Prediction of Net Uplift Capacity of Single Pile with Variation of $K_s$ Value

K. Shanker$^1$, P. K. Basudhar$^2$, N. R. Patra$^3$

$^1$Principal, Kamala Institute of Technology & Science, Singapur, Huzurabad, AP, 505 468,
$^2$Retd. Professor, Civil Engineering Department, Indian Institute of Technology Kanpur,
$^3$Assoc. Professor, Civil Engineering Department, Indian Institute of Technology Kanpur, Kanpur 208016, India.

Abstract—The paper pertains to the development of semi empirical approach to predict the net uplift capacity of single pile embedded in sand based on the fundamental concept of evaluating the unit shaft friction and then summing it up over the length of the pile. Two main factors that are important in using this approach namely the coefficient of lateral earth pressure coefficient ($K_p$) that may vary widely from Rankine passive to active earth pressure coefficients, $K_p$ and $K_a$ respectively and the pile –soil interface friction angle ($\delta$). As such to the developed method the coefficient $K_a$ is assumed to vary with depth from $K_p$ to $K_a$, the variation being parabolic, linear or constant (average of $K_p$ and $K_a$). And for interpreting field tests, $\delta$ is considered to be 0.84. A comparative assessment of the net ultimate uplift capacity of pile so obtained from experimental and field data reported in literature and also from model tests conducted as a part of the present investigation show that the proposed method has an excellent potential in predicting the uplift capacity of piles embedded in sand.

Keywords—model tests, piles, sand, uplift capacity

I. INTRODUCTION

Piles are quite often required to resist uplift forces. Resistance to uplift is due to the shaft friction developed between the pile shaft and the surrounding soil. Some foundation engineers, mostly based on the database available in the literature, have concluded that the magnitude of shaft friction is independent of the direction of loading. Others site evidence to the contrary. Mohan et al (1963), Rao and Venkatesh (1985), O’Neill (2001), Ramaswamy et al (2004), have shown that pull-out shaft friction is significantly less than the push-in shaft friction. However, according to Vesic (1970) there is practically no difference between the two. The uplift resistance of a single pile in sand is usually assumed to be dependent on the peak local shaft friction which is related to the lateral effective stress at failure, $\sigma'_{th}$. Generally, an equation of the following form is used to evaluate the net ultimate uplift capacity of a vertical circular pile in sand:

$$P_{nu} = \pi d \int_{0}^{L} (\sigma'_{th} \tan \delta) dz$$

(1)

$$\sigma'_{th} = K_a (x) \sigma'_{v}$$

(2)

In which,

$P_{nu}$ is the net ultimate uplift capacity, $d$ is the pile diameter, $L$ is the embedded length of the pile, $\delta$ is the pile-soil interface friction angle, $K_a$ is the lateral earth pressure coefficient and $\sigma'_{v}$ is the effective vertical stress.

It is seen from Eq.1 and Eq.2 that the uplift resistance of piles in sandy soil is very much dependent on the lateral earth pressure coefficient, $K_a$ that is governed by factors such as friction angle of soil, soil density, method of installation, length to diameter ratio of pile, and roughness of pile. Thus, an estimate of $K_a$ on the basis of these factors becomes difficult (Meyerhofer’s, 1976). Literature on the subject showed that the reported $K_a$ values vary over a wide range from Rankine’s passive earth pressure coefficient , $K_p$ to Rankine’s active earth pressure coefficient , $K_a$ and, in some cases may even be higher than $K_p$ (Rao and Venkatesh,1985). In reality, the magnitude of $K_a$ varies with depth; it is approximately equal to the $K_a$ at the top of the pile and may be less than the at-rest pressure coefficient, $K_r$, at greater depth(Das, 2003). Nevertheless, due to lack of sufficient evidence conservative values of $K_r$ equal to $K_a$ are used predict the shaft capacity that differs greatly from the actual value.

As, such an attempt has been made here to estimate the uplift capacity of a single pile embedded in sand more effectively assuming a $K_s$ to vary with depth along the pile length. A comparative assessment of the ultimate uplift capacity of piles predicted by using the proposed method and the measured values obtained from model tests conducted in the laboratory and field tests have been presented to validate the developed method.

II. ANALYSIS

The net uplift capacity of a pile is estimated using Eq.1 and Eq.2 assuming linear and parabolic variation of lateral earth pressure coefficient $K_a$ ranging from $K_p$ to $K_a$ with depth as shown in Fig.1 (a) and Fig. 1(b) respectively and also assuming a constant value for the lateral earth pressure coefficient equal to the average of $K_p$ and $K_a$.

III. EXPERIMENTAL INVESTIGATION

Apart from collecting data on the subject from the literature (Das, 1983; Chattopadhyay and Pise, 1986; Das and Pise, 2003), model tests were conducted in the laboratory to find the uplift capacity of square piles of size 20mm x 20mm at different L/d ratios of 10,20,30 and 40 placed in sand bed of medium to loose state. The model piles were held vertically in the tank of size 990mm x 975mm x 970mm and sand was placed by rainfall technique maintaining uniform density. The measured values of soil parameters like unit weight ($\gamma$), angle of friction ($\phi$), pile – soil interface friction angle ($\delta$) and relative density ($D_r$) being 15.8 kN/m$^3$, 38°, 26° and 54.3% in medium dense state and 15.4 kN/m$^3$, 34°, 22° and 34.35% in medium loose state. The measured values of soil parameters like unit weight ($\gamma$), angle of friction ($\phi$), pile – soil interface friction angle ($\delta$) and relative density ($D_r$) being 15.8 kN/m$^3$, 38°, 26° and 54.3% in medium dense state and 15.4 kN/m$^3$, 34°, 22° and 34.35% in
IV. RESULTS AND DISCUSSIONS

In Fig.2, Fig.3 and Fig.4 data obtained from the present investigation and collected from the literature on the subject are compared with the predicted values of net uplift capacity of piles made with the assumption of linear, constant and parabolic variation of $K_s$ respectively. Fig.2 shows that the assumption of linear variation of the lateral earth pressure results in reasonable values of the predicted uplift capacity with 65% of the data having an error less than 30% on the safer side. Thus most of the predicted values (27 of 32) with the above assumption under estimate the uplift capacity while the remaining data (5 out of 28) marginally differ from the measured values on the higher side.

It is seen from Fig. 3 and Fig. 4 that the predictions are similar with constant and parabolic distribution of lateral earth pressure coefficient and most of the data (68%) are close to the ideal line having an error less than 30% that may be considered to be inherent and admissible in such experimental study. The data are scattered on either side of the ideal line. Thus, parabolic distribution appears to provide better predictions of the uplift capacity. To check if it is true under field condition also, the following study is undertaken. Ismael and Klyam (1979) and Vesic (1970) conducted field test to measure the uplift capacity of piles. Using the present approach the values of the uplift capacity of those piles for the given site conditions were estimated and compared with the measured values as follows.

Ismael and Klyam (1979) reported a full-scale pull out test of a cylindrical pier of diameter 1.2m and length of 6.4m embedded in a soil medium composed of compact fine to medium sand with some silt and traces of clay. The average standard penetration number (N) reported was 20 and $\phi=34^0$. Submerged unit weight was $11kN/m^3$. For theoretical prediction, $\delta=27^0$, i.e., 80% of the value of $\phi$ was used (Potyondy, 1961). The values of the predicted gross uplift capacity of the pier using linear, constant and parabolic variation of $K_s$ are $779 kN$, $1003 kN$ and $1057kN$ respectively. Out of all the three variations, the prediction with linear variation is closest to the measured value of $890kN$ with an error of 12.5% on the safer side while the corresponding value using the Meyerhof’s (1973) method is $953kN$ with an error of 7.1% on the unsafe side.

Vesic (1970) reported a full scale uplift test on a driven pile along the banks on the Ogeechee River. The relevant data for this uplift test are as follows:

- **Pile:** L=15.01m, d=0.453m
- **Soil:** Classification – primarily SW to SP, $D_s =87\%$, Location of ground water table: Approximately 1.8m below the ground surface. Average saturated unit weight ($\gamma_{wu}$) =19.96kN/m$^3$, Average effective unit weight ($\gamma'$) =10.15kN/m$^3$.

For theoretical prediction, corresponding to an $N$ value of 43, $\phi$ was taken to be $39^0$ (Peck et al, 1974) and the value of $\delta$ was chosen to be equal to $28^0$, i.e., 72% of $\phi$ (average for polished and rusted steel surface, Potyondy, 1961). The values of the predicted gross ultimate uplift capacity with the linear, constant and parabolic variation of $K_s$ are $1461kN$, $2055kN$ and $2192kN$ respectively. Here also the prediction is better with linear variation of $K_s$ in comparison of other two variations, the error (15% safe side) between the predicted and the measured value of $1539kN$ being the least while the corresponding value using the Meyerhof’s (1973) method is $1648kN$ with an error of 7% on the unsafe side. It has been seen in the earlier part of the discussion that parabolic distribution of variation of $K_s$
lateral earth pressure coefficient results in better prediction of uplift capacity of model test piles. The linear distribution gave results with majority of the test points producing conservative predictions. The error in predictions ranged from -15% to 69% with majority of the data with an error less than 40%. For Parabolic variation the error ranged from -76% to 53% and for majority of the data the absolute error fell below 30%. But, in this case 18 data out of 32 were found to be on unsafe side.

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Linear variation of $K_s$ with depth resulted in better prediction for field problems with the predictions lying on the safer side in comparison to the parabolic and constant variation of $K_s$ with the corresponding predictions being on the unsafe side. Thus, considering all the aspects it is concluded that net uplift capacity of pile if estimated by using the proposed simple method with linear variation of $K_s$ with depth would result in better and safe prediction and, as such, may be adopted in practice.

V. CONCLUSIONS

Based on the studies conducted and presented in this paper it is found that the proposed simple method with linear variation of lateral earth pressure with depth along the length of the pile, results in safe and better predictions of net uplift capacity of piles. The above conclusion has been demonstrated to be valid by comparing those with pile load test results (obtained both from laboratory and field test) reported in literature.

NOTATIONS

The following symbols are used in this paper.

- $d$ = Pile diameter
- $D_r$ = Relative density
- $K_s$ = Lateral earth pressure coefficient
- $K_a$ = Rankine active earth pressure coefficient
- $K_p$ = Rankine passive earth pressure coefficient
- $L$ = Embedded length of pile
- $P_{nu}$ = Net ultimate uplift capacity of pile
- $\phi$ = Angle of internal friction of the soil
- $\theta$ = Angle of failure surface with horizontal
- $\delta$ = Pile-soil friction angle
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Fig. 3 Predicted and measured value of uplift capacity: Constant $K_s$

Fig. 4 Predicted and measured value of uplift capacity: Parabolic variation of $K_s$. 

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