

Poisson's Ratio of Soilcrete Blocks

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Abstract

Tropical countries are subject to extreme weather conditions and as such require special building materials to accommodate this. Soilcrete blocks (made with laterite, cement and water) can effectively be used in these regions because of their thermal insulating properties and greater resistance to extreme weather conditions. Poisson's ratio, which is required for structural computations, has been ignored to an extent with regards to block moulding technology. In this work, Poisson's ratio was determined for the soilcrete blocks produced. Modified regression theory was used to generate a model for prediction of the Poisson's ratio of soilcrete blocks. The model was subjected to statistical tests which proved its adequacy.

1. Introduction

Block can be generally described as a solid mass used in construction. It can be made from a wide variety of materials ranging from binder, water, sand, laterite, coarse aggregates, and clay to admixtures. The constituent materials determine the type of block which includes soilcrete, sandcrete, mud blocks, clay bricks, etc. Soilcrete blocks are made of cement, laterite and water. Sandcrete blocks (made of river sand, cement and water) are used in most places but they are not considered the best for building in tropical countries because of their poor environmental and thermal insulating properties as a result of high degree of porosity. Soilcrete blocks can effectively be used in tropical areas because of their good thermal insulating properties. They are advantageous in hot dry climates where extreme temperature can be moderated inside buildings of compressed stabilized earth blocks [1].

Several researchers have reported that laterite can be used in good quality block production, road and building construction. Boeck et. al., produced cement stabilized laterite blocks using 4-6% cement [2]. Good laterite blocks were produced from different sites in Kano when laterite was stabilized with 3 to 7% cement [3]. Laterite stabilized with cement was used successfully to produce bricks in Sudan [1]. Aguwa produced laterite –cement blocks using 0-10% cement content by weight of the soil [4-5]. Alutu and Oghenejobo used 3% to 15% of cement to produce cement-stabilised laterite hollow blocks [6]. It is worthy of note here that most of these researchers' work revolved around compressive strength of blocks and the cost effectiveness of using laterite in block production. Other properties/characteristics like Poisson's ratio have not been handled adequately. Model for prediction of Poisson's ratio using mix ratio and vice versa has not been formulated.

Knowledge of Poisson's ratio (an elastic constant) is necessary for structural design computations. In all engineering materials, the elongation produced by an axial tensile load in the direction of the force is accompanied by a contraction in any transverse (lateral) direction. The ratio of the lateral contractive strain to axial strain in a material is referred to as Poisson's ratio [7] and it is given as:

$$\mu = \text{lateral strain/axial strain} = \epsilon_1/\epsilon \quad (1)$$

where ϵ_1 is the lateral strain, ϵ is the axial strain and μ , the Poisson's ratio.

Neville proposed another way of estimating Poisson's ratio. It is the ratio of tensile stress at cracking in flexure to compressive stress at cracking in compression specimen [8]. Hence,

$$\mu = \sigma_t / \sigma_c \tag{2}$$

where μ = Poisson's ratio

σ_t = tensile stress at cracking in flexure

σ_c = compressive stress at cracking in compression specimen

Ability to predict this vital structural characteristic/property of blocks is of utmost importance. Modified regression method proposed by Osadebe [9] was used in this work to formulate a model for prediction of Poisson's ratio of soilcrete blocks using specified mix ratio. The model can also yield all the possible mix ratios for a desired Poisson's ratio. The formulation of the regression equation was done from first principles using the so-called absolute volume (mass) as a necessary condition. This principle assumes that the volume (mass) of a mixture is equal to the sum of the absolute volume (mass) of all the constituent components. Osadebe assumed that the response function is continuous and differentiable with respect to its predictors. By making use of Taylor's series, the response function could be expanded in the neighborhood of a chosen point. The modified regression theory has been applied successfully with good results by various scholars [9-14].

2. Methodology

The materials used for this work are Eagle cement brand of Ordinary Portland Cement, laterite, sourced from Ikeduru LGA, river sand from Otamiri river in Imo State and potable water. All the materials conform to British Standard/ specifications [15-17].

Here, analytical and experimental procedures were used in formulating a mathematical model for predicting the Poisson's ratio of soilcrete blocks. The model is based on the modified regression theory.

2.1 Formulation of model based on modified regression theory

The polynomial equation as given by Osadebe [9] is $Y = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_{12} z_1 z_2 + \alpha_{13} z_1 z_3 + \alpha_{23} z_2 z_3$ (3)

In general, Eqn (3) is given as:

$$Y = \sum \alpha_i z_i + \sum \alpha_{ij} z_i z_j \tag{4}$$

where $1 \leq i \leq j \leq 3$

Eqns (3) and (4) are the optimization model equations. Y is the response function at any point of observation, z_i , the predictors are the ratios of the actual portions to the quantity of soilcrete (fractional portions) and α_i are the coefficients of the optimization model equations.

Different points of observation will have different responses with different predictors at constant coefficients. At the n th observation point, $Y^{(n)}$ will correspond with $Z_i^{(n)}$. That is to say that:

$$Y^{(n)} = \sum \alpha_i z_i^{(n)} + \sum \alpha_{ij} z_i^{(n)} z_j^{(n)} \tag{5}$$

where $1 \leq i \leq j \leq 4$ and $n = 1, 2, 3, \dots, 10$

Eqn (5) can be put in matrix form as

$$[Y^{(n)}] = [Z^{(n)}] \{\alpha\} \tag{6}$$

Rearranging Eqn (6) gives:

$$\{\alpha\} = [Z^{(n)}]^{-1} [Y^{(n)}] \tag{7}$$

The actual mix proportions, $s_i^{(n)}$ and the corresponding fractional portions, $z_i^{(n)}$ are presented on Tables 1 and 2. These values of the fractional portions $Z^{(n)}$ were used to develop $Z^{(n)}$ matrix and the inverse of $Z^{(n)}$ matrix. The solution of Eqn (7) with known $Z^{(n)}$ matrix and $Y^{(n)}$ matrix from laboratory tests gives the unknown constant coefficients α_i .

Table 1. Values of actual mix proportions and their corresponding fractional portions for a 3-component mixture

N	S ₁	S ₂	S ₃	RESPONSE	Z ₁	Z ₂	Z ₃
1	0.8	1	8	Y ₁	0.081633	0.102041	0.816327
2	1	1	12.5	Y ₂	0.068966	0.068966	0.862069
3	1.28	1	16.67	Y ₃	0.067546	0.05277	0.879683
4	0.9	1	10.25	Y ₁₂	0.074074	0.082305	0.843621
5	1.04	1	12.335	Y ₁₃	0.072348	0.069565	0.858087
6	1.14	1	14.585	Y ₂₃	0.068161	0.059791	0.872048

S₁ = Actual water cement ratio Z₁ = Fractional water/cement ratio
 S₂ = Actual cement quantity Z₂ = Fractional portion of cement
 S₃ = Actual laterite quantity Z₃ = Fractional portion of laterite

Table 2. Z⁽ⁿ⁾ matrix for a 3-component mixture

Z ₁	Z ₂	Z ₃	Z ₁ Z ₂	Z ₁ Z ₃	Z ₂ Z ₃
0.081633	0.102041	0.816327	0.00833	0.066639	0.083299
0.068966	0.068966	0.862069	0.004756	0.059453	0.059453
0.067546	0.05277	0.879683	0.003564	0.059419	0.046421
0.074074	0.082305	0.843621	0.006097	0.06249	0.069434
0.072348	0.069565	0.858087	0.005033	0.062081	0.059693
0.068161	0.059791	0.872048	0.004075	0.05944	0.05214

2.2 Experimental Investigation

The mix proportions from Table 1 were used to measure out the quantities of water (S₁), cement (S₂), laterite (S₃), for production of soilcrete blocks. A total of twelve mix ratios were used to produce thirty six solid blocks that were cured and tested on the 28th day. Six out of the twelve mix ratios were used as control mix ratios to produce eighteen blocks for the confirmation of the adequacy of the mixture design

model. The initial cracking load in flexure was recorded and used to calculate tensile stress at cracking in flexure.

The initial cracking load in compression specimen was recorded and used to calculate compressive stress at cracking in compression specimen. With these two parameters known, Poisson's ratio was calculated using

Eqn (2). Three blocks were tested for each point and the average taken as the Poisson's ratio of the point.

3. Results and Discussions

The experimental values of Poisson's ratios of the soilcrete blocks are presented on Table 3 while the replication variances of the test result are presented on Table 4.

Table 3. Experimental values of Poisson's ratio of soilcrete blocks

Exp. No	Mix ratios (w/c: cement: laterite)	Repli-Cates	Initial Cracking Load in Flexure (KN)	Tensile Stress at Cracking in Flexure σ_t (N/mm ²)	Initial Cracking Load in Compression (KN)	Compressive Stress at Cracking in Flexure σ_c (N/mm ²)	Poisson's Ratio $\mu = \sigma_t/\sigma_c$	Average Poisson's Ratio μ
1	0.8:1:8	A	15.5	0.230	80	1.185	0.194	0.174
		B	17.5	0.259	90	1.333	0.194	
		C	16.0	0.237	120	1.778	0.133	
2	1:1:12.5	A	2.5	0.037	20	0.296	0.125	0.135
		B	3.5	0.052	25	0.370	0.141	
		C	2.8	0.041	20	0.296	0.139	
3	1.28:1:16.67	A	2.5	0.037	20	0.296	0.125	0.110
		B	2.3	0.034	40	0.593	0.057	
		C	3.0	0.044	20	0.296	0.149	
4	0.9:1:10.25	A	3.5	0.052	30	0.444	0.117	0.095
		B	3.0	0.044	35	0.518	0.085	
		C	2.9	0.043	35	0.518	0.083	
5	1.04:1:12.335	A	2.1	0.031	30	0.444	0.070	0.074
		B	4.0	0.059	50	0.741	0.080	
		C	2.9	0.043	40	0.593	0.073	
6	1.14:1:14.585	A	2.2	0.033	20	0.296	0.111	0.091
		B	2.5	0.037	40	0.593	0.062	
		C	3.0	0.044	30	0.444	0.099	
7	1.09:1:13.46	A	2.5	0.037	20	0.296	0.125	0.099
		B	2.0	0.030	20	0.296	0.101	
		C	2.1	0.031	30	0.444	0.070	
8	1.02:1:12.417	A	2.1	0.031	30	0.444	0.070	0.105
		B	2.1	0.031	20	0.296	0.105	
		C	2.8	0.041	20	0.296	0.139	
9	0.866:1:9.485	A	2.5	0.037	50	0.741	0.050	0.054
		B	2.5	0.037	50	0.741	0.050	
		C	2.8	0.041	45	0.667	0.061	
10	1.0924:1:13.8761	A	4.0	0.059	40	0.593	0.099	0.079
		B	2.5	0.037	45	0.667	0.055	
		C	3.8	0.056	45	0.667	0.084	
11	1.052:1:12.818	A	4.0	0.059	30	0.444	0.133	0.092
		B	2.5	0.037	30	0.444	0.083	
		C	3.8	0.031	35	0.518	0.060	
12	1.1:1:13.685	A	2.0	0.030	20	0.296	0.101	0.098
		B	2.0	0.030	20	0.296	0.101	
		C	2.3	0.034	25	0.370	0.092	

Table 4. Poisson's ratio test results and replication variance

Expt. No.	Replicates	Response Y_i	Response Symbol	$\sum Y_i$	Y	$\sum Y_i^2$	S_i^2
1	1A	0.194	Y_1	0.521	0.174	0.093	0.0012
	1B	0.194					
	1C	0.133					
2	2A	0.125	Y_2	0.405	0.135	0.055	0.000
	2B	0.141					
	2C	0.139					
3	3A	0.125	Y_3	0.331	0.110	0.041	0.002
	3B	0.057					
	3C	0.149					
4	4A	0.117	Y_{12}	0.285	0.095	0.028	0.000
	4B	0.085					
	4C	0.083					
5	5A	0.070	Y_{13}	0.223	0.074	0.017	0.000
	5B	0.080					
	5C	0.073					
6	6A	0.111	Y_{23}	0.272	0.091	0.026	0.001
	6B	0.062					
	6C	0.099					
Control							
7	7A	0.125	C_1	0.296	0.099	0.031	0.001
	7B	0.101					
	7C	0.070					
8	8A	0.070	C_2	0.314	0.105	0.035	0.0012
	8B	0.105					
	8C	0.139					
9	9A	0.050	C_3	0.161	0.054	0.0087	0.000
	9B	0.050					
	9C	0.061					
10	10A	0.099	C_4	0.238	0.079	0.020	0.001
	10B	0.055					
	10C	0.084					
11	11A	0.133	C_5	0.276	0.092	0.0282	0.001
	11B	0.083					
	11C	0.060					
12	12A	0.101	C_6	0.294	0.098	0.0289	0.000
	12B	0.101					
	12C	0.092					
						\sum	0.0084
Legend: $y = \sum y/n$ $S_y^2 = [1/(n-1)]\{\sum y_i^2 - [1/n(\sum y_i)^2]\}$ where $1 \leq i \leq n$ y_i = the responses y = the mean of responses for each control point n = control points, $n-1$ = degree of freedom							

Considering all the design points, number of degrees of freedom,

$$V_e = \sum(N_{i-1}) \quad (8)$$

where $1 \leq i \leq 12$

$$V_e = 12 - 1 = 11$$

Replication variance,

$$S_y^2 = 1/V_e \sum S_i^2 \quad (9)$$

$$S_y^2 = 0.0084/11 = 0.0007636$$

where S_i^2 is the variance at each point

$$\text{Replication error, } S_y = \sqrt{S_y^2} \quad (10)$$

$$= \sqrt{0.0007636} = 0.028$$

This replication error value was used below to determine the t-statistics values for the model.

3.1 Determination of Osadebe's mathematical model for Poisson's ratio of soilcrete blocks

Substituting the values of $Y^{(n)}$ from test results (given in Tables 3 and 4) into Eqn (7) gives the values of the coefficients, α as:

$$\alpha_1 = 8790.55199, \alpha_2 = 1164.1268, \alpha_3 = 27.3908, \alpha_4 = -16111.1579, \alpha_5 = -9787.0468, \alpha_6 = -866.3843$$

Substituting the values of these coefficients, α into Eqn (3) yields:

$$Y = 8790.55199Z_1 + 1164.1268Z_2 + 27.3908Z_3 - 16111.1579Z_4 - 9787.0468Z_5 - 866.3843Z_6 \quad (11)$$

Eqn (11) is the Osadebe's mathematical model for optimisation of Poisson's ratio of soilcrete block based on 28-day strength.

3.1.1 Test of adequacy of Osadebe's model for Poisson's ratio of soilcrete blocks

The model equation was tested for adequacy against the controlled experimental results. The statistical hypothesis for this mathematical model is as follows:

Null Hypothesis (H_0): There is no significant difference between the experimental and the theoretically expected results at an α -level of 0.5.

Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected results at an α -level of 0.05.

The student's t-test and fisher test statistics were used for this test. The expected values ($Y_{\text{predicted}}$) for the test control points were obtained by substituting the values of Z_1 from (Table 2) into the model equation i.e. Eqn (11). These values were compared with the experimental result (Y_{observed}) given in (Table 3).

(i) Student's t-test

For this test, the parameters Δ_y , ϵ and t are evaluated using the following equations respectively

$$\Delta_y = Y_{(\text{observed})} - Y_{(\text{predicted})} \quad (12)$$

$$\epsilon = (\sum a_i^2 + \sum a_{ij}^2) \quad (13)$$

$$t = \Delta_y \sqrt{n} / (S_y \sqrt{1 + \epsilon}) \quad (14)$$

where ϵ is the estimated standard deviation or error,

t is the t-statistics,

n is the number of parallel observations at every point

S_y is the replication error

a_i and a_{ij} are coefficients while i and j are pure components

$$a_i = X_i(2X_i - 1)$$

$$a_{ij} = 4X_i X_j$$

$$Y_{\text{obs}} = Y_{(\text{observed})} = \text{Experimental results}$$

$$Y_{\text{pre}} = Y_{(\text{predicted})} = \text{Predicted results}$$

Using Eqns (12), (13), (14), the student's t-test computations are presented on Table 5.

T-value from table

For a significant level, $\alpha = 0.05$, $t_{\alpha/(v_e)} = t_{0.05/6}(5) = t_{0.01}(5) = 3.365$. The t-value is obtained from standard t-statistics table.

This value is greater than any of the t-values obtained by calculation (as shown in Table 5). Therefore, we accept the Null hypothesis. Hence the model equation is adequate.

(ii) Fisher Test

For this test, the parameter y , is evaluated using the following equation:

$$y = \sum Y/n \quad (15)$$

where Y is the response and n the number of responses.

The Fisher test computations are presented on Table 6.

Using variance, $S^2 = [1/(n-1)][\sum (Y-y)^2]$ and $y = \sum Y/n$ for $1 \leq i \leq n$, $S^2_{(\text{obs})}$ and $S^2_{(\text{pre})}$ are calculated as follows:

$$S^2_{(\text{obs})} = 0.001763/5 = 0.0003526 \text{ and } S^2_{(\text{pre})} = 0.000695/5 = 0.000139$$

The fisher test statistics is given by:

$$F = S_1^2 / S_2^2 \quad (16)$$

where S_1^2 is the larger of the two variances.

Hence, $S_1^2 = 0.0003526$ and $S_2^2 = 0.000139$

Therefore, $F = 0.0003526 / 0.000139 = 2.54$

From standard Fisher table, $F_{0.95}(5,5) = 5.1$ which is higher than the calculated F-value. Hence the regression equation is adequate.

Table 5. T-statistics test computations for Osadebe’s Poisson’s ratio model

N	CN	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	ϵ	$Y_{(observed)}$	$Y_{(predicted)}$	Δ_y	t
1	C_1	1	2	-0.125	0.25	0.01562	0.0625	0.6094	0.099	0.075	0.024	1.170
		1	3	-0.125	0.5	0.01562	0.25					
		2	3	-0.125	0.5	0.01562	0.25					
		3	-	0	-	0	-					
				Σ		0.04686	0.5625					
Similarly												
2		-	-	-	-	-	-	0.6094	0.105	0.079	0.026	1.268
3		-	-	-	-	-	-	0.899	0.054	0.101	0.047	2.110
4		-	-	-	-	-	-	0.8476	0.079	0.096	0.017	0.774
5		-	-	-	-	-	-	0.640	0.092	0.073	0.019	0.918
6		-	-	-	-	-	-	0.6208	0.098	0.078	0.020	0.972

Table 6. F-statistics test computations for Osadebe’s Poisson’s ratio model

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)} - Y_{(obs)}$	$Y_{(pre)} - Y_{(pre)}$	$(Y_{(obs)} - Y_{(obs)})^2$	$(Y_{(pre)} - Y_{(pre)})^2$
C_1	0.099	0.075	0.011167	-0.00867	0.000125	7.51E-05
C_2	0.105	0.079	0.017167	-0.00467	0.000295	2.18E-05
C_3	0.054	0.101	-0.03383	0.017333	0.001145	0.0003
C_4	0.079	0.096	-0.00883	0.012333	7.8E-05	0.000152
C_5	0.092	0.073	0.004167	-0.01067	1.74E-05	0.000114
C_6	0.098	0.078	0.010167	-0.00567	0.000103	3.21E-05
Σ	0.527	0.502			0.001763	0.000695
	$y_{(obs)}=0.087833$	$y_{(pre)}=0.083667$				

Legend: $y = \sum Y/n$
 where Y is the response and n the number of responses.

4. Conclusion

1. Poisson's ratio of soilcrete blocks was determined
2. Modified regression theory proposed by Osadebe was used to generate a model for prediction of Poisson's ratio of soilcrete blocks
3. The efficacy of the model was proved using student's t test and fisher test
4. The model can predict the Poisson's ratio of soilcrete blocks if the mix ratio is specified and vice versa

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