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Practical Determination of Geoidal Undulation and Geoidal Map of Part of Mubi, Adamawa State, Nigeria

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***Abstract* - Geoid is the reference surface for Orthometric height. This is the height preferred by users because of its relationship with Mean Sea Level which approximate the geoid. If the geoid is known, it can be used to produce the geoidal map. Geoidal Maps are essential tool in all spheres of our day- to- day activities most especially in geophysical studies, because they portrayed the geopotential configurations of any given place. However, as important as such maps have the same instrument for geospatial exploration purposes are lacking for many places including of Mubi north. The aim of the research is to determine the Geoidal Undulation and produce the Geoidal Map of part of Mubi North Local Government Area Adamawa State, Nigeria. This work involves determination of Geoidal undulation for production of Geoidal map of part of Mubi north, Adamawa state. Single Frequency Global Positioning System and Geodetic Level (Wild N3) instruments were used to obtain ellipsoidal and Orthometric heights of the areas. The adjusted Orthometric heights obtained from Geodetic Levelling and the Ellipsoidal heights which is part of the geodetic coordinates obtained from GNSS were post processed using Leica Geo-Office Software. The Geoidal lines and Digital Geoidal Model (DGM) were created using Surfer 7 Software. The Microsoft Office Excel was used to deduce the Ellipsoidal height, Orthometric height and Geoidal Undulations, for production of Geoidal Map of the study Area. The statistical analysis of the result met the Geodetic specifications and therefore can be used for any work required the use of geoidal undulation in the study area.**

Keywords: Geoid, Geoidal Map, Global Positioning System (GPS), Orthometric height and Ellipsoidal height.

I. INTRODUCTION

1.1 Geoid

Geoid comes from the word "geo" which literarily means earth-shaped. Geoid is an empirical approximation of the figure of the earth (minus topographic relief). It is defined as the equipotential surface of the earth's gravity field which best fits, in the least square sense; the mean sea level (Deakin, 1996). Geoidal height (or Geoid undulation) can be defined as the separation of the reference ellipsoid with the Geoid surface measured along the ellipsoidal normal. (Erol and Celik, 2004). Geoid is also defined as the equipotential surface of the earth's gravitation field which coincides with the sea surface, in the absence of disturbing factors like tsunamis, ocean currents, salinities, wind, and so on (Vaníček and Krakiwsky, 1986).

Geoid is an equipotential surface of the gravity field which can be approximated by an idealized Mean Sea-Level. It is used to determine the shape of the earth in Geodesy. Geoid also plays an important role in many other geosciences. It is a fundamental reference surface for surveying and mapping from which the natural elevation of a point is measured. It is also an equipotential surface that manifests the true distribution of masses in the interior of the earth; and it is use by geophysicist to obtain gravity anomalies in order to learn about the density variations of the earth interior (Anthony, 2011). Aleem (2013a) listed a number of applications of Geoid.

Though, geoid is much smoother than the actual earth surface, unlike the ellipsoid, it is still too complicated to serve as the computational surface on which to solve geometrical problems why complied but it is suitable as a vertical datum. Vertical Datum is a reference surface for elevation of points. When height is determined with reference to the Geoid, the height so determined is called "Orthometric height". The datum adopted for Orthometric height is the Geoid. The benchmark elevations used in surveying and engineering are referenced to the surface of geoid and are called orthometric heights or Mean Sea Level (MSL) elevation. Many Surveyor's and geodetic applications are interested in orthometric height, which is the height above the geoid, not above the ellipsoid (Featherstone et al., 1998). The transformation between the two heights could be obtained directly knowing the geoid undulations (Fotopoulos, 2003)

Geoid determination has been, and still is, one of the main objectives of geodesy. Determination of geoid has been one of the main research areas in Science of Geodesy for decades. According to wide spread use of Global Positioning System (GPS) in geodetic applications, great attention is paid to the precise determination of local/regional geoid with an aim to replace the geometric levelling, which is very onerous measurement with GPS surveys. GPS technique provides the surveyor with three-dimensional coordinates including ellipsoidal heights (h) with respect to its reference geocentric ellipsoid. That is World Geodetic System 1984 (WGS84). As in GPS measurements, geodesists have chosen an oblate ellipsoid of revolution, flattened at the poles, to approximate the geoid in order to simplify survey data reduction and mapping. However, most surveying measurements are made in relation to the geoid, which is the equipotential surface of the earth gravitational field, not ellipsoid because the equipment is aligned with the local gravity vector, which is perpendicular to the geoid surface, usually through the use of a spirit bubble (Featherstone, 1998).

Geoid is an equipotential surface of the earth that coincides with the undisturbed MSL. Therefore one might say that it describes the actual shape of the earth. Geoid is also the reference surface for most height networks since levelling gives the heights above the geoid. Besides this geometrical aspect of the geoid, it also related to the gravitational field of the Earth. It is actually possible to calculate the gravity accelerations everywhere outside the earth through analytical continuation if we know the gravity at the geoid. The geoid is usually described by the separation between itself and a mathematical surface. This separation is called geoidal height or geoidal undulation or Geoid separation (Aleem, 2013b). The mathematical surface is a biaxial ellipsoid with the dimension chosen so that it describes the Earth's shape as closely as possible and it is the same ellipsoid that is used as a reference surface for measuring geodetic latitude and geodetic longitude.

The reason to use the ellipsoid instead of the geoid for the geodetic coordinates is that it is too difficult to carry out the necessary calculations on such an irregular surface as the geoid. The Geoid is a mathematical model of the earth measured with gravity that corresponds with the mean ocean surface level on the earth- such as if the water were extended over the land. The surface is highly irregular. However, there are different local geoids that are used to get the most accurate mathematical model possible for use in measuring vertical distances. The geoidal undulation varies globally between ±110 m, when referred to the GRS 80 ellipsoid. The geoid model will give geoidal undulation at every point of observation and the fundamental relationship, to first approximation, that binds the ellipsoidal heights obtained from GPS measurements and heights with respect to a vertical (local) datum established from conventional spirit levelling (Heiskanen and Moritz, 1967; Featherstone et al., 1998; Olaleye et al,. 2011). Ellipsoidal heights can't satisfy the aim in practical surveying, engineering or geophysical applications as they have no physical meaning and must be transformed to orthometric heights (H), which are referred to geoid, to serve the geodetic and surveying applications. To accomplish this transformation between the ellipsoidal heights and orthometric heights it is just to know the undulation of geoid from the ellipsoid (N).

Basically a WGS84 ellipsoidal height (h) is transformed to an orthometric height (H) by subtracting the geoid-WGS84-ellipsoid separation (N), which is called geoid undulation. The fundamental relationship, that binds the ellipsoidal heights obtained from GPS measurements and heights with respect to a vertical (local) datum established from conventional spirit levelling and gravity data is given by numerous authors as (Heiskanen and Moritz, 1967; Featherstone et al., 1998)

$$H = h - N \qquad (1)$$

From Equation 1, Geoidal height (N) can be computed which for points will be plotted on map in form of spot heights. The Geoid height on each point can be interpolated to produce the Geoidal map.

$$N = h - H \qquad (2)$$

Where:

N = Geoidal height or Geoidal undulation

h = Ellipsoidal height with respect to a reference ellipsoid

H = Orthometric height based on geoid

The geometrical relationship between the triplets of the height types is illustrated below: Source: (Aleem, 2013a)

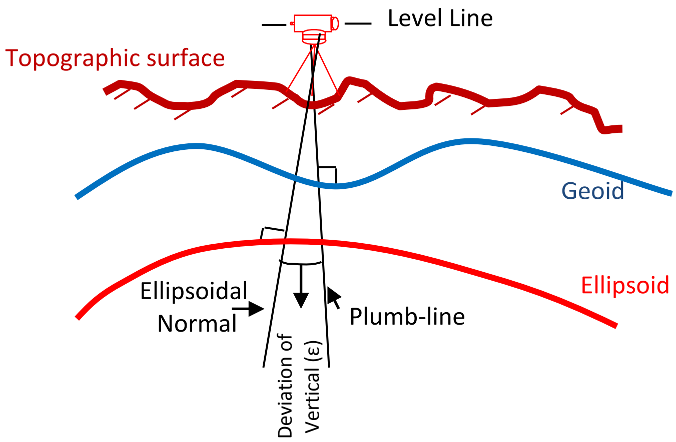

Figure 1: The Relationship between Orthometric, Geoid and Ellipsoidal Heights

II THE STUDY AREA

The study area of the research is located in Mubi North Local Government area of Adamawa State, Nigeria. The area is located geographically between latitude 10^{0} 30'N and10^{0} 05'N and longitude 13^{0} 10' E and 13^{0} 30' E

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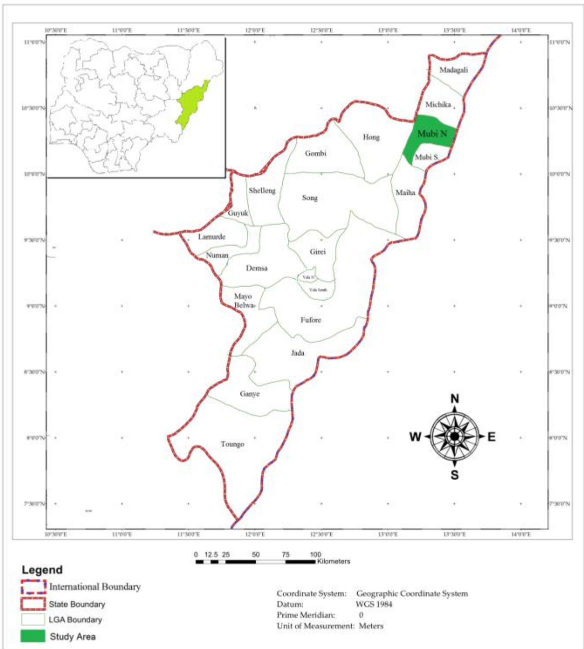

Figure 2: Adamawa State Map Showing Mubi North

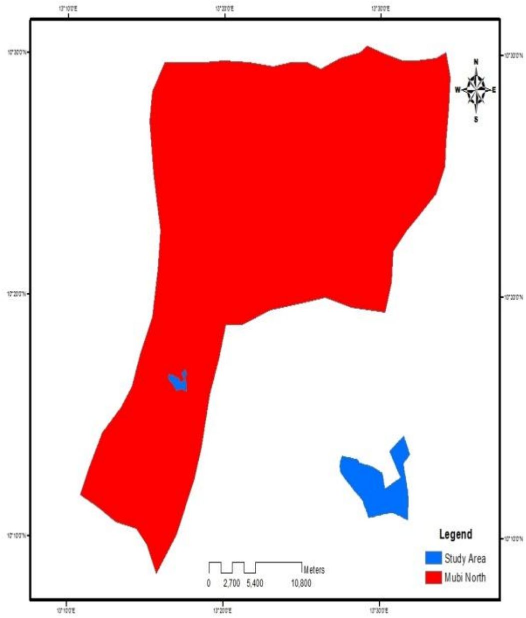

Figure 3: Mubi North Map Showing Study Area

III. METHODOLOGY

3.0 General

The basic data include Ellipsoidal heights which was acquired using Single Frequency Global Positioning System (GPS) Leica SR 20 and use of the Geodetic level (Wild N3) to determine the Orthometric heights. Geoidal maps are produced using a number of stages which includes field data acquisition, field data processing, processed result analysis, final result presentation and storage.

3.1 Data Acquisition

The data acquired includes latitude (ϕ), longitude (λ) and ellipsoidal heights (h) of points, using Single Frequency Global Positioning System (GPS) Receivers. This set of data was obtained from the site by means of direct field observation. The second quantity orthometric heights (H) of these points were acquired from Geodetic levelling using Wild N3 geodetic level. A total number of 101 points and 17 (bench marks) were occupied. This becomes necessary due to the topographical nature of the area.

3.1.2 Equipment Used

The equipment needed for the exercise are:

The Hardware:

1. Single Frequency Global Positioning System (GPS) Leica SR 20 and its accessories
2. Geodetic Level (Wild N3) and its accessories

The Software:

1. Surfer software
2. Microsoft Office Excel
3. Leica geo-offices

3.20 Observation:

Observational procedures were follow using the guideline for the control of geodetic surveying in Nigeria as published by the Surveyors Council of Nigeria (SURCON). The field work started with reconnaissance/

3.2.1 Reconnaissance

The basic principle of surveying requires a thorough planning and cursory examination of the area to be surveyed. Reconnaissance must be done before proceeding on the actual field work in any survey job. In this research work, reconnaissance was carried out in two phases; office and field reconnaissance and the following factors were considered namely: the nature of the terrain, the inter-visibility of the stations in geodetic levelling, sky visibility in GPS observation, the suitability of the station and the method to be adopted, as well as the general information available about the task to be carried out

3/2/2 Field Reconnaissance: This is one of the most important aspects of survey and must be undertake before work commences on site. The aim of the reconnaissance is for the surveyor to locate suitable positions. When carrying out reconnaissance, the surveyor should walk around the site keeping in mind the Purpose and requirements of the survey.

3.2.3 Office Reconnaissance: This refer to the process of obtaining plan map of the area, from the office or authority concerned to enable one know the position and the area to be survey and also collection of coordinates of control points and benchmarks

3.30 Data Processing

Two set of data are involved in this work: the one obtained by Single Frequency GPS instrument and the one obtained by Geodetic Level. The GPS observation was post

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processed using Leica geo-offices software and the final coordinates and the heights of the study area were determined as shown in Table 1. On the other hand, Geodetic Levelling observations were computed sample of the final results were as shown in Table 2.

Table 1: Observed Latitudes, Longitudes and Ellipsoidal Height using Single Frequency (GPS)

Stations	Latitudes (ϕ) (Dec. Min. Sec)	Longitudes (λ) (Dec. Min. Sec)	Ellipsoidal Height (h) (M)
AKF 03	10 16 48.9887	13 17 17.4456	610.493
AKF 02	10 16 41.9542	13 17 21.5565	612.860
AKF 01	10 16 49.2961	13 17 35.1107	606.449
FPM 01	10 16 32.7309	13 17 34.1378	611.098
FPM 02	10 16 16.1142	13 17 40.4336	622.000
FPM 03	10 16 1.9140	13 17 41.2380	618.524
FPM 04	10 16 3.6405	13 17 14.1126	590.300
FPM 05	10 16 3.2912	13 16 49.0172	600.600
FPM 06	10 16 24.4535	13 16 40.4612	597.678
FPM 07	10 16 34.1576	13 16 29.0256	578.700
FPM 08	10 16 47.0107	13 16 27.8783	605.984
FPM 09	10 16 43.5946	13 16 46.5200	610.069
FPM 10	10 16 35.0793	13 17 1.6528	598.800
FPM 11	10 16 30.4124	13 17 11.5367	584.600
FPM 12	10 16 20.0883	13 17 16.3902	578.000
FPM 13	10 16 24.0449	13 17 26.0632	593.900
FPM 14	10 16 27.0971	13 17 30.8775	617.500

Table 2: Sample Geodetic Levelling

TABLE SAMPLE GEODETIC LEVELLING FIELD SHEET

ASPECT	BACK STAFF				STAFF NO	FORWARD STAFF				STAFF NO.	REMARK
	STADIA HAIRS		MIDDLE HAIRS			MIDDLE HAIRS		STADIA HAIRS		TP 01	
	LEFT	RIGHT	LEFT	RIGHT	**AKF 03**	LEFT	RIGHT	LEFT	RIGHT		
UPPER	5.24760	2.22760	5.00761	1.98762	50m	4.00761	0.98761	4.23760	1.23761	50m	100m
LOWER	4.77760	1.74760	5.00761	1.98761		4.00761	0.98761	3.76761	0.74761		
SUM	10.0252	3.97520	10.01522	3.97523		8.01522	1.97522	8.00521	1.98522		
DIFF.	0.47000	0.48000	0.00000	0.00001		0.00000	0.00000	0.46999	0.49000		
MEAN	5.01261	1.98760	5.00761	1.98762		4.00761	0.98761	4.00261	0.99261		
UPPER	5.09762	2.08762	4.86765	1.84765	TP 01	4.22762	1.21762	4.46765	1.44765	TP 02	
LOWER	4.63762	1.60762	4.86765	1.84765	50m	4.22762	1.21762	3.98762	0.96765	50m	200m
SUM	9.73524	3.69524	9.73530	3.69530		8.45524	2.43524	8.45527	2.41530		
DIFF.	0.46000	0.48000	0.00000	0.00000		0.00000	0.00000	0.48000	0.48000		
MEAN	4.86762	1.84762	4.86765	1.84765		4.22762	1.21762	4.22764	1.20765		
UPPER	4.85025	1.83025	4.72025	1.71025	TP 02	4.36025	1.35025	4.50022	1.48022	**AKF 02**	
LOWER	4.59025	1.59023	4.72022	1.71024	25m	4.36022	1.35023	4.22023	1.21023	25m	**250m**
SUM	9.44050	3.42048	9.44047	3.42049		8.72047	2.70048	8.72045	2.69045		
DIFF.	0.26000	0.24002	0.00003	0.00001		0.00003	0.00002	0.27999	0.26999		
MEAN	4.72025	1.71024	4.72024	1.71025		4.36024	1.35024	4.36022	1.34523		

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IV. RESULTS AND DISCUSSIONS

4.0 Results The results of determination of Geoidal Undulation for Production of Geoidal Map of part of Mubi north are hereby presented. The result includes: final computed orthometric heights of the study area as shown in Table 3,

Table 3: Sample Typical Field Sheet of the Geodetic Levelling

STN	LEFT RIGHT	BACK-SIGHT	FORE-SIGHT	RISE	FALL	MEAN		REDUCED LEVEL	DIST.	RMK
						RISE	FALL			
AKF 03								**593.868**	50m	
TP 1		5.00761 1.98762	4.00761 0.98701	1.00000 1.00061		1.00031		594.86831	100m	
TP 2		4.86765 1.84765	4.22762 1.21762	0.64003 0.63003		0.63503		595.50334	200m	
AKF 02		4.72024 1.71025	4.36024 1.35024	0.36000 0.36001		0.36001		**595.86335**	**250m**	
TP 3		4.33020 1.33019	4.88020 1.86019		0.55000 0.53000		0.54000	595.32335	100m	
TP 4		4.26017 1.24014	4.91016 1.89017		0.64999 0.65003		0.65001	594.67334	200m	
TP 5		4.16019 1.14018	4.96018 1.94020		0.79999 0.80002		0.80001	593.87333	300m	
TP 6		4.10020 1.09020	4.76020 1.74020		0.66000 0.65000		0.65500	593.21833	400m	
AKF 01		4.18020 1.16019	4.85020 1.83019		0.67000 0.67000		0.67000	**592.54833**	**470m**	
TP 7		4.87514 1.85513	4.16514 1.15514	0.71000 0.69999		0.70500		593.25333	100m	
TP 8		4.96494 1.94494	3.94494 0.94494	1.02000 1.00000		1.01000		594.26333	200m	
TP 9		5.07497 2.05496	4.15497 1.13496	0.92000 0.92000		0.92000		595.18333	 300m	
TP 10		4.79491 1.77492	4.22492 1.21492	0.56999 0.56000		0.56500		595.74833	380m	
TP 11		4.25491 1.23492	4.31491 1.29492		0.06000 0.06000		0.06000	595.68833	410m	

The field sheet was reduced and Reduced levlled of all the point were computed using Rise and Fall method. Alternately, if the height of Instruments were taken at every set up station height collimation method can equally be used Orthometric Correction were applied using the method suggested by Aleem (2013a). The Table 4 below shows the Orthometric Heights of the Stations.

Table 4: Orthometric Heights of the Stations

Stations	**Orthometric Height**
AKF 03	593.86800
AKF 02	595.86335
AKF 01	592.54833
FPM 01	596.46834
FPM 02	602.57463
FPM 03	605.30961
FPM 04	604.85963
FPM 05	586.85964
FPM 06	583.63464
FPM 07	582.92463
FPM 08	592.05462
FPM 09	593.08460
FPM 10	595.82457
FPM 11	597.62955
FPM 12	600.84456
FPM 13	599.11456
FPM 14	598.28456

The Universal Traverse Mercator (UTM) Coordinates of the stations were also processed. The Table 5 below shows the list of UTM Coordinates of the Stations

Table 5: UTM Coordinates of the Stations

Stations	Northing's (N) (M)	Easting's (E) (M)	Ellipsoidal Heights (h) (M)
AKF 03	1136900.80819	312528.85184	610.493
AKF 02	1136684.00256	312652.79272	612.860
AKF 01	1136907.39031	313066.44242	606.449
FPM 01	1136398.57432	313034.13089	611.098
FPM 02	1135887.00100	313223.00031	622.000
FPM 03	1135450.56396	313245.16287	618.524
FPM 04	1135508.00131	312419.99912	590.300
FPM 05	1135501.34749	311656.26807	600.600
FPM 06	1136152.96782	311399.38686	597.678
FPM 07	1136453.00107	311052.99916	578.700
FPM 08	1136848.10972	311020.20946	605.984
FPM 09	1136740.10264	311586.90949	610.069
FPM 10	1136475.99995	312045.99872	598.800
FPM 11	1136331.00047	312346.00023	584.600
FPM 12	1136012.99924	312492.00142	578.000
FPM 13	1136132.99994	312787.00074	593.900
FPM 14	1136226.00077	312933.99947	617.500

The Geoidal Undulations of the stations were computed. The Table 6 below shows the list of Geoidal Undulations of the Stations

Table 6: Computed Geoidal Undulations.

Stations	Ellipsoidal Heights (h) (m)	Orthometric Heights (H) (m)	Geoid Heights (N) (m) N = h – H
AKF 03	610.493	593.86800	16.625
AKF 02	612.860	595.86335	16.997
AKF 01	609.449	592.54833	16.901
FPM 01	613.098	596.46834	16.630
FPM 02	619.000	602.57463	16.425
FPM 03	621.524	605.30961	16.214
FPM 04	621.300	604.85963	16.440
FPM 05	603.600	586.85964	16.740
FPM 06	599.678	583.63464	16.043
FPM 07	599.300	582.92463	16.375
FPM 08	608.984	592.05462	16.929
FPM 09	610.069	593.08460	16.984
FPM 10	611.850	595.82457	16.025
FPM 11	613.630	597.62955	16.000
FPM 12	617.001	600.84456	16.156
FPM 13	615.900	599.11456	16.785
FPM 14	614.500	598.28456	16.215

The results of Ellipsoidal height in Table and Orthometric Heights in Table 4 were used to plot the Chart in Figure to shows the relationship between Ellipsoidal and Orthometric Heights

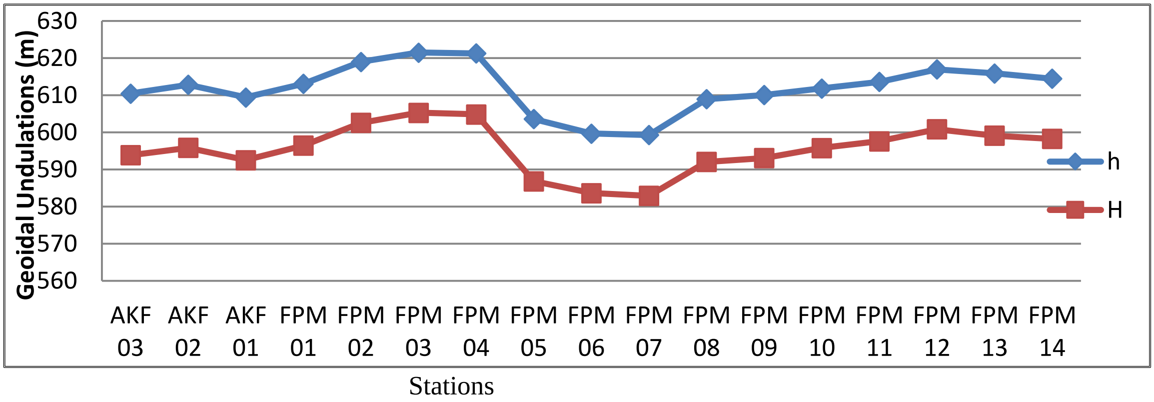

Figure 4: Relationship between Ellipsoidal and Orthometric Heights of part of Mubi North.

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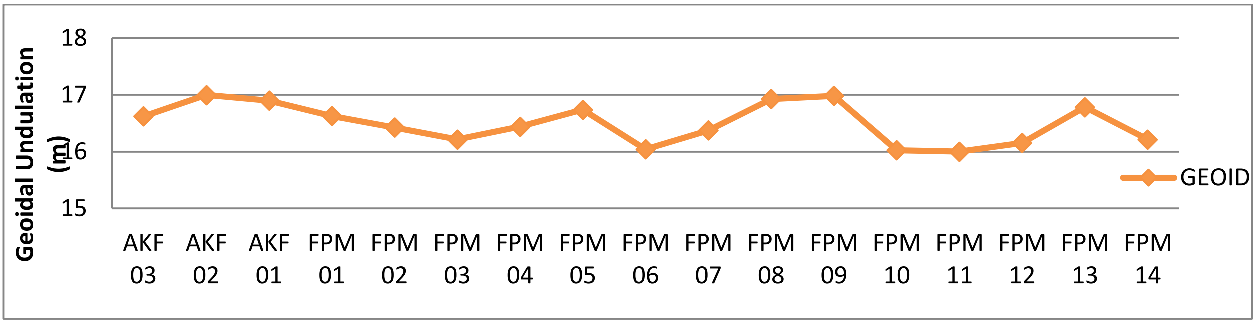
The 'h' represents the Ellipsoidal Height and the 'H' represents the Orthometric Height of the study area as shown in Figure 4.

The results of Geoidal Undulation Table 6 was used to plot the Figure 5 below to show the profile of the study area



Stations

Figure 5: Chart showing the Geoidal Profile of part of Mubi North.

Digital Geoidal Model (DGM)

The results of Geoidal Undulation Table 6 and the Geodetic coordinates from Table 1 were used to create Digital Geoidal Model (DGM) Figure 6 of the study area using Surfer Software

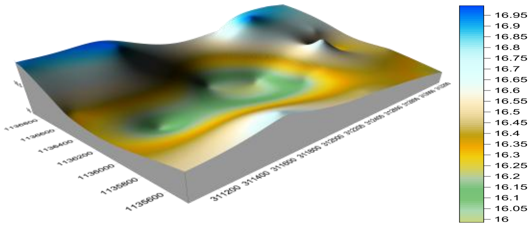


Figure 6: Digital Geoidal Model of part of Mubi North

Geoidal Map of part of Mubi North

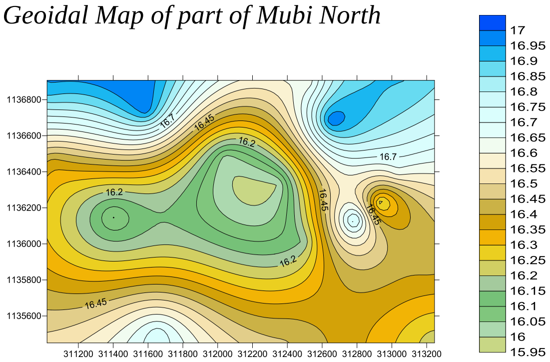


Figure 7: Geoidal Map of part of Mubi North

Figure 7 shows the Geoidal Map of the study area produced. Geoidal Undulation values at different points are used for producing the Geoidal Map. Therefore, any line on a Geoidal Surface is an imaginary line drawn on the Geoidal Map to connect points of the same Geoidal Height on, above or below the Geoidal surface.

4.2 Discussion of Results

The adjusted Orthometric heights obtained from Geodetic Levelling are shown on Table 4. The coordinates and Ellipsoidal heights obtained were post processed by the Leica Geo-offices software and the final adjusted coordinates and heights presented on Table 1. Heights obtained from the two methods were Ellipsoidal height and Orthometric height and Geoidal heights were tabulated Table 6. The Geoidal height profile were plotted and shown in figures 4 and 5 respectively. Digital Geoidal Model was equally produced as shown in figure 6 Geoidal map of the study area is as shown in Figure 7. The trends of the results of orthometric and ellipsoidal heights followed the same pattern. This is an indication that the two height systems are true representation of the same terrain. The Geoidal map and the Geoidal Model slope towards the same direction of the sea. Through, this was expected which is an indication that Geoidal height and Orthometric are natural height systems.

V. SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

In carrying out Determination of Geoidal undulation for production of Geoidal Map of part of Mubi North, the coordinates and the ellipsoidal and orthometric heights were determined with the aid of Single Frequency (GPS) instruments and Geodetic Level Wild N3 instruments respectively. The heights determined by Geodetic Levelling were reduced. The Single Frequency (GPS) coordinates and heights determined were post processed using the Leica Geo- Offices software and the final adjusted coordinates and heights are determined. Heights of the study area were obtained using orthometric and ellipsoidal heights. Geoidal undulations were determined and used to plot the Geoidal map of the study area. The Digital Geoidal Model was also created.

5.2 Conclusion

Data were acquired for Orthometric and Ellipsoidal height using Single Frequency (GPS) instruments and Geodetic Level Wild N3 instruments in part of Mubi North Local Government Area; Geoidal Undulations were computed and used for the production of Geoidal map of the area. The Geoidal Model was produced as 3D surface Geoidal Model for the study area.

5.3 Recommendations

In view of the foregoing results, it is necessary therefore to recommend as follows:

i. The research should be repeated using observations with Differential Global Positioning System in full static mode with more time spent on each station to see if the accuracy of the result could be improved.
ii. In order to attain the vision of Geospatial development in Nigeria the Nigeria government should make some efforts towards the production of a National Geoid Model through the office of Surveyor General of Federation in order to keep in pace with the other developing countries.
iii. Engineering firms as well as survey firms should endeavour to determine the Orthometric height within their project areas so that the height information needed for their projects will be adequate.

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