it is important to mention that because the synthetization process is performed according with the functional relationship that the synthetized variables have among them, e.g. addition, subtraction, product, division, square root, etc., then in the synthetization process, the Table 4.2 given in [5] p.162 and p.163 were used.

The analysis to determine the corresponding modified endurance limit S_e is as follows. From (8) the surface factor k_a and the size factor k_b are synthesized. It is done by using *"x"* to represent k_a and *"y"* to represent k_b. Hence their corresponding mean are μ_x for k_a and μ_y for k_b and their mean values are $\overline{k_a} = \mu_x = 0.8$ and $\overline{k_b} = \mu_y = 0.85$. Thus, since from (8), the relation between k_a and k_b is a product, then from the operation number 6 of Table 4.2 in [5] p.162, the mean of the synthesized binary variable is

$$xy = \mu_x\mu_y = New\ mean\ of\ synthetized\ factor \quad (16)$$

Numerically it is

$$(\overline{k_a})(\overline{k_b}) = (\mu_x)(\mu_y) = (0.8)(0.85) = 0.68$$

Similarly, by using σ_x to represent the standard deviation of k_a and σ_y for k_b, their numerical values are; σ_x=(0.8)(0.1)=0.08 and σ_y=(0.85)(0.1)=0.085. Therefore, following the operation number 6 (product) of Table 4.2 of [5] p.162, with ρ=0 (the finished surface k_a is independent of the size element k_b), the standard deviation of the synthesized variable (k_ak_b) is

$$\sigma_{k_ak_b} = \left[\left(\mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2\right)(1+\rho^2)\right]^{\frac{1}{2}} \quad (17)$$

Numerically it is

$$\sigma(k_ak_b) = \{[(0.8)^2(0.85)^2 + (0.85)^2(0.08)^2 + (0.08)^2(0.085)^2][1+(0)^2]\}^{1/2}$$

$$\sigma(k_ak_b) = 0.0964$$

The next step is to synthesize the load factor k_c with the synthesized variable (k_ak_b). Doing this, the mean of $(\overline{k_a})(\overline{k_b})$is taken as μ_x=0.68, and from sec. IVA $\left(\overline{k_c}\right) = 1$ is taken as μ_y. Thus, since the relation of k_c with (k_ak_b) is also a product, then following the operation number 6 (product) of Table 4.2 of [5] p.162, the mean of the synthesized variable is

$$\left(\overline{k_ak_b}\right)\left(\overline{k_c}\right) = \mu_x\mu_y = (0.68)(1) = 0.68$$

Similarly, taking the 10% of μ_y as its deviation, the standard deviations to be synthesized are

$$\sigma_{k_ak_b} = \sigma_x = 0.0964$$

and $$\sigma_{k_c} = \sigma_y = (1)(0.1) = 0.1$$

And following the operation number 6 (product) of Table 4.2 of [5] p.162 with ρ=0 (the load is independent of k_ak_b), given in (16), the standard deviation of the new synthesized binary variable ($k_ak_bk_c$) is

$$\sigma_{k_ak_bk_c} = \{[(0.68)^2(0.1)^2 + (1)^2(0.0964)^2 + (0.0964)^2(0.1)^2][1+(0)^2)]\}^{\frac{1}{2}} = 0.1183$$

The next step is to synthesize the temperature factor k_d with the synthesized variable ($k_ak_bk_c$). Thus, since the relation of k_d with ($k_ak_bk_c$) is also a product, then because

$$\left(\overline{k_a}\right)\left(\overline{k_b}\right)\left(\overline{k_c}\right) = \mu_x = 0.68$$

and from sec. IVA $(\overline{k_d}) = \mu_y = 1$ then the mean of the synthesized variable is

$$(\overline{k_a k_b k_c})(\overline{k_d}) = \mu_x \mu_y = (0.68)(1) = 0.68$$

And by using $\sigma_x = \sigma(k_a k_b k_c) = 0.1183$ and σ_y=σ(k_d)=(1)(0.1)=0.1, the standard deviation of the new synthesized binary variable ($k_a k_b k_c k_d$) is

$$\sigma_{k_a k_b k_c k_d} = \{[(0.68)^2(0.1)^2 + (1)^2(0.1183)^2 + (0.1183)^2(0.1)^2][1 + (0)^2)]\}^{\frac{1}{2}} = 0.1369$$

The next step is to synthesize the fatigue factor K_F with the synthesized variable ($k_a k_b k_c k_d$). Thus, since the relation of K_F with ($k_a k_b k_c k_d$) is a quotient then by using $\overline{k_a k_b k_c k_d} = \mu_x = 0.68$

and from form sec. IVA $\overline{K_F} = \mu_y = 2$, then by using operation 7 (quotient) of Table 4.2 by [5] (17) p.163, the mean of the synthesized binary variable is

$$\frac{\mu_x}{\mu_y} = New\ synthetized\ variable \qquad (18)$$

Numerically it is

$$\mu_{k_a k_b k_c k_d}/\overline{K_F} = \frac{\mu_x}{\mu_y} = \frac{0.68}{2} = 0.34$$

Similarly, by using

$$\sigma(k_a k_b k_c k_d) = \sigma_x = 0.1369$$

and $\sigma(K_F) = \sigma_y = (2)(0.1) = 0.2$

in the operation 7 (quotient) of Table 4.2 of [5] (17) p.163, the standard deviation of the new synthesized binary variable ($k_a k_b k_c k_d/K_F$) is given as

$$\sigma = \frac{1}{\mu_y}\left(\frac{\mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2}{\mu_y^2 + \sigma_y^2}\right)^{\frac{1}{2}} \qquad (19)$$

Numerically it is

$$\sigma_{\frac{k_a k_b k_c k_d}{K_F}} = \frac{1}{2}\left[\frac{(0.68)^2(0.2)^2 + (2)^2(0.1369)^2}{(2)^2 + (0.2)^2}\right]^{\frac{1}{2}} = 0.07604$$

Finally, to obtain the modified endurance limit S_e, the binary synthesis method is performed between the synthesized binary variable $x = (k_a k_b k_c k_d)/\overline{K_F}$ and the endurance limit y=S_e' of the used material. The endurance material limit is Se'=0.5S_{ut}=0.5(438MPa) =219MPa. Thus, the means to be synthesized are $\mu_x = (k_a k_b k_c k_d)/\overline{K_F} = 0.34$ and S_e'=μ_y=219MPa. And because the relation of S_e' with $x = (k_a k_b k_c k_d)/\overline{K_F}$ is a product, then from operation 6 of Table 4.2 of [5] p.162, the mean of the synthesized variable is

$$S_e = \mu_x \mu_y = (\mu_{\frac{k_a k_b k_c k_d}{K_F}})(S_e') \qquad (20)$$

Numerically it is

$$S_e = (0.34)(219\ MPa) = 74.46MPa$$

Similarly, by taking

$$\sigma_{\frac{k_a k_b k_c k_d}{K_F}} = \sigma_x = 0.07604$$

and

$$\sigma_{S_e} = \sigma_y = (219MPa)(0.1) = 21.9MPa$$

the corresponding standard deviation of the new synthesized binary variable with ρ=0 (variables are independent each other) is

$$\sigma_{\frac{k_a k_b k_c k_d S_e'}{K_F}} = \left\{\left[(\mu_x)^2(\sigma_y)^2 + (S_e')^2(\sigma_x)^2 + (\sigma_x)^2(\sigma_y)^2\right][1 + (\rho)^2]\right\}^{1/3} \qquad (21)$$

$$\sigma_{\frac{k_a k_b k_c k_d S_e'}{K_F}} = \{[(0.34)^2(21.9MPa)^2 + (219MPa)^2(0.07604)^2 + (0.07604)^2(21.9MPa)^2][1 + (0)^2)]\}^{\frac{1}{2}} = 18309786.41Pa$$

Therefore, the modified endurance limit S_e value to be used to determine the corresponding S_a and S_m values is S_e=74428904.94Pa, with expected standard deviation of σ(S_e)= 18309786.41Pa. The numerical analysis to determine the S_a and S_m values as well as the shaft diameter is as follows.

D. Binary synthesis diameter determination

Since from the plotted Soderberg diagram (see Fig.15), no mathematical function exists to determine the S_a and S_m values directly from the known S_e = 74428904.97 Pa, S_y= 320 MPa and γ = 65.75°, then the SolidWorks routine was used (any other routine can be used). The found S_a and S_m values are S_a =66945197.51Pa and S_m=30156634.18Pa.

Therefore, by using a safety factor of 2 [14,19] and the estimated S_a value in (10) the designed diameter is

$$d_b = \sqrt[3]{\frac{10.18\ M}{Sa/\mathrm{SF}}} \qquad (22)$$

Numerically it is

$$d_b = \sqrt[3]{\frac{113922419.49Pa}{66945197.51\mathrm{Pa}/2}} = 0.038m$$

Here it is important to notice that although from (22) the shaft diameter was determined by considering S_e to be random and we used a SF=2, as in the static, fatigue and rigidity approach, in this binary synthesis analysis is not possible to determine the reliability that the designed shaft presents. Therefore, now let present the proposed method which will let us both to perform the analysis by considering a probabilistic behavior and to determine the reliability of the designed shaft. The proposed Probabilistic method is as follows.

VI. PROPOSED SHAFT DESIGN METHOD

In this section the method to probabilistically to determine the diameter of the designed shaft as well as its corresponding reliability is given. In the method the applied alternating stress S_a and the material strength S_e, are random variables. And based on the minimum expected strength S_{emin} value, the maximum allowed S_{amax} value, for which zero failure are expected is determined. However, first let start showing that in our analyzed case, because the addressed S_a value is higher than the minimum expected S_{emin} value ($S_a > S_{emin}$), then failures are expected. The analysis is as follows.

A. Overlapping of S_a and S_{emin}

Since in the analyzed case, the addressed alternating value was S_a =66945197.51Pa, and because the endurance limit value is S_e = 74428904.97Pa, with standard deviation of σ_{Se} =

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18309786.41Pa, then the minimum expected S_e value is S_{emin} $=S_e - \sigma_{Se}$, which numerically is S_{emin} = 56119118.56Pa. Thus, we have that because 66945197.51Pa >56119118.56Pa (see Figure 15), then failures are expected [21]. And therefore, a lower S'_a value which does not overlapping with S_{emin} has to be determined and used to determine the shaft diameter. The steps are as follows.

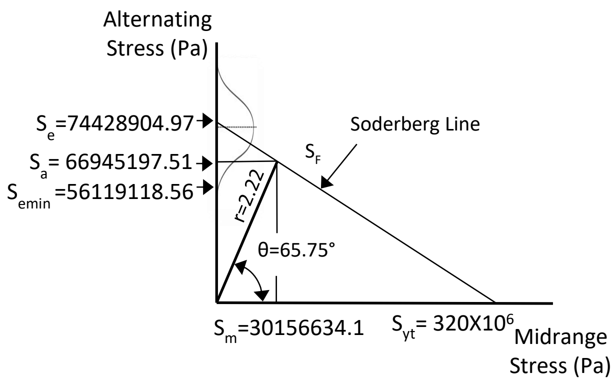

Fig. 15. Overlap of S_a and S_{emin}

B. *Steps of the Proposed Method*

Step 1. By using the modified factors, determine the mean value of S_e and its respective standard deviation (σ_{Se}).

Step 2. Determine the minimum expected value of S_e as

$$S_{emin} = \mu_{Se} - \sigma_{Se} \qquad (23)$$

Step 3. Draw a new Soderberg line parallel to the original one, but now starting at the S_{emin} value of step 2 (see Fig.16).

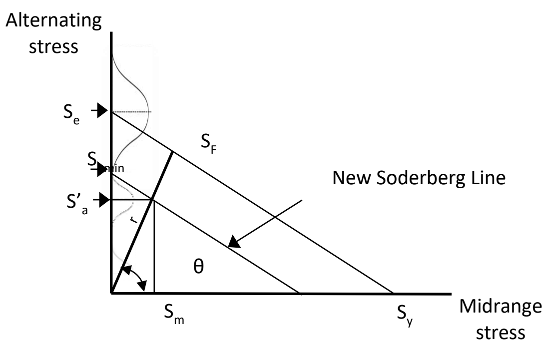

Fig. 16. New Soderberg Line

Step 4. Based on the Semin value of step 2, determine the maximum allowed S'_a value as

$$S'_a = S_{emin}/1.1 \qquad (24)$$

Step 5. Determine the standard deviation of S'_a as

$$\sigma_{Sa} = 0.10 S'_a \qquad (25)$$

Here notice that 1) by its own construction S_{amax} and S_{emin} are always equals

$$S'_a + \sigma_{Sa} = S_{emin} \qquad (26)$$

And that 2) as shown in Fig. 16, it implies we are using a safety factor of 1.

Step 6. If a different safety factor value is used, determine the corresponding S_{a2} value to be use as

$$S_{a2} = S'_a/SF \qquad (27)$$

Step 7. Determine the shaft's diameter as

$$d = \sqrt[3]{\frac{10.18M}{Sa2}} \qquad (28)$$

Step 8. Determine the reliability of the designed shaft by using (29) and (30)

$$Z = \frac{\mu_{sa2} - \mu_{se}}{\sqrt{\sigma_{se}^2 + \sigma_{s'a}^2}} \qquad (29)$$

$$R(t) = 1 - P(Z) \qquad (30)$$

Now let present the application of the proposed method.

C. *Numerical Application of the Proposed Method*

Step 1. From (21) the mean of S_e is $\mu(S_e)$=74428904.97Pa. And from (20) and (21) its standard deviation is $\sigma(S_e)$=18309786.41Pa.

Note 1: Notice that although here $\mu(S_e)$ and $\sigma(S_e)$, were both determined by using the synthesis binary method, in practice they can be determined by any other method.

Step 2. From (23) the minimum expected endurance limit (S_{emin}) is

S_{emin} = 74428904.97Pa – 18309786.41Pa = 56119118.56Pa.

Step 3. The new Soderberg line is drawn starting at S_{emin}=56119118.56Pa and being parallel to the original one (see Fig.17).

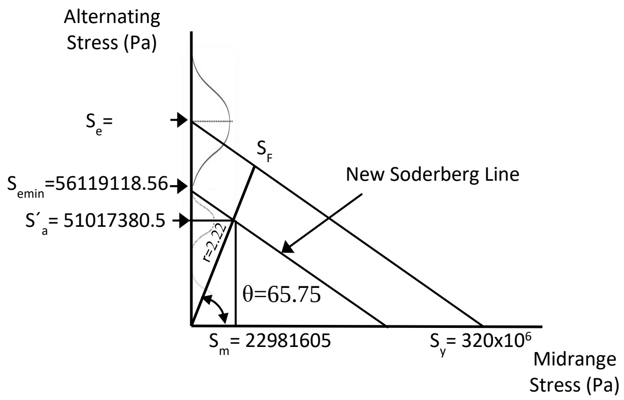

Fig. 17. New Soderberg Line with values

Step 4. From (24) the maximum allowed alternating stress value is S'_a = (56119118.56Pa) /1.1 = 51017380.5Pa

Step 5. From (25) the standard deviation of S'_a is $\sigma S'_a$ = 0.10 S'_a= 0.10 (51017380.5Pa) = 5101738.05Pa.

Step 6. From (27), and by using a safety factor of 2 [14,19] the alternating stress (S_{a2}) value to determine the shaft diameter is

$$S_{a2} = 51017380.5Pa/2 = 25508690.25Pa$$

Step 7. From (28) the shaft's diameter is

$$d = \sqrt[3]{\frac{1866.84Nm}{S_{a2}}} = \sqrt[3]{\frac{1866.84Nm}{25508690.25N/m^2}} = 0.0418m$$

Step 8. From (29) and (30), the shaft's reliability is as follows

$$Z = \frac{\mu_{sa2} - \mu_{se}}{\sqrt{\sigma_{se}^2 + \sigma_{s'a}^2}} = \frac{25508690.25 - 74428904}{\sqrt{(18309786.41)^2 + (5474023.6)^2}} = -2.57.$$

Therefore, since this Z value corresponds to P(Z)=0.005 cumulative failure probability, then from (29) the deigned reliability is R(t)=1-P(Z) = 1 – 0.005=0.9995.

As a summary of this section, we have that by considering that the applied stress S'_a and inherent strength Se are both random, by applying the proposed probabilistic method, the designed shaft's diameter is d=0.0418m(1.647in). And that because it was determined by using the S'_a value which does not overlap with the Semin value, then shaft 2 can be considered safe. Even more, from step 8, we conclude the designed reliability of the shaft is R(t)=99.95%.

Now let compare the designed R(t) index of the proposed method with those given by the static, fatigue, and binary synthesis approaches.

D. A Reliability Comparison Among the Shaft Design Methodologies

The comparison among the design approaches is given in Table 2, where the given reliability indices were determined by using (29) and (30). The corresponding standard deviation were determined by using (25). In all approaches, the mean value of the endurance limit (μs_e) of 74428904Pa and standard deviation (σs_e) of 18309786.41 were used.

From Table 2, we observe that higher the shaft diameter, the higher the reliability. And that the higher diameter is generated by considering the standard deviation in the analysis. Also note that without using (29) and (30), for the static, fatigue, and binary synthesis approach it is not possible to determine their designed reliability. Now let generalize the proposed probabilistic method to use the lognormal and the Weibull distribution in the analysis. The generalization is as follows.

Table 2. Reliability of the shaft diameter determined by different approaches.

Approach	Average (s_a)	S. Dev.	Diameter	R(t) (29-30)
Static	142549382.8	28509871	0.0235	0.0222
Fatigue	36605852.26	7321172.5	0.037	0.9724
Synthesis	51853608.01	10370660	0.3302	0.8583
Proposed	25508690.25	5101706.7	0.0418	0.9950
Torsional Rigidity	25360503.11	5072095.1	0.4191	0.9951

VII. GENERALIZATION OF THE PROPOSED METHOD TO THE WEIBULL DISTRIBUTION

Since a random variable is said to follows a normal distribution only when its variation coefficient given as

$$CV = \sigma/\mu \tag{31}$$

is equal or lower than 0.10 (CV≤0.10) [5] pg.159, then because from the above analysis the C.V of S'_a of CV_a=5101706.7/25508690.25=0.20 and the C.V of S_e of CV_e=0.24, are both higher than 0.10, then in this section the proposed method is generalized to use the Weibull distribution to determine the designed shaft's reliability. This is done because due to its flexibility, the Weibull distribution can model also the normal distribution behavior. Weibull distribution has been applied to solve a variety of problems in different areas [22] and is frequently adopted to reflect lifetime distribution to assess system reliability [23]. This approximately occurs for a Weibull shape parameters β close to β=3.44. The Weibull density function is given by

$$f(S_j) = \frac{\beta_j}{\eta}\left(\frac{S_j}{\eta_j}\right)^{\beta_j - 1} exp\left\{-\left(\frac{S_j}{\eta_j}\right)^{\beta_j}\right\} \tag{32}$$

Where η is the scale parameter and *j=a,e*, with *a* representing the alternating stress S'_a, and e representing the modified endurance limit S_e. The corresponding Weibull reliability function is given as

$$R(S_j) = exp\left\{-\left(\frac{S_j}{\eta_j}\right)^{\beta_j}\right\} \tag{33}$$

Therefore, because as demonstrated in [24] the Weibull shape β and scale η parameters, both can be determined directly from the maximum and minimum stress values, then in this section, the mean and the standard deviation of S'_a are used to determine the maximum and the minimum expected stresses values of S'_a as

$$S'_{amax} = S'_a + \sigma_{sa} \tag{34}$$

and $$S'_{amin} = S'_a - \sigma_{sa}$$

Here observe that this imply that 1) because the failure will be generated by fatigue, and not by the first cycle then the addressed S_m value is not being considered in the analysis, and therefore. 2) we are taking $S'_{a\max}$, and $S'_{a\min}$ as the principal stresses σ_1 and σ_2 values to perform the Weibull analysis (*if S_m is going to be considered, then the corresponding σ_1 and σ_2 values must be estimated using the S_m also*) and 3) that the standard deviation value is being used as the amplitude stress value as plotted in Fig.16. Thus, following [24], the corresponding Weibull stress β_a and η_a parameters are given as

$$\beta_a = \frac{-4\mu_y}{(0.97161)(S'_{amax}/S'_{amin})} \tag{35}$$

$$\eta_a = exp\left\{\sqrt{(S'a_{max})(S'a_{min})} - \mu_y/\beta_a\right\} \tag{36}$$

In (35), the constant 0.971611 value was determined following the method given in sec.4.1 in [24], pg.236 (there the key formula is (48)). And μ_Y is the mean value of the Y vector given by linearizing (33). Its linear form is given as

$$Y_i = ln\left(-ln\left(1 - F(S_i)\right)\right) = \beta_j\left[ln(S_i) - ln(\eta_j)\right] \tag{37}$$

Where F(S*i*)=1-R(S*i*) is the cumulated failure percentile estimated by using the median rank approach [25] given by

$$F(S_i) = (i - 0.3)/(n + 4) \tag{38}$$

With *n* being estimated as [26]

$$n = \frac{-1}{ln\left(R(S_j)\right)} \tag{39}$$

In (39), R(S) is the desired reliability of the analysis. Here observe that although here R(S)=0.9535 was used to n be an integer (n=21), any desired percentile can be used. Additionally, observe from (39), that because n only depends on R(S), then μ_y is constant end for n=21 its value is μ_y= –0.545624125.

On the other hand, by using the maximum and minimum S_e values given by

$$S_{emax} = S_e + \sigma_{se} \tag{40}$$

and

$$S_{emin} = S_e - \sigma_{se}$$

And by using the method given in the sec.4.1 of [20] in pg.236, the corresponding β_e and η_e parameters are determined as

$$\beta_e = \frac{-4\mu_y}{(0.971611)ln(S_{emax}/S_{emin})} \tag{41}$$

$$\eta_e = exp\left\{\sqrt{(S_{emax})(S_{emin})} - \mu_y/\beta_e\right\} \tag{42}$$

Therefore, by using the Weibull stress parameters of (35) and (36) or the Weibull strength parameters of (41), (42) and (38), the corresponding logarithm of the expected stress (or strength) values are given as

$$ln(t_{ij}) = x_{ij} = Y_j/\beta_j + ln(\eta_j) \tag{43}$$

And therefore, the expected stress (or strength) values are given as

$$S_{ij} = exp\{x_{ij}\} \tag{44}$$

Finally, by using the addressed Weibull families W(β_a, η_a), called Weibull stress family, and the W(β_e, η_e), called Weibull strength family in the general reliability stress/strength function given by

$$R(t|S_a', S_e) = \int_0^\infty f(s)\left[\int_S^\infty f(S)dS\right]dS \tag{45a}$$

The corresponding Weibull/Weibull reliability stress/strength function is given by

$$R(t|S_a', S_e) = \int_0^\infty \frac{\beta_a}{\eta_a}\left(\frac{S_a}{\eta_a}\right)^{\beta_a-1} exp\left\{\left(\frac{s}{\eta_a}\right)^{\beta_a}\right\}\left[\int_S^\infty \frac{\beta_e}{\eta_e}\left(\frac{S_e}{\eta_e}\right)^{\beta_e-1} exp\left\{-\left(\frac{S_e}{\eta_e}\right)^{\beta_e}\right\}dS_a\right]dS_e \tag{45b}$$

Here, it is very important to notice that for $\beta_a \neq \beta_e$, (45b) has not a close solution and thus, it must be solved by using a numerical method as the one given in [27], and that for $\beta_a = \beta_e = \beta$, the solution of (45b) [21] is given as

$$R(t|S_a', S_e) = \frac{\eta_{Se}^{\beta}}{\eta_{Se}^{\beta}+\eta_{S_a'}^{\beta}} \tag{45c}$$

Therefore, based on the formulation from the above, the steps to determine the reliability of the designed shaft by using the Weibull distribution are as follows.

A. Steps of the proposed generalization form of the normal to the Weibull distribution

Step 1. Based on the mean and standard deviation of S'_a and S_e, from (34) and (40) determine their corresponding maximum and minimum expected values.

From (35)

$$S_{amax} = 25508690.25 + 5101738.5 = 30610428.75Pa$$

$$S_{amin} = 25508690.25 - 5101738.5 = 20426951Pa$$

And from (41)

$$S_{emax} = 74428904.97 + 18309786.41 = 92738691.37Pa$$

$$S_{emin} = 74428904.97 - 18309786.41 = 56119118.56Pa$$

Step 2. By using $n=21$ in (38) and then the $F(t_i)$ values in (37) determine the Y_i elements and its corresponding mean μ_Y value.

Note. Remember, that for $n=21$, the analysis is performed with a reliability of $R(t_i)=95.35\%$ and that for $n=21$ the μ_Y value is $\mu_Y= -0.545624125$. Hence, $R(t_i)=95.35\%$ could be seen as the confidence interval used in quality. For example, since in Table 2 we used $n=21$, then as can be seen from (39) the analysis in Table 2 was performed with $R(t_i)=95.35\%$. Therefore, from Table 2 we say with reliability of 95.35 that the reliability of the shaft is of $R(t)=99.50\%$.

Step 3. By using the maximum and minimum S'_a and S_e values of step 1, and the μ_Y value from step 2 in (35) and (36), determine the corresponding stress Weibull family, and by using them in (41) and (42), determine the corresponding strength Weibull family.

The Weibull stress family is W(β_a=5.5399696, η_a=27580143.64Pa (4000.16164 lb/in^2)). And the Weibull strength family is $W(\beta_e=4.45324405,\ \eta_e=81545114.98Pa\ (11827.119\ lb/in^2))$ (see Table 3).

Step 4. By using the addressed Weibull parameters in (45b) if $\beta_a \neq \beta_e$, or in (45c) if $\beta_a = \beta_e$, determine the designed reliability of the designed shaft.

Since $\beta_a \neq \beta_e$, then by solving (45b) by using the Weibull++ software routine, the designed reliability is R(t)=0.992579.

As a conclusion we have that because the variation coefficient of S'_a and S_e are both higher than 0.10, then the shaft reliability must be determined by using the Weibull distribution. Hence the shaft's reliability is of R(t)=0.992579, and due to the torsional rigidity approach, the shaft's diameter has to be of d=0.4191m (1.65 in).

Now let show that the addressed Weibull families completely represents the normal parameters from which they were derived. The analysis is as follows.

B. Weibull and Normal Parameters Relationships

The Weibull and normal parameters relationship are direct. Therefore, they can easily be addressed by using the expected stresses values defined in (43) and (44), and numerically given in Table 3.

1) From Table 3 we observe the normal averages values of $\mu(S'_a)$= 25508690.25Pa (3699.72 lb/in^2) and $\mu(S_e)$= 74428904.97Pa (10,795lb/in^2) of the S'_a and S_e variables, are given as the average values of columns S'_a and S_e. Therefore, the addressed Weibull families completely represent the normal average values.

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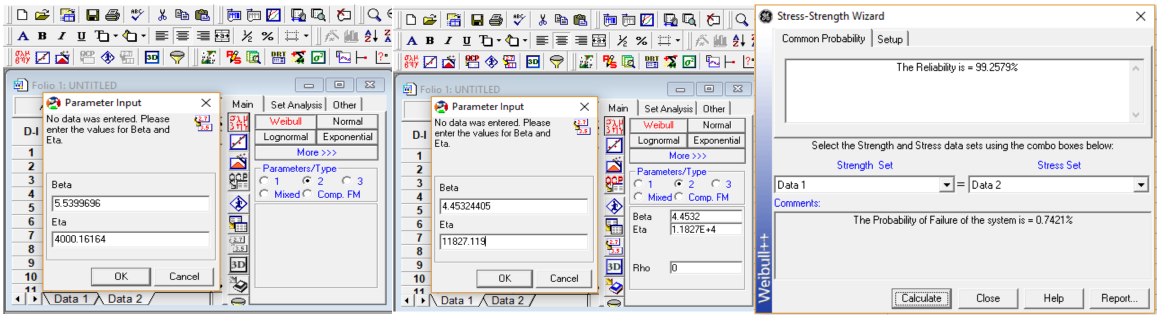

Fig. 18. Input and output of the Weibull++ routine

Table 3. Weibull and Normal Data relationships

Stress Data W(βe =4.45324405, ηe =11827.11901 lb-in^2)				Strength Data W(βa =5.5399696, ηa =4000.16164 lb-in^2)			
n Eq.(40)	Y_i Eq.(38)	$ln(S'a_i)$ Eq.(44)	$S'a_i$ Eq.(45)	n Eq.(40)	Y_i Eq.(38)	$ln(Se_i)$ Eq.(44)	$S'e_i$ Eq.(45)
1	-3.40348334	8.613879831	5507.575741	1	-3.40348334	7.679739504	2164.0559714
2	-2.49166198	8.818634241	6759.027118	2	-2.49166198	7.844329101	2551.2254736
3	-2.00346322	8.928261909	7542.145047	3	-2.003463219	7.932452104	2786.2506118
4	-1.66164593	9.005018828	8143.854136	4	-1.661645928	7.994152315	2963.5771517
5	-1.39439830	9.065030726	8647.544842	5	-1.394398299	8.042392224	3110.0442202
6	-1.17205365	9.114959418	9090.265709	6	-1.172053652	8.082526857	3237.4033649
7	-0.97938116	9.158225066	9492.194100	7	-0.97938116	8.117305476	3351.9765838
8	-0.80744734	9.196833734	9865.841677	8	-0.807447338	8.148340632	3457.6368110
9	-0.65049212	9.232078875	10219.765059	9	-0.650492124	8.176672054	3556.9974447
10	-0.50450882	9.264860215	10560.334314	10	-0.504508816	8.203022976	3651.9734647
11	-0.36651292	9.295847939	10892.698037	11	-0.366512921	8.227932119	3744.0834221
12	-0.23412230	9.325576967	11221.388989	12	-0.234122302	8.251829474	3834.6347711
13	-0.10528508	9.354508062	11550.777869	13	-0.105285078	8.275085418	3924.8578688
14	0.02192840	9.383074535	11885.501023	14	0.021928399	8.298048265	4016.0265213
15	0.14952577	9.411727213	12230.978252	15	0.149525769	8.321080408	4109.5976526
16	0.27984500	9.440991101	12594.192842	16	0.279845003	8.344603864	4207.4155917
17	0.41596210	9.47155693	12985.088370	17	0.415962097	8.369173872	4312.0722665
18	0.56250196	9.504463248	13419.487868	18	0.562501963	8.395625256	4427.6544636
19	0.72761583	9.541540458	13926.384115	19	0.727615827	8.425429366	4561.6029571
20	0.92931067	9.586832124	14571.635212	20	0.929310672	8.461836578	4730.7383938
21	1.22965981	9.654277145	15588.319227	21	1.22965981	8.516051521	4994.2949067
μy=	-0.545624125	μx= 9.2556276	μt= 10795.0000	μy=	-0.545624125	μx= 8.1956014	μt= 3699.7200
σy=	1.175116938	σx= 0.2638789	σt= 2629.6350	σy=	1.175116938	σx= 0.2121161	σt= 734.8660

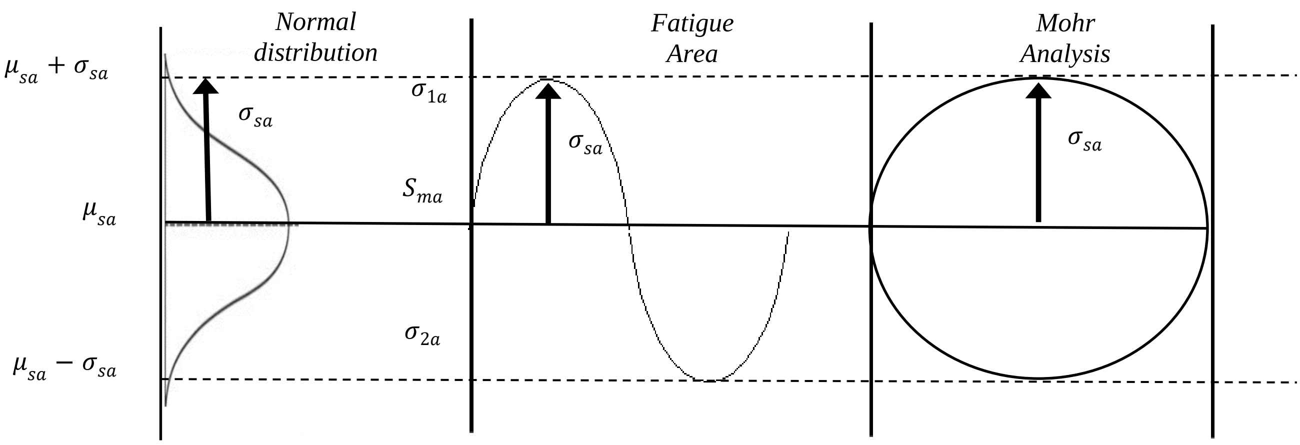

Fig. 19. The normal standard deviation as the alternating stress and Mohr radio

2) Since the standard deviation $\sigma(S'_a)$=5101738.05Pa (739.9444lb/in^2) and $\sigma(S_e)$=18309786.41Pa (2,655.51lb/in^2) values of the S'_a and S_e variables where used as the alternating stress value (radio of the corresponding Mohr Circle) to determine the principal stress values (See Fig. 19) used to estimate β_a and β_e in (35) and (41), then from (14) of [24], by using data of Table 3, they are given as

$$\sigma(S'_a) = \sqrt{\mu(S'_a)^2 - (S'_{amax})(S'_{amin})} = \sqrt{\mu(S'_a)^2 - (\mu_{xa})^2} \quad (46a)$$

$$\sigma(S_e) = \sqrt{\mu(S_e)^2 - (S_{emax})(S_{emin})} = \sqrt{\mu(S_e)^2 - (\mu_{xe})^2} \quad (46b)$$

Numerically

$$\sigma(S'_a) = \sqrt{(25508690.25Pa)^2 - (exp\{8.1956014\})^2} = 51017380.5Pa$$

and

$$\sigma(S_e) = \sqrt{(74428904.97Pa)^2 - (exp\{9.2556276\})^2} = 18309786.41Pa$$

Therefore, we conclude that the addressed Weibull families completely represent the normal standard deviation values also.

3) The general conclusion is that the Weibull distribution always can be used to model the normal behavior, and that the Weibull distribution should be used instead of the normal one, when the variation coefficient of the analyzed data is higher than 0.10.

Now since the Weibull and the lognormal distribution are both completely related each other [29], then let present the corresponding analysis to base on the Weibull parameters to determine the log-normal ones.

C. *Weibull and Log-Normal Parameters Relationships*

Here it is important to mention that because the Weibull distribution is generated by a non-homogeneous Poisson process [27], and the lognormal distribution is generated by a geometric Brownian motion [29], then one distribution should not be used instead of the other to determine the reliability of the analyzed element. However, because in practice the environment on which the element is performing its function, generally is lognormal [30], then in this section the determination of the log-normal parameters, directly from the Weibull analysis is given. The analysis is based on the (41) and (44) formulated in section 3.2 in [28]. Equations on which the Weibull β and η parameters and the log-mean μ_x and the log-standard deviation σ_x parameters are directly related as

$$\beta = \frac{\sigma_y}{\sigma_x} \quad (47)$$

$$\eta = exp\left\{\mu_x - \frac{\mu_y}{\beta}\right\} = exp\left\{\mu_x - \frac{\sigma_x \mu_y}{\sigma_y}\right\} \quad (48)$$

Therefore, since σ_y is constant (for $n=21$ its value is σ_y=1.175116938), then given any Weibull parameters, the corresponding lognormal μ_x and σ_x parameters can directly be determined. From Table 3 for the S'_a variable they are μ_{xa}= 56506.68Pa (8.1956014 lb/in^2), and σ_x=1462.48Pa (0.2121161 lb/in^2). And for the S_e variable they are μ_{xe}=63815.30Pa (9.2556276 lb-in^2) and σ_{xe}=1819.38Pa (0.2638789 lb/in^2).

As a summary we have that due to the direct relationship between the Weibull and the lognormal distribution parameters given in (47) and (48), given any Weibull analysis the corresponding log parameters always can be determined also. However, it is important to mention that because in the lognormal distribution the damage is cumulated in multiplicative form, then the lognormal distribution is generally used when a chemistry reaction is involved. Or also when the variation coefficient of the analyzed data tends to be the log-standard deviation σ_x. Therefore, since the CV_a= 0.27864493 and σ_x=1462.48Pa (0.2121161 lb/in^2) and CV_e= 0.25379319 and σ_{xe}=1819.38Pa (0.2638789 lb/in^2), then the use of the lognormal distribution to determine the reliability of the designed shaft is not recommended.

Even though, in the next section the reliability of the designed shaft using the normal, Weibull, lognormal, and the combination among them is given.

D. *Reliability Comparison Among the Normal, Weibull and Log-Normal Distributions*

Using the stress/strength formulation given in (46a) with the normal, Weibull and log-normal distributions, as well as with their combinations, the designed reliability of the shaft are given in Table 4. From Table 4, and as a conclusion we have the reliability of the shaft designed is the one given by the Weibull/Weibull approach of R(S)=0.992579. And that ones we know the mean and standard deviation of the stress and strength analyzed data; we can perform either of the above stress/strength reliability analysis. The final conclusions are as follows.

Table 4. Reliability indices for the normal, Weibull and Log-Normal approaches.

Approach	Stress Parameters		Strength Parameters		*R(S)* index
Normal/Normal	μ=3699.72	σ=739.944	μ=10795	σ=2655.51	0.994881
Weibull/Weibull	**β=5.5399696**	**η=4000.16164**	**β=4.45324405**	**η=11827.1190**	**0.992579**
Log-Normal/Log-normal	μx=8.1956014	σx=0.2121161	μx=9.2556276	σx=0.2638789	0.999126
Normal/Weibull	μ=3699.72	σ=739.944	β=4.45324405	η=11827.1190	0.990113
Weibull/Normal	β=5.5399696	η=4000.16164	μ=10795	σ=2655.51	0.994979
Norma/Log-Normal	μ=3699.72	σ=739.944	μx=9.2556276	σx=0.2638789	0.999558
Log-Normal/Normal	μx=8.1956014	σx=0.2121161	μ=10795	σ=2655.51	0.994469
Log-Normal/Weibull	μx=8.1956014	σx=0.2121161	β=4.45324405	η=11827.1190	0.989532
Weibull/Log-Normal	β=5.5399696	η=4000.16164	μx=9.2556276	σx=0.2638789	0.999613

VIII. CONCLUSION

From the above methodologies the following conclusion were drawn.

1. The static analysis let us to efficiently determine the maximum and minimum stresses values on which the shaft's diameter is determined.

2. The failure modes of a shaft can be either, by yielding when the mean stress S_m is higher than the yield material strength S_y, or from fatigue, generated by both the S_m and the alternating stress Sa values, or from plastic deformation given by the torsional forces.

3. The dynamic fatigue analysis let us to determine the shaft's diameter by considering the endurance limit modified factors.

4. The binary synthesis approach let us to determine the shaft's diameter by considering the endurance limit Se to be random, as well as to determine the mean S_m and maximum alternating stress S'_a values at which the shaft will be subjected.

5. From section VII, we observe that the proposed method can be used to easily determine the reliability of the deigned shaft by using the Weibull distribution. And that this is made by simple using the addressed mean and standard deviation of S'_a and S_e as the maximum and minimum principal stresses values, used to determine the Weibull parameters of the stress and strength distributions.

6. The proposed method given in section VI, let us to determine the shaft's diameter by considering that S'_a and Se are both random. As well as to determine the designed reliability by using the selected stress/strength approach as in Table 4.

7. For variation coefficients higher than 0.10, the Weibull/Weibull stress/strength function defined in (44b) and (44c) is recommended.

8. Notice that a random variable follows a lognormal distribution only if it is generated by a geometric Browmian motion [29], or equivalently when the generated damage cannot be cumulated by using an additive damage model [31]. Or in practice, if the variation coefficient of the collected data tends to be the log-standard deviation.

9. Due to the variation coefficient of the stress and strength data were not lower than 0.10 nor close to the log-standard deviation, then the reliability of the designed shaft is the one given by the Weibull/Weibull stress/strength approach.

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