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Peristaltic Transport Of A Conducting Bingham Fluid In A Channel

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Abstract

Peristaltic transport of a conducting Bingham fluid in a channel under long wavelength and low Reynolds number assumptions is studied. This model is probably suitable for the blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions. Further the blood is experimentally proved to be a conducting fluid in several physiological applications. Motivated by this fact, it is proposed to study the peristaltic transport of a Bingham fluid. It is observed that for a Bingham fluid the pressure difference and the mechanical efficiency of pumping depend on the yield stress and the magnetic field.

1. Introduction

Peristaltic pumping is a form of fluid transport generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion, which propagates along the length of a tube-like structure. Peristalsis occurs naturally as a means of pumping physiological fluids from one place in the body to another. Some electro-chemical reactions are held responsible for this phenomenon. This mechanism occurs in swallowing of food through oesophagus, in the ureter, the gastro intestinal tract, the bile duct and even in small blood vessels.

2 Nomenclature

a: Half width of the wave
B_0: Applied magnetic field
b: Amplitude of the wave
F: Dimensionless friction force
h: Height of the wave in the direction of y
M: Hartmann number
P: Pressure
p: Dimensionless pressure
ΔP: Pressure rise/drop
q: Volume flux
Q: Time averaged flow rate
R_e: Reynold's number
t: Time
U,V: Axial and transverse velocity components in laboratory frame
u,v: Axial and transverse velocity components in wave frame
u_p: Velocity in plug flow region
x,y: Axial and transverse co ordinates
X,Y: Dimensionless axial and transverse co ordinates
y_0: Plug flow region
λ: wave length
σ_e: Electrical conductivity
Ψ: Stream function
φ: Amplitude ratio (b/a)
τ_0: Yield stress
μ: Viscosity index

3 Mathematical formulation and solution

Consider the peristaltic pumping of a conducting Bingham fluid in a channel of half-width (a). A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half width of the channel as shown in figure 4.1. The region between Y = 0 and Y = y_0 is called plug flow region. In this region, $|\tau_{y,x}| \le \tau_o$. In the region between Y =

Y_0 and $Y = H$, $|\tau_{y,x}| > \tau_0$. The wall deformation is given by

$$H(x, t) = a + b \sin \frac{2\pi}{\lambda}(x - ct) \qquad (1)$$

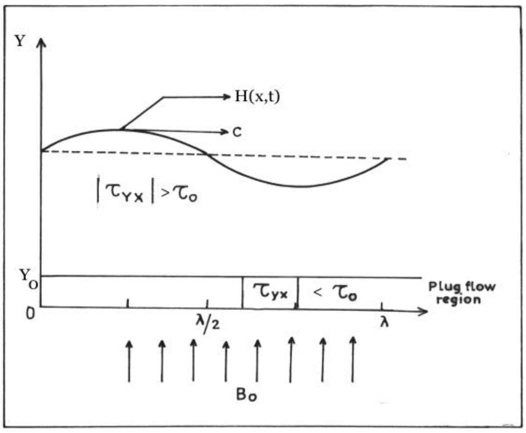

Fig. 1 Physical Model

4. Equations of Motion

Under the assumptions that the tube length is an integral multiple of the wave length λ and the pressure difference across the ends of the tube is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y). The transformation between these two frames is given by

$$x = X\text{-}ct, \qquad y = Y$$

$$u(x,y) = U(x\text{-}ct, Y) - c$$

$$v(x,y) = V(x\text{-}ct, Y) \qquad (2)$$

In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small (for example in ureter, $R_e \cong 1$). So, we assume that the flow is inertia – free. Further, we assume that the wavelength is infinite. So the flow is of Poiseuille type at each local cross-section. Using the non-dimensional quantities,

$$\bar{u} = \frac{u}{c};\ \bar{x} = \frac{x}{\lambda};\ \bar{y} = \frac{y}{a};\ \bar{h} = \frac{h}{a};\ \bar{t} = \frac{ct}{\lambda}$$

$$\bar{F} = \frac{Fa}{\lambda\mu c};\ \bar{\tau}_0 = \frac{a\tau_0}{\mu c};\ \bar{\psi} = \frac{\psi}{ac};\ \bar{p} = \frac{Pa^2}{\lambda\mu c}$$

$$\bar{y}_0 = \frac{y_0}{a};\ \bar{q} = \frac{q}{ac};\ \varphi = \frac{b}{a}$$

The non-dimensional form of equations governing the motion (dropping the bars) is

$$\frac{d}{dy}(\tau_0 - \Psi yy) + M^2 \Psi y = -\frac{dp}{dx} \qquad (3)$$

$$0 = \frac{\partial p}{\partial y} \qquad (4)$$

where $M = \sqrt{\frac{B_o^{\,2} a^2 \sigma_e}{\mu}}$, the Hartmann number

The non-dimensional boundary conditions are

$$\psi = 0; \quad \psi_{yy} = \tau_0 \quad \text{at } y = 0$$

$$u = \psi_y = -1 \quad \text{at } y = h \qquad (5)$$

$$\text{Where } h = 1 + \varphi \operatorname{Sin} 2\pi x \qquad (6)$$

5. Solution

Solving equation (4) with the boundary conditions (5), we obtain the velocity as

$$u = A_1 e^{My} + B_1 e^{-My} + \frac{p}{M^2} \qquad (7)$$

where $A_1 = a_1 p + a_2$, $B_1 = a_3 p + a_4$

$$a_1 = \left[\frac{-\frac{1}{M^2}+\frac{1}{M^2}e^{-Mh}}{2 Sinh M}\right]$$

$$a_2 = \frac{-1-\tau_0 e^{-Mh}}{2 Sinh M}$$

$$a_3 = \frac{-\frac{1}{M^2}e^{Mh}+\frac{1}{M^2}}{2 Sinh M}$$

$$a_4 = \frac{\tau_0 e^{Mh}+1}{2 Sinh M}$$

$$p = -\frac{\partial p}{\partial x} \qquad (8)$$

Taking y = y_0 in equation we get the velocity in plug flow region as

$$u_p = A_1 e^{My_0} + B_1 e^{-My_0} + \frac{p}{M^2} \quad (0 \le y \le y_0) \qquad (9)$$

the volume flux q through each cross-section in the wave frame is given by

$$q = \int_0^{y_0} u_p dy + \int_{y_0}^{y} u dy = a_5 P + a_6 \qquad (10)$$

where a_5 =

$$a_1 e^{My_0}\left(y_0 - \frac{1}{M}\right) + a_3 e^{-My_0}\left(y_0 + \frac{1}{M}\right) + \frac{a_1}{M}e^{Mh} - \frac{a_3}{M}e^{-Mh} + \frac{h}{M^2}$$

a_6 =

$$a_2 e^{My_0}\left(1 - \frac{1}{M}\right) + a_4 e^{-My_0}\left(1 + \frac{1}{M}\right) + \frac{a_2}{M}e^{Mh} - \frac{a_4}{M}e^{-Mh}$$

From the above equation

$$\frac{\partial p}{\partial x} = \frac{a_6 - q}{a_5} \qquad (11)$$

Averaging equation over one period yields the time mean flow (time-averaged volume flow rates) $\bar{Q}$ as

$$\bar{Q} = \frac{1}{T}\int_0^T Q dt$$

$$= q + 1 \qquad (12)$$

6. The Pumping characteristics

Integrating the equation (10) with respect to x over one wavelength, we get the pressure rise over one cycle of the wave as

$$\Delta P = \int_0^1 \left(\frac{a_6 - \bar{Q} + 1}{a_5}\right) dx \qquad (13)$$

The pressure rise required to produce zero average flow rate is denoted by ΔP_0 and is given by

$$\Delta P_0 = \int_0^1 \left(\frac{a_6 + 1}{a_5}\right) dx \qquad (14)$$

The dimensionless friction force F at the wall across one wave length is given by

$$\mathbf{F} = \int_0^1 h\left(-\frac{dp}{dz}\right) dx$$

$$= \int_0^1 h\left(\frac{\bar{Q} - 1 - a_6}{a_5}\right) dx \qquad (15)$$

7. Mechanical Efficiency:

The mechanical efficiency of pumping for one wave length is given by

$$E = \frac{\bar{Q}\Delta p}{\int_0^1 p_x (h-1) dx}$$

$$= \frac{\bar{Q}\left[I_1 - (\bar{Q}-1)I_2\right]}{Q\left[I_3 - (\bar{Q}-1)I_4\right]} \qquad (16)$$

Where $I_1 = \int_0^1 \frac{a_6}{a_5} dx$

$I_2 = \int_0^1 \frac{1}{a_5} dx$

$$I_3 = \int_0^1 \frac{a_6}{a_5} Sin\, 2\pi x dx$$

$$I_4 = \int_0^1 \frac{1}{a_5} Sin\, 2\pi x\, dx$$

8 Discussion of the Results.

From the equation 13 we have calculated the pressure difference as a function of $\overline{Q}$ for different values of yield stress τ_0 for a fixed φ and shown in Fig. 2. It is observed that for a given ΔP the flux $\overline{Q}$ depends on yield stress and it decreases with increase in yield stress. For a given flux $\overline{Q}$ the pressure rise ΔP decreases with increase in yield stress τ_0. For free pumping, the flux $\overline{Q}$ increases with decreasing yield stress.

From the variation of pressure rise with time averaged flow rate is calculated from equation (13) for different amplitude ratios and shown in Fig. 3. For given ΔP the flux $\overline{Q}$ for Bingham fluid depends on φ and we find that the larger the amplitude ratio the greater the pressure rise against which the pump works. This observed for small values of ΔP ($\Delta P < 0001$) and free pumping cases.

From equation (13) we have calculated the variation of pressure rise with time averaged flow rate $\overline{Q}$ for different magnetic parameters M and is shown in Fig. 4 we observed that the larger the magnetic parameter the greater the pressure rise against which the pump works. For free pumping the flux $\overline{Q}$ depends on magnetic field and it increases with increasing magnetic parameter M.

From equation (15), we have calculated the frictional force as a function of $\overline{Q}$ for fixed amplitude ratio, and is depicted in Fig. 5. It is observed that the frictional force 'F' has the opposite behaviour compared to pressure rise (ΔP). As τ_0 increases the frictional force also increases with the flux $\overline{Q}$. For a given $\overline{Q}$ the frictional force increases with decrease in yield stress.

From equation (16) we have calculated the mechanical efficiency as a function of $\overline{Q}$ and is depicted in Fig. 6 and Fig. 7. for different values of magnetic parameters M, yield stress and amplitude ratio φ. For the chosen parameter the mechanical efficiency is positive for $\overline{Q}$ less than or equal to 6 and is negative for $\overline{Q}$ greater than 6. Further we observed that the pumping efficiency is greater for higher values of magnetic parameters. It is also found that the mechanical efficiency decreases with the increasing yield stress. For lower value flux $\overline{Q}$, the efficiency remains constant and is unaffected by variation in M and τ_0.

From equation (14), we have calculated the pressure rise required to produce zero average flow rate ΔP_0 as a function of amplitude ratio φ for different values of yield stress τ_0 and is shown in Fig. 8. It is observed that for a conducting Bingham fluid, the value of ΔP_0 increases with decrease in yield stress τ_0. It is also observed that the increase in the value of ΔP_0 is more for the amplitude ratio beyond 0.7.

From equation (14), we have calculated the pressure rise required to produce zero average flow rate ΔP_0 as a function of amplitude ratio φ for different values of magnetic parameter m and is shown in Fig. 9. It is observed that for a conducting

Bingham fluid, the value of ΔP_0 increases with increase in magnetic parameter.

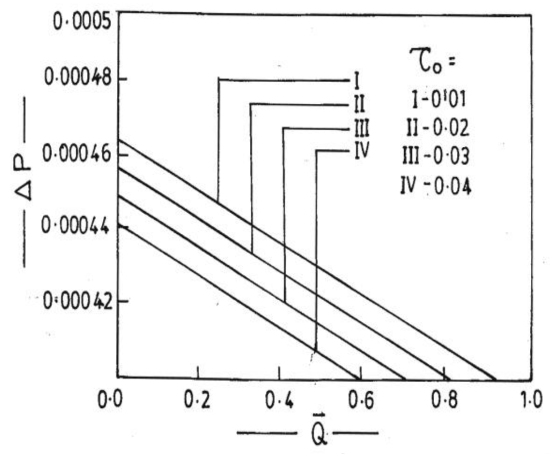

Fig. 2 The variation of ΔP with averaged flow rate $\overline{Q}$ for different τ_0 with φ = 0.6, m = 0.02, y_0 = 0.2

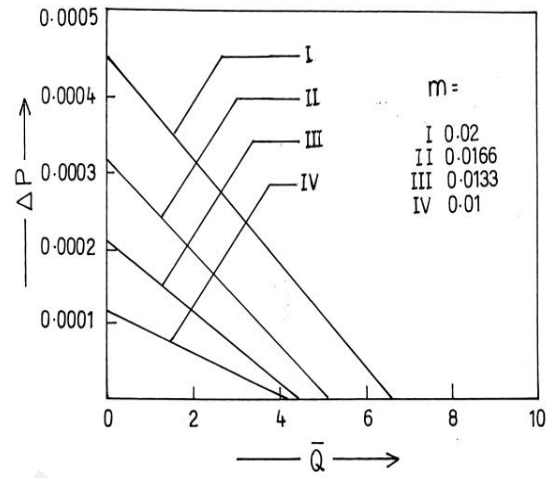

Fig. 4 The variation of ΔP with averaged flow rate $\overline{Q}$ for different m with τ_0 = 0.01, φ = 0.6, y_0 = 0.2

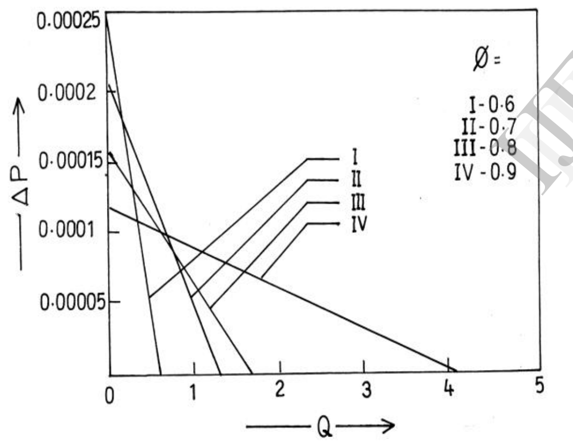

Fig. 3 The variation of ΔP with averaged flow rate $\overline{Q}$ for different φ, with τ_0 = 0.01, m = 0.01, y_0 = 0.2

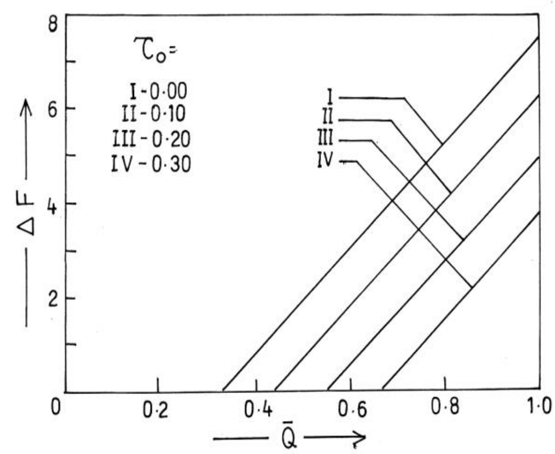

Fig. 5 The variation of ΔF with averaged flow rate $\overline{Q}$ for different τ_0 with φ = 0.6, m = 0.02, y_0 = 0.2

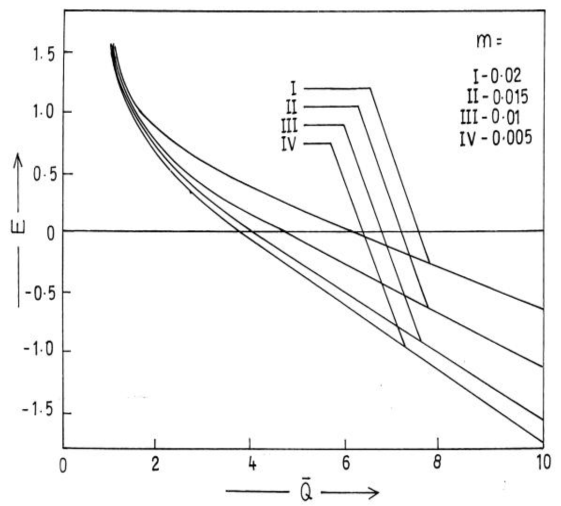

Fig. 6 The variation of Mechanical Efficiency E with averaged flow rate $\overline{Q}$ for different m with τ_0 = 0.01, φ = 0.6, y_0 = 0.1

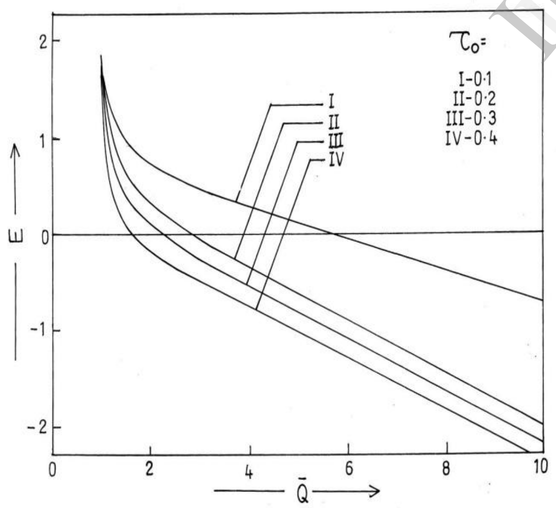

Fig. 7 The variation of Mechanical Efficiency E with averaged flow rate $\overline{Q}$ for different τ_0 with φ = 0.6, m = 0.02, y_0 = 0.1

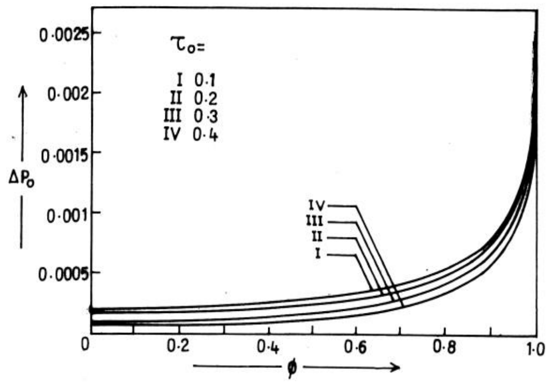

Fig. 8 The variation of ΔP_0 with Amplitude ratio φ for different τ_0 with m = 0.02, y_0 = 0.2

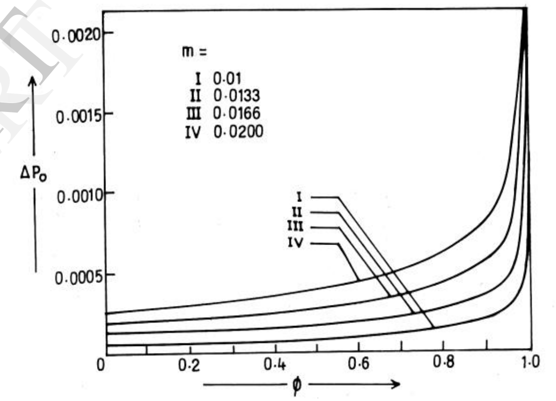

Fig. 9 The variation of ΔP_0 with Amplitude ratio φ for different m with τ_0 = 0.01, y_0 = 0.2

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