Power System State Estimation by use of WLS with Phasor Measurement Unit

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Abstract

The traditional techniques are too bulky and iterative in nature for system power flow measurement. The new technique of Phasor Measurement Unit is now applied for measurement of bus voltage and current. The state estimator by use of full weighted least square is nonlinear in nature, but with PMUs and traditional technique it will improves the accuracy of the measurement with minimum use of the iteration process. In this paper the formation of measurement by using WLS state estimation and PMU are incorporated with use of traditional technique will be investigated. Several parameters in different cases are tested by launching of PMUs on specific buses and their effect on accuracy. The impact of PMU on bus voltage, bus current and real power flows are illustrated. The parameter assessment obtained on IEEE 6 bus system and IEEE 9 bus system will be discussed.

Keywords – *full weighted least square (WLS) state; state estimation; traditional technique; phasor measurement units.*

1. Introduction

Phasor Measurement Unit is related with the electrical signal and their respective waves on a grid to measure the effective health of the system. The phasor value is a complex quantity which includes both the magnitude and phase angle of the sine waves. The Phasor measurements on a bus at the same time are called "Synchrophasor". The PMU [4] [8] is a device that allows us for such continuous signal measurement. In power system engineering, traditionally we use various devices for measurement purposes on every substation. The recently developed PMU [6] [9] [11] will help in deciding to stall such devices at proper location:-

- Accurate and comprehensive planning.
- Better congestion tracking,
- Visualization and advanced warning systems,
- Information sharing over a wide region,
- Improvements of System Integrity Protection Schemes (SIPS),
- Grid restorations,
- More reliable,
- Efficient,
- Cost effective grid operation.

The traditional technique of measurement is too bulky and iterative in nature for measurement of instantaneous current and instantaneous voltage on a bus of power system. The full weighted least square state [1] [7] [10] incorporation with first order Taylor series is become a linear equation. The WLS state estimation equations with PMU measurement are linear in nature. Some research already conducted in formulation of a relation between full weighted least square state and PMUs. The traditional approach of parameter measurement will treat PMU as additional computational burden on measurement and calculation. The problem of finding optimal placement of PMU devices for state estimation of power system is investigated in the literature. This paper reproduces the accuracy of measurement with or without using PMU on state estimation parameters. The system state estimation with traditional technique without using any PMU as case 1 and with PMU only as case P is discussed.

2. Full WLS State Estimation Technique

Consider the set of measurements given by the vector z :

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ . \\ . \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, x_3, \ldots\ldots x_n) \\ h_2(x_1, x_2, x_3, \ldots\ldots x_n) \\ h_3(x_1, x_2, x_3, \ldots\ldots x_n) \\ . \\ . \\ h_m(x_1, x_2, x_3, \ldots\ldots x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ e_m \end{bmatrix} \qquad (1)$$

Where:

$$h^T = [h_1(x), h_2(x), h_3(x), ...h_m(x)] \qquad (2)$$

$h_i(x)$ is the nonlinear function relating measurement i to the state vector x

$x^T = [x_1, x_2, x_3 .... x_n]$ is the system state vector

$e^T = [e_1\ e_2\ e_3 .... e_m]$ is the vector of measurement errors.

The WLS estimator [1], [2] will minimize the following objective function:

$$J(x) = [z - h(x)]^T R^{-1} [z - h(x)] \qquad (3)$$

For economic value of the objective function, the first-order optimality conditions have to be satisfied. These can be expressed in compressed form as follows:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \qquad (4)$$

The non-linear function g(x) can be prolonged into its Taylor series around the state vector x^k neglecting the advanced terms.

$$g(x) = g(x^k) + G(x^k)(x - x^k) + ... = 0 \qquad (5)$$

The Gauss-Newton technique is used to resolve the above equation:

$$x^{k+1} = x^k - [G(x^k)]^{-1} . \, g(x^k) \qquad (6)$$

where, k is the iteration index and x^k is the solution vector at iteration k . G(x) is called the gain matrix, and expressed by:

$$G(x) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) R^{-1} H(x^k) \qquad (7)$$

$$g(x^k) = -H^T(x^k) R^{-1} [z - h(x^k)] \qquad (8)$$

Usually, the gain matrix is decomposed and sparse into its triangular factors. At each iteration k, the following sparse linear set of equations are solved using forward/backward substitutions, where

$$\Delta x^{k+1} = x^{k+1} - x^k$$

$$[G(x^k)] \, \Delta x^{k+1} = H^T(x^k) R^{-1} [z - h(x^k)] = H^T(x^k).R^{-1} \Delta z^k \qquad (9)$$

These iterations are going on until the maximum variable difference satisfies the condition, ' Max $|\Delta x^k| < \varepsilon$ '.

3. Traditional Technique

The traditional technique [1] of measurement is mainly consider relation of power injection or power flow with respect to line voltage and line current are as

$$I_{ij} = \sqrt{(g_{ij}^2 + b_{ij}^2)(V_i^2 + V_j^2 - 2V_i V_j \, Cos \, \theta_{ij})}$$
$$= \frac{\sqrt{P_{ij}^2 + Q_{ij}^2}}{V_i} \qquad (10)$$

Real and reactive power injection at bus i can be expressed by,

$$P_i = |V_i| \sum_{j=1}^{N} |V_j| (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \qquad (11)$$

$$Q_i = |V_i| \sum_{j=1}^{N} |V_j| (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \qquad (12)$$

Real and reactive power flow from bus i to bus j are,

$$P_{ij} = |V_i|^2 (g_{si} + g_{ij}) - |V_i||V_j| (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}) \qquad (13)$$

$$Q_{ij} = -|V_i|^2 (b_{si} + b_{ij}) - |V_i||V_j| (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij}) \qquad (14)$$

So the structure of the measurement of Jacobian H will be as

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix} \qquad (15)$$

4. WLS with Traditional Technique

A PMU will be able to measure multiple current with one voltage phasors. The transmission line usually formed as pie network due to their special benefit on system parameters.

Fig. 1 shows an example of 4-bus system which has single PMU at bus 1. It has one voltage phasor measurement and three current phasors measurement, namely $V_1 \angle \theta_1$, $I_1 \angle \delta_1$, $I_2 \angle \delta_2$ and $I_3 \angle \delta_3$

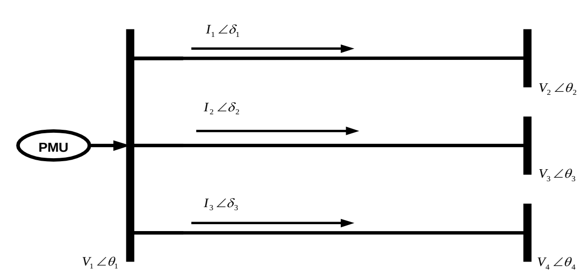

Fig 1. Single PMU Measurement Model

If we define y_{shunt} as the shunt admittance and y as the series admittance, the current phasor measurements can be framed in rectangular coordinates as shown in Fig 2.

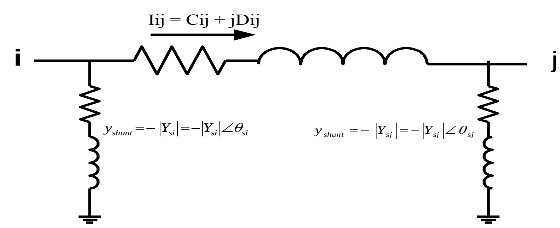

Fig 2. Transmission Line Model

The expressions for complex quantity C_{ij} and D_{ij} are:

$$C_{ij} = |V_i Y_{si}|\cos(\delta_i + \theta_{si}) + |V_j Y_{ij}|\cos(\delta_j + \theta_{ij}) - |V_i Y_{ij}|\cos(\delta_i + \theta_{ij}) \quad (16)$$

$$D_{ij} = |V_i Y_{si}|\sin(\delta_i + \theta_{si}) + |V_j Y_{ij}|\sin(\delta_j + \theta_{ij}) - |V_i Y_{ij}|\sin(\delta_i + \theta_{ij}) \quad (17)$$

where, the state vector is given as:

$$x = [|V_1|\angle 0^0, |V_2|\angle\delta_2, |V_3|\angle\delta_3 .... |V_N|\angle\delta_N]^T \quad (18)$$

The Jacobian H measurement corresponding to the real and reactive parts of the current phasors is:

$$\frac{\partial C_{ij}}{\partial V_i} = |Y_{si}|\cos(\delta_i + \theta_{si}) - |Y_{ij}|\cos(\delta_i + \theta_{ij}) \quad (19)$$

$$\frac{\partial C_{ij}}{\partial V_j} = |Y_{ij}|\cos(\delta_j + \theta_{ij}) \quad (20)$$

$$\frac{\partial C_{ij}}{\partial \delta_i} = -|V_i Y_{si}|\sin(\delta_i + \theta_{si}) + |V_i Y_{ij}|\sin(\delta_i + \theta_{ij}) \quad (21)$$

$$\frac{\partial C_{ij}}{\partial \delta_j} = -|V_j Y_{ij}|\sin(\delta_j + \theta_{ij}) \quad (22)$$

$$\frac{\partial D_{ij}}{\partial V_i} = |Y_{si}|\sin(\delta_i + \theta_{si}) - |Y_{ij}|\sin(\delta_i + \theta_{ij}) \quad (23)$$

$$\frac{\partial D_{ij}}{\partial V_j} = |Y_{ij}|\sin(\delta_j + \theta_{ij}) \quad (24)$$

$$\frac{\partial D_{ij}}{\partial \delta_i} = |V_i Y_{si}|\cos(\delta_i + \theta_{si}) - |V_i Y_{ij}|\cos(\delta_i + \theta_{ij}) \quad (25)$$

$$\frac{\partial D_{ij}}{\partial \delta_j} = |V_j Y_{ij}|\cos(\delta_j + \theta_{ij}) \quad (26)$$

The measurement vector z contains δ, C_{ij}, and D_{ij} as well as the power injections, power flows and voltage magnitude measurements.

$$z = [P_{inj}{}^T, Q_{inj}{}^T, P_{flow}{}^T, Q_{flow}{}^T, |V|^T, \delta^T, C_{ij}{}^T, D_{ij}{}^T]^T \quad (27)$$

Usually, measurements obtained from PMUs are more accurate and precise as compared to the traditional measurements. Therefore, measurements done with the help of PMUs are expected to generate more accurate and precise result as estimated by traditional technique.

5. State Estimation with PMUs

The state vector and measurement data can be articulated in rectangular coordinates [1]. The voltage measurement ($V = |V|\angle\theta$) can be expressed as (V = E + jF), and the current measurement can be expressed as (I = C + jD). Where (g_{ij} +jb_{ij}) is the series admittance of the line and (g_{si} + jb_{si}) is the shunt admittance of the transmission line. Line current flow I_{ij} can be expressed as a linear function of voltages.

$$\begin{aligned} I_{ij} &= [(V_i - V_j)\times(g_{ij} + jb_{ij})] + [V_i \times (g_{si} + jb_{si})] \\ &= V_i \times [(g_{ij} + jb_{ij}) + (g_{si} + jb_{si})] - V_j \times (g_{ij} + jb_{ij}) \end{aligned} \quad (28)$$

The measurement vector z is expressed as z = h(x) + e, (where x is a state vector, h(x) is a matrix of the linear equations, and e is an error vector). In rectangular coordinates:

z = (Hr + jHm)(E + jF) + e (29)

where, H = Hr + jHm , x = E + jF and z = A + jB.

A and B are expressed by:

A = Hr × E – Hm × F (30)

B = Hm × E + Hr × F (31)

In matrix form,

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} Hr & -Hm \\ -Hm & Hr \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} + e \quad (32)$$

Then, the estimated value $\hat{x} = \hat{E} + j\hat{F}$ can be obtained by solving the linear equation below:

$$\Delta\hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z = G^{-1} H^T R^{-1} \Delta z \quad (33)$$

If we define the linear matrix H_{new} as

$H_{new} = \begin{bmatrix} Hr & -Hm \\ -Hm & Hr \end{bmatrix}$, then the above equation can be rewritten by:

$$\hat{x} = \begin{bmatrix} \hat{E} \\ \hat{F} \end{bmatrix} = (H_{new}{}^T R^{-1} H_{new})^{-1} H_{new}{}^T R^{-1} \begin{bmatrix} A \\ B \end{bmatrix} \quad (34)$$

Hence, the equation for rectangular formed variable $\hat{x}$ can be given by the rectangular forms of H matrix and z vector. They are all real numbers.

In respect of the system reliability and accuracy, PMU can deliver more accurate measurement data. Several cases to be tested with PMUs added to the traditional measurement set. The simulations and analysis of different cases are as shown in Table 1 are done with several IEEE bus systems in the next section.

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Table 1. Different cases PMU addition in IEEE System

Cases	Measurements
1	Traditional with No PMUs
P	Only PMUs

6. Simulation Results

For examine the system accuracy with or without PMU on system variables, some cases are tested with the help of MATLAB simulink software. The traditional method parameters are tested without PMU and then their performance to be tested in comparison with the PMU as per the Table will shown location of PMU at Bus : -

Table 2. PMU Locations for Each IEEE System

Type of System	PMU locations at Bus
IEEE 6 System	Bus 2
IEEE 9 System	Bus 2

The circuit diagram will be shown as in Fig.3 and Fig.4 for IEEE 6 and IEEE 9 bus system respectively.

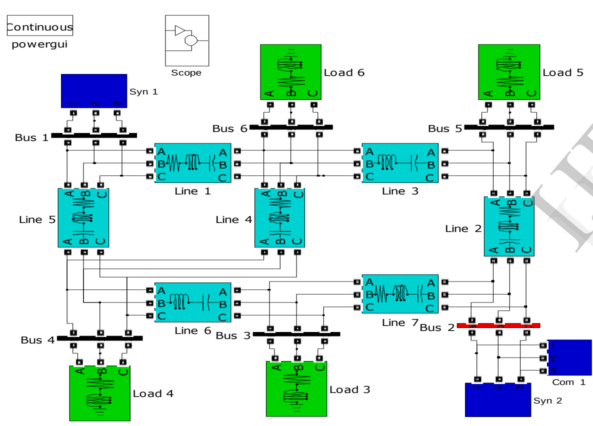

Fig 3. IEEE 6 Bus System

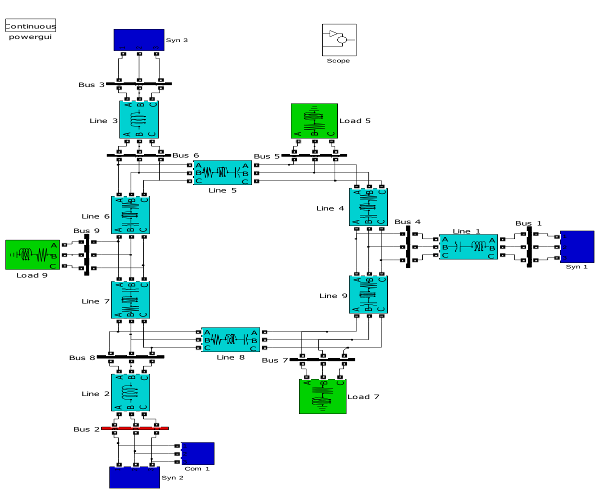

Fig 4. IEEE 9 Bus System

In this segment, IEEE bus systems as IEEE 6 bus system [3] and IEEE 9 bus system [5] are tested with their respective cases to find out the consequences of the PMUs to the precision of the estimated variables.

The parameters measured are voltage, current and real power (flow & injected) measurements. The variation of parameters with or without PMU easily reflected in the fig.5 – 16:

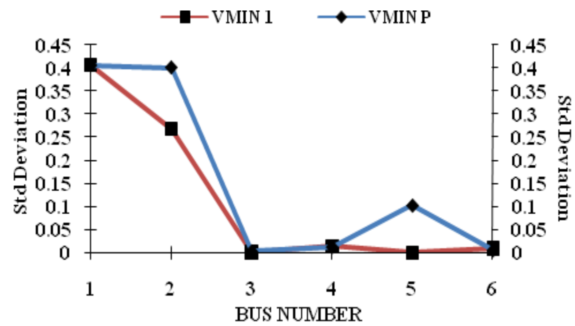

Fig 5. Graph Bus Voltage vs Bus Number

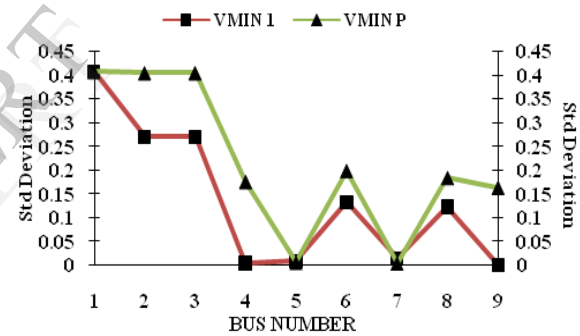

Fig 6. Graph Bus Voltage vs Bus Number

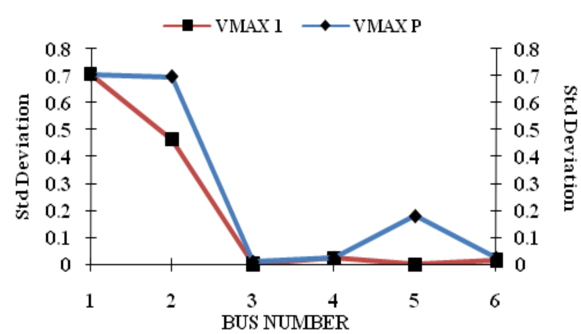

Fig 7. Graph Bus Voltage vs Bus Number

The graph to be designed on the basis of inconsistency exist in Standard Deviation (S D) of each parameter at each Bus which is planned for 20 cycles of operation for each IEEE system, it has standard deviation categorized in two categories i.e. Minimum & Maximum variation on actual parameters received at each Bus. The figure 5 & figure 7 will illustrate the variation in Standard Deviation at

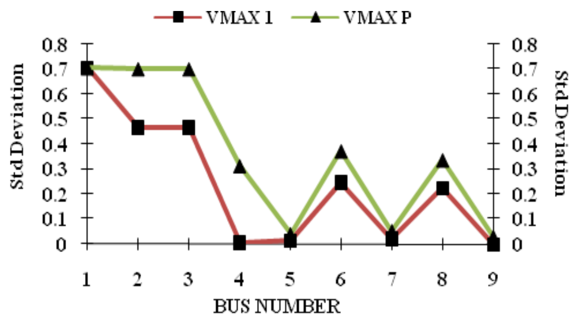

Fig 8. Graph Bus Voltage vs Bus Number

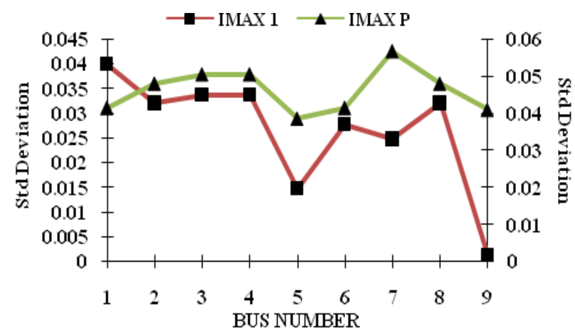

Fig 12. Graph Bus Current vs Bus Number

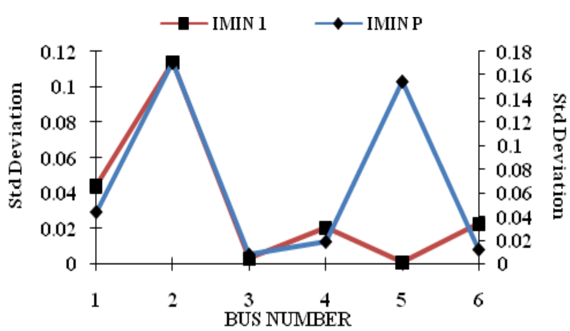

Fig 9. Graph Bus Current vs Bus Number

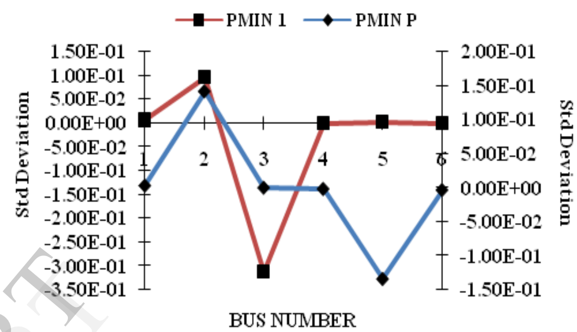

Fig 13. Graph Bus Power vs Bus Number

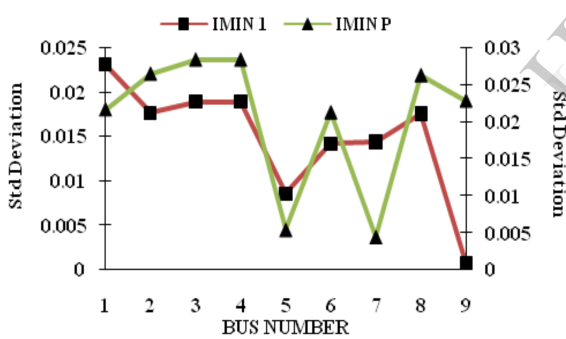

Fig 10. Graph Bus Current vs Bus Number

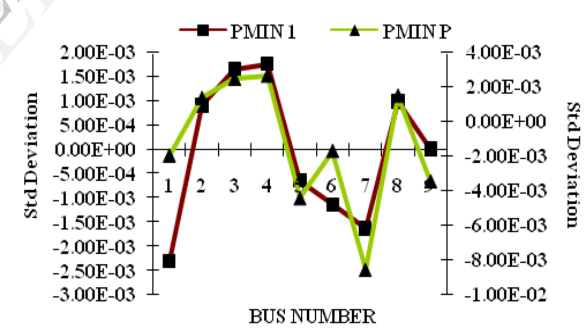

Fig 14. Graph Bus Power vs Bus Number

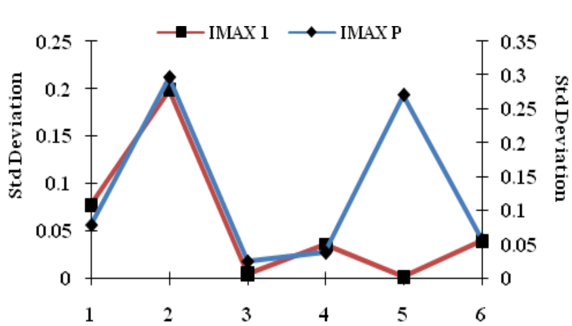

Fig 11. Graph Bus Current vs Bus Number

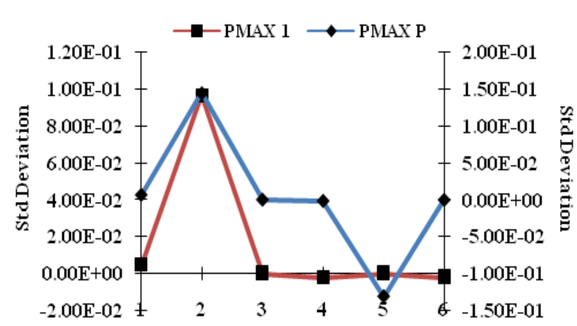

Fig 15. Graph Bus Power vs Bus Number

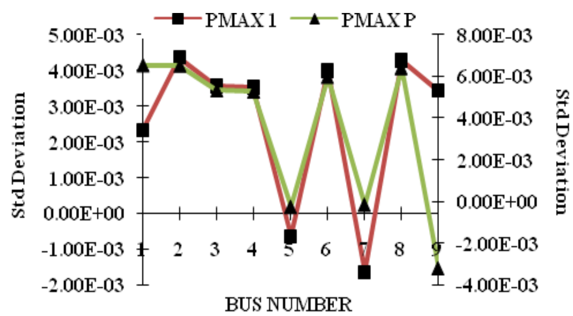

Fig 16. Graph Bus Power vs Bus Number

Minimum & Maximum of Voltage (V) with respect to Each Bus on varying the number of PMU accretion from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 6 bus system. Similarly Figure 6 & Figure 8 will illustrate the variation in Standard Deviation at Minimum & Maximum of Voltage (V) with respect to Each Bus on varying the number of PMU accretion from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 9 bus system. Similarly for Standard Deviation in Current (I) & Real Power (P) will be illustrated in Figure 9 – 12 & Figure 13 – 16 respectively, from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 6 bus system and IEEE 9 bus system.

Table 3. Average Std. Dev. of the Estimated Variables

Type of Var.	Type of System	**Case 1**		**Case P**	
		Min	**Max**	**Min**	**Max**
Voltage (V)	6 Bus	0.1164	0.201	0.1552	0.273
	9 Bus	0.1363	0.239	0.2161	0.360
Current (I)	6 Bus	0.0341	0.059	0.0682	0.127
	9 Bus	0.0148	0.026	0.0205	0.046
Power (P)	6 Bus	-0.0366	0.016	0.0001	0.002
	9 Bus	-5.4E-05	0.002	-0.0013	0.003

The table 3 will illustrates the average variation of Standard Deviation in Voltage, Current and Power at the place of Minimum & Maximum values on each bus in case of IEEE 6 bus system and IEEE 9 bus system as in Fig 5 – 16. The Std. Deviation will improved while compared to the of 'Case 1' to 'Case P' either for the IEEE 6 bus system or IEEE 9 bus system. In IEEE 9 bus system, the Std. Deviation of the estimated voltage is approximately 0.239584 when there is no PMUs, but subsequent addition of PMUs in the system, it build up to 0.3607. It means that the Std. Deviation of 'No PMUs' is improved by addition of PMUs. The interesting thing which is investigate during the process of operation is that the standard deviation growing as escalating the PMU. Therefore, this result will illustrate that the effectively establishing of PMUs in system, facilitate us in reducing the chances of error in measurement of estimated variables.

7. Conclusion

In this paper traditional technique has been discussed incorporation of WLS and PMU. The study carried out to ascertain the correlation between traditional technique and full weighted least square state estimation technique with the PMU. Additionally it is envisage that the result obtained will provide more accurate and precise information of voltage, current & power variation on each bus in each system as compared to the traditional techniques. Such formulation will provide us a more accurate and precise information of voltage, current & power variation without doing any bulky iteration as in traditional technique. By using a single PMU will lead to enhance the result in respect of accuracy.

The advantages of using PMU will advances the accuracy of the estimated variables. The Case I and Case P are tested while progressively increasing the PMUs which are added to the measurement set. With the help of advanced accuracy of PMU, it was seen that the estimated accuracy is also increases. One of the motivating thing is that the accuracy improves most effectively when the number of implemented PMUs are around '17 %' of the system buses. It is proved that the quality of the estimation is enhanced by adopting PMU data to the set of measurement. The PMU measurements will provide us improved accuracy and redundancy for the system.

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