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Prime Labeling Of Friendship Graphs

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Abstract:

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integer 1,2,3…$|V|$ such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for some fan related graphs. We also discuss prime labeling in the context of some graph operations namely fusion and duplication in fan Fn

Keywords: Prime Labeling, Fusion, Duplication.

1. Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to Bondy and Murthy [1].

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A (1982 P 365-368). [2] Many researches have studied prime graph for example in Fu.H.(1994 P 181-186) [5] have proved that path Pn on n vertices is a Prime graph.

In Deretsky.T(1991 p359 – 369) [4] have proved that the C_n on n vertices is a prime graph. In Lee.S (1998 P.59-67) [2] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all tress have prime labeling which is not settled till today. The prime labeling for planner grid is investigated by Sundaram.M(2006 P205-209) [6]

In (2010) S.K.Vaidhya and K.K.Kanmani have proved the prime labeling for some cycle related graphs [7]

Definition 1.1

Let G = (V(G), E(G)) be a graph with p vertices. A bijection f:V(G)→ {1,2,…p} is called a prime labeling if for each edge e=uv, gcd{f(u), f(v)}=1. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Let u and v be two distinct vertices of a graph G. A new graph G1 is constructed by identifying (fusing) two vertices u and v by a single vertex x in such that every edge which was incident with either u or v in G now incident with x in G.

Definition : 1.3

Duplication: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_{k'}$ with N($v_{k'}$)=N(v_k)

In other words a vertex $v_{k'}$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_{k'}$

In this paper we prove that the graphs obtained by identifying any two vertices of degree 2 in the fan graph Fn and two vertices which are adjacent to vertices of degree 2 (u_2 and v_2 or u_{n-1} or v_{n-1}) and the graph obtained by duplication the vertex of degree 2 admit prime labeling.

Definition : 1.4 (Friendship graph)

The friendship graphs T_n is a set of n triangles having a common central vertex.

Theorem.1

The friendship graph T_n is a prime graph

Proof:

Let $V(T_n) = \{ v_1, v_2, \ldots v_{2n+1}\}$ with v_1 as the centre vertex.

$E(T_n) = \{ v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{v_{2i} v_{2i+1} / 1 \leq i \leq n\}$

Define a labeling $f: v(T_n) \rightarrow \{1,2, \ldots 2n+1\}$ as follows:

$f(v_i) = i$ for $1 \leq i \leq 2n+1$

Clearly f is a prime labeling

T_n is a prime graph.

Theorem.2

The graph obtained by identifying any two vertices other than centre vertex of a friendship graph T_n is a prime graph.

Proof:

Let $V(T_n) = \{ v_1, v_2, \ldots v_{2n+1}\}$

$E(T_n) = \{v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{v_{2i} v_{2i+1} / 1 \leq i \leq n\}$

Where v_1 is the centre vertex of the friendship graph.

Let the vertex v_j be fused with u_k where $2 \leq j \leq k \leq 2n+1$ and let the newe vertex be v and let G_v be the graph obtained by fusion of v_i and v_j

Now define a function f: $V(G_v) \rightarrow \{1,2, \ldots 2n\}$ as follows

$f(v_1) = 1$

$f(v_i) = i$ for $2 \leq i \leq j-1$

$f(v_i) = i-2$ for $j+2 \leq i \leq k-1$

$f(v) = k-1$

$f(v_{j+1}) = k-2$

$f(v_i) = i-1$ for $k+1 \leq i \leq 2n$

Clearly f is a prime labeling.

Thus G_v is a prime graph

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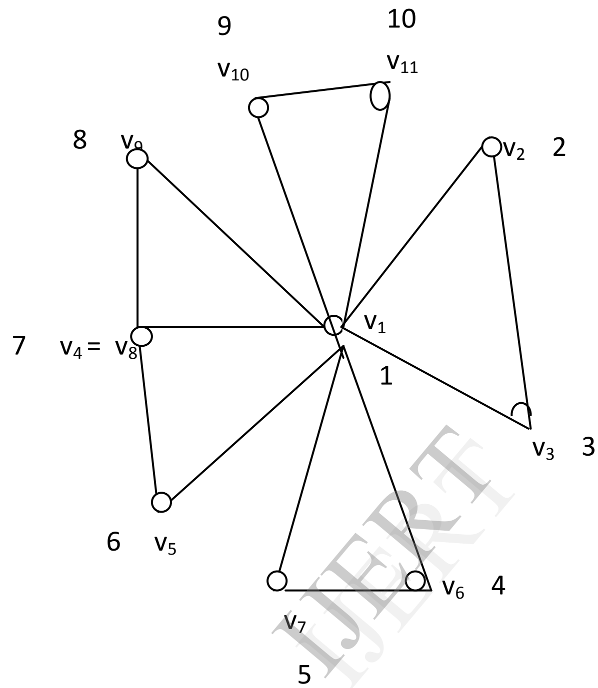
Example

Consider T_5 and let G_v be the graph obtained by identifying v_j and v_k where j=4, K=8.



Theorem.3

The graph obtained by duplicating a vertex v_k except the centre vertex of the friendship graph T_n produces a prime graph.

Proof:

Let $V(T_n) = \{v_1, v_2, \ldots\ldots v_{2n+1}\}$ with v_1 as the centre vertex and let

$$E(T_n) = \{v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{ v_{2i} v_{2i+1} / 1 \leq i \leq n\}$$

Let G_R be the graph obtained by duplicating the v_k by the new vertex $v_k^{1.}$

Case (i):

If k is odd

Define a labeling f: $V(G_K) \rightarrow \{1,2, \ldots.. 2n+2\}$ as follows:

$f(v_1) = 1$

$v(v_i) = i+1$ for $2 \leq i \leq 2n+1$

$f(v_k^1) = 2$

Then clearly f is a prime labeling.

Case (ii):

If k is even

Define a labeling f: $V(G_k) \rightarrow \{1,2,\ldots..2n+1\}$ as follows:

$f(v_1) = 1$

$f(v_i) = i$ for $1 \leq i \leq 2n+1$

$f(v_k^1) = 2n+2$

Then clearly f is a prime labeling.

Thus G_k is a prime graph .

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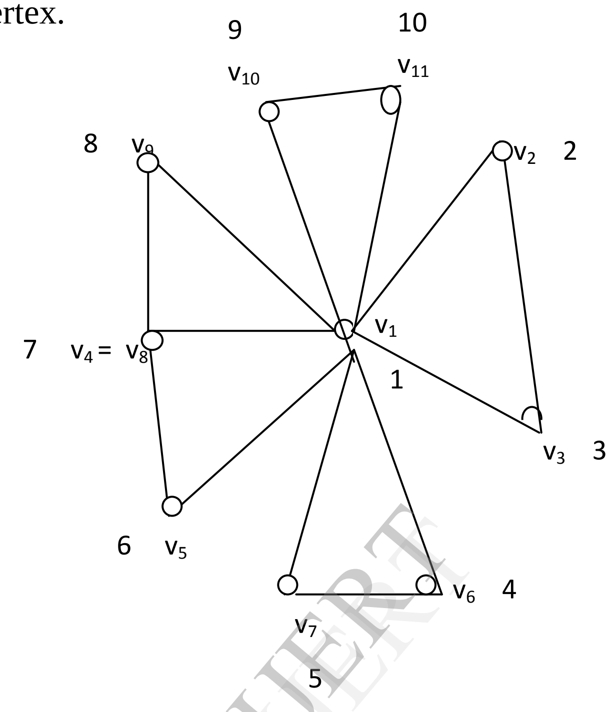
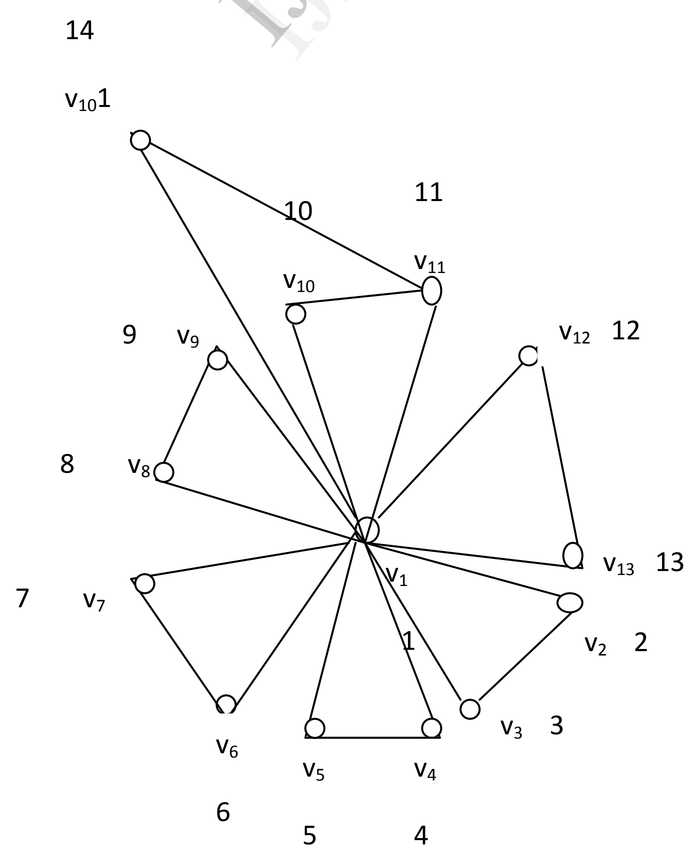
Example:

Consider T_6 and let G_k be the graph obtained by duplicating v_k and let v_k^1 be the new vertex.

K=7



K = 10



Theorem.4

The graph obtained by switching any vertex v_k in a friendship graph T_n produces a prime graph.

Proof:

Let $V(T_n) = \{ v_1, v_2, \ldots v_{2n+1}\}$

$E(T_n) = \{v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{ v_{2i} v_{2i+1} / 1 \leq i \leq n\}$

Where v_1 is the centre vertex of T_n

Let G_k be the graph obtained by switching any arbitrary vertex v_k in T_n.

If k=1 then the function

f: $v(G_1) \rightarrow \{1,2,\ldots 2n+1\}$ defined by $f(v_i) = i$ for $1 \leq i \leq 2n+1$ is a prime labeling for G_1

Assume that k>1

Case (i):

2n+1 is prime

Define a labeling f: $v(G_k) \rightarrow \{1,2,\ldots.2n+1\}$ as follows:

$f(v_i) = i$ for $1 \leq i \leq k-1$

$f(v_k) = 2n+1$

$f(v_i) = i-1$ for $k+1 \leq i \leq 2n+1$

Then f admits prime labeling.

Case (ii):

If 2n+1 is not prime let P be the largest prime number such that $p < 2n+1$

Define a labeling $f: V(G_k) \rightarrow \{1, 2, \ldots 2n+1\}$ by

$f(v_i) = i$ for $1 \leq i \leq k-1$

$f(v_k) = p$

$f(v_i) = i-1$ for $k+1 \leq i \leq p$

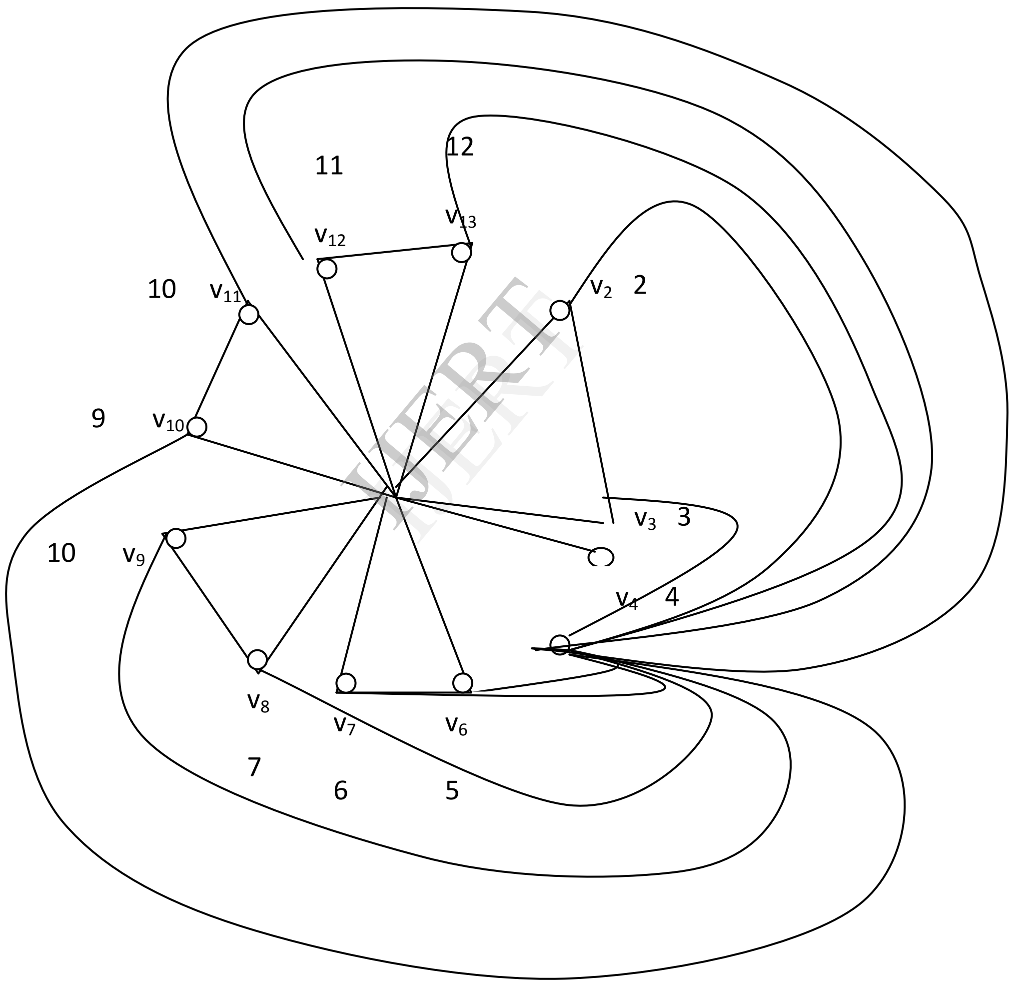
$f(v_i) = i$ for $p+1 \leq i \leq 2n+1$

then f admits prime labeling

Thus G_k is a prime graph.

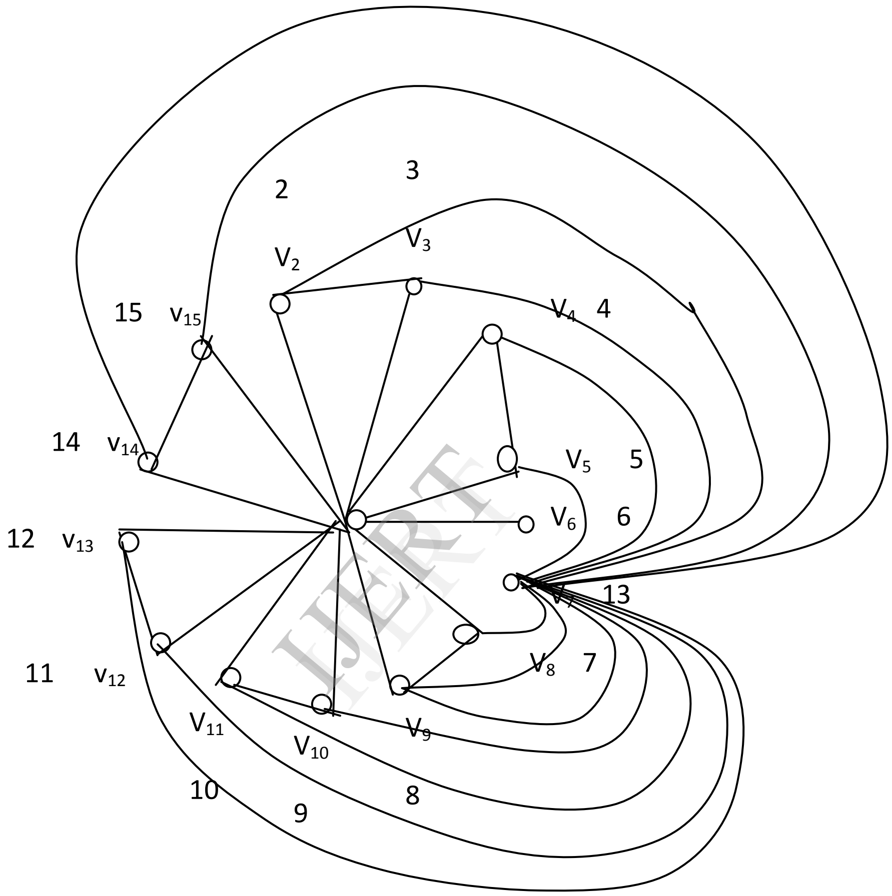
Example.1

Let G_5 be the graph obtained by switching vertex v_5 in T_6 (2n+1 is not prime)



Example.2

Let G_7 be the graph obtained by switching a vertex v_7 in T_7 (2n+1 is not prime)



Remark:

Path union of two copies of T_n is not prime. Since in the graph obtained by path union of two copies of T_n the number of vertices is 4n+2

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