

Perishable Inventory Model With Finite Rate Of Replenishment Having Mixture Of Weibull Lifetime And Demand As Function Of Both Selling Price And Time

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ABSTRACT

In this paper we develop and analyse an inventory model for deteriorating items with mixture of Weibull rate of decay having finite rate of replenishment and demand as function of both selling price and time with shortages. Using the differential equations, the instantaneous state of inventory at time 't', the amount of deterioration etc. are derived. With suitable cost considerations the total cost function and profit rate function are also obtained by maximizing the profit rate function, the optimal ordering and pricing policies of the model are derived. The sensitivity of the model with respect to the parameters is discussed through numerical illustration. It is observed that the deteriorating parameters have a tremendous influence on the optimal selling price and ordering quantity.

Key Words: Perishability, Instantaneous rate of deterioration, Total cost function, Profit rate function, the optimal ordering and pricing policies.

1. INTRODUCTION

Much work has been reported regarding inventory models for deteriorating items in recent years. In many of the inventory systems the major consideration is regarding the pattern of demand and supply. Goyal and Giri[4] have reviewed inventory models for deteriorating items. They classified the literature by the self life characteristic of the inventory of goods. They also developed on the basis of demand variations and various other conditions or constraints. Various functional forms are considered for describing the demand pattern. Aggarwal and Goel[1] have developed an inventory model with the weibull rate of decay having selling price dependent demand. In reality, the demand rate of any product may vary with time or with price or with the instantaneous level of inventory displayed in a supermarket. Mathew[8] Roy and Chaudhuri [10] and Sana [11] ,Tripathy and Mishra [13] have studied inventory models with demand rates depending on selling price of the item. Inventory problems involving time dependent demand patterns have received the attention of several researchers in recent years. Mathew [7] Chun, *et al.* [2], Giri, *et al.* [3], Manna, *et al.* [6], Mahata and Goswami [5] and Skouri, *et al.* [12] are among those who studied inventory models for deteriorating items having time dependent demand.

However, no serious attempt was made to develop inventory models for deteriorating items having mixture weibull rate of decay and both selling price and time dependent demand with finite rate of replenishment, which are very useful in many practical situations arising at oil and natural gas

industry, photo chemical industries, chemical processes, etc. Hence, in this paper we develop and analysed an inventory model with the assumption that the lifetime of the commodity is random and follows a three-parameter mixture weibull distribution having demand as a function of both selling price and time with finite rate of replenishment and shortages. Using differential equations the instantaneous state of on hand inventory is obtained. With suitable cost considerations the total cost function is derived. The optimal ordering policies are also obtained. The sensitivity of the model is analysed though numerical illustration. This model includes some of the earlier models as particular cases for specific or limiting values of the parameters.

2. ASSUMPTIONS AND NOTATIONS

We adopt the following assumptions and notations for the models to be discussed.

2.1 Assumptions

Assumption 1: The demand is known and is a function of unit selling price $\lambda(s)$.

Assumption 2: Replenishment rate is finite.

Assumption 3: Lead time zero

Assumption 4: Shortages are allowed and fully back logged

Assumption 5: A deteriorating item is lost

Assumption 6: The production rate is finite

Assumption 7: T is the fixed duration of a production cycle

Assumption 8: The lifetime of the commodity is random and follows a three parameter weibull distribution of the form $f(t) = \alpha\beta(t-\gamma)^{\beta-1} e^{-\alpha(t-\gamma)^\beta}$ for $t > \gamma$ where α, β, γ are parameters.

$$\text{Hence, the instantaneous rate of deterioration } h(t) \text{ is } h(t) = \alpha\beta(t-\gamma)^{\beta-1} \quad (2.1)$$

2.2 Notations

Q	: The ordering quantity I one cycle of length T.
A	: The cost of placing an order.
C	: The cost price of one unit.
h	: The inventory holding cost per unit per unit time.
Π	: The shortage cost per unit for unit time.
S	: The selling price of unit.
$\lambda(s)$: The demand rate, which is a function of unit selling price
r	: The rate of demand.
n	: The pattern index.

3. INVENTORY MODEL

Consider an inventory system in which the amount of stock is zero at time $t = 0$. Replenishment starts at $t = 0$ and stops at $t = t_1$. The deterioration of the item starts after a certain fixed lifetime ' γ '. Since the perishability starts after ' γ ' the decrease in the inventory is due to demand during the period $(0, \gamma)$, demand and deterioration during the period (γ, t_1) . During (t_1, t_2) the inventory level gradually decreases mainly to meet up demand and partly due to deterioration. By this process the stock reaches zero level at $t = t_2$. Now shortages occur and accumulate to the level $t = t_3$. Replenishment starts again at $t = t_3$ and the backlogged demand is cleared at $t = T$. The cycle then repeats itself after time ' T '.

The schematic diagram representing the inventory system is shown in Fig.1.

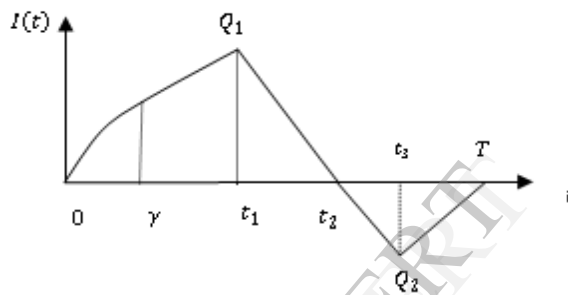


Fig.1 schematic diagram representing the inventory level of the system

Consider an inventory system in which the lifetime of the commodity is random and follows a mixture of three parameters weibull distribution with parameters $\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma$ and p

Hence the instantaneous rate of deterioration $h(t)$ as

$$H(t) = \frac{p\alpha_1\beta_1(t-\gamma)^{\beta_1-1}e^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)\alpha_2\beta_2(t-\gamma)^{\beta_2-1}e^{-\alpha_2(t-\gamma)^{\beta_2}}}{pe^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)e^{-\alpha_2(t-\gamma)^{\beta_2}}}$$

for $t \geq \gamma$; $\alpha_1, \alpha_2, \beta_1, \beta_2, > 0$; and $0 \leq p \leq 1$ (3.1)

Along with all other assumptions made, assume that the demand rate is known and is a function of both selling price and time, say $\lambda(s, t)$. This functional form for the demand rate includes several functions involving selling price and / or time. To mention a few,

- (i) $\lambda(s,t) = \lambda_1(s)$ like $\lambda(s,t) = a + b \cdot s$
- (ii) $\lambda(s,t) = \lambda_2(t)$ like $\lambda_2(t) = \frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$
- (iii) $\lambda(s,t) = a \lambda_1(s) + b \lambda_2(t)$ like $= \frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} + as$
- (iv) $\lambda(s,t) = a \lambda_1(s) \lambda_2(t)$ like se^{-at} and $= \frac{sr t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$
- (v) $\lambda(s,t) = \theta$, a constant, etc.

Where, s is the selling price per unit, T is the cycle length, r is the demand during the period T , n is the pattern index, a and b are known constants

Let 'Q' be the ordering quantity in one cycle, 'A' be the cost of placing an order, 'C' be the cost of one unit, 'h' be the inventory holding cost per unit per unit time. Also assume that shortages are not allowed. $\lambda(s,t)$ is assumed such that the total cost function is convex. In this model the stock decreases due to (i) the demand of items in the interval $(0,\gamma)$, (ii) the combination of deteriorating and demand of items in the interval (γ,t) , (iii) during (t_1,t_2) the inventory level gradually decreases mainly to meet up demands and partly for deterioration, by this process the stock reaches zero level at $t=t_2$, Now shortages occur and accumulate to the level $t + t_3$. Production starts again at $t = t_3$ and the backlog is cleared at $t = T$. The cycle then repeats itself after time T .

Let $I(t)$ be the inventory level of the system at time t ($0 \leq t \leq T$). Then, the differential equations describing the instantaneous states of $I(t)$ over the cycle of length T are

$$\frac{d}{dt} I(t) = k - \lambda(s,t) \quad \text{for} \quad 0 \leq t \leq \gamma \quad (3.2)$$

$$\frac{d}{dt} I(t) + h(t) \cdot I(t) = k - \lambda(s,t) \quad \text{for} \quad \gamma \leq t \leq t_1 \quad (3.3)$$

$$\frac{d}{dt} I(t) + h(t) \cdot I(t) = -\lambda(s,t) \quad \text{for} \quad t_1 \leq t \leq t_2 \quad (3.4)$$

$$\frac{d}{dt} I(t) = -\lambda(s,t) \quad \text{for} \quad t_2 \leq t \leq t_3 \quad (3.5)$$

$$\frac{d}{dt} I(t) = k - \lambda(s,t) \quad \text{for} \quad t_3 \leq t \leq T \quad (3.6)$$

with the initial condition $I(0)=0; I(t_2)=0; I(T)=0$.

Substituting $h(t)$ and solving the above differential equations (3.2) to (3.6), the on-hand inventory at time t can be obtained as

$$I(t) = (k - \lambda(s, t))t \quad 0 \leq t \leq \gamma \quad (3.7)$$

$$I(t) = (k - \lambda(s, t))t \left[pe^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)e^{-\alpha_2(t-\gamma)^{\beta_2}} \right] \\ \left\{ \int_{\gamma}^t \left[pe^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)e^{-\alpha_2(u-\gamma)^{\beta_2}} \right]^{-1} du + \gamma \right\}, \quad \gamma \leq t \leq t_1 \quad (3.8)$$

$$I(t) = (k - \lambda(s, t)) \left[pe^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)e^{-\alpha_2(t-\gamma)^{\beta_2}} \right] \\ \left\{ \int_{t_1}^{t_2} \left[pe^{-\alpha_1(t-\gamma)^{\beta_1}} + (1-p)e^{-\alpha_2(u-\gamma)^{\beta_2}} \right]^{-1} du + \int_{t_1}^{t_1} \left[pe^{-\alpha_1(u-\gamma)} + (1-p)e^{-\alpha_2(u-\gamma)^{\beta_2}} \right]^{-1} du \right\}, \\ t_1 \leq t \leq t_2 \quad (3.9)$$

$$I(t) = \lambda(s, t)(t_2 - t), \quad t_2 \leq t \leq t_3 \quad (3.10)$$

$$I(t) = (k - \lambda(s, t))(t - T), \quad t_3 \leq t \leq T \quad (3.11)$$

The stock loss due to deterioration in the interval $(0, T)$ is

$$L(T) = (kt_1 - \lambda(s, t)t_2), \quad (3.12)$$

The backlogged demand at t time is

$$B(t) = \lambda(s, t)(t_3 - t_2), \quad (3.13)$$

The ordering quantity in a cycle of length T is obtained as

$$Q = kt_1 + k(T - t_3) \quad (3.14)$$

Let $K(t_1, t_2, s)$ be the expected total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost and the shortage cost. $K(t_1, t_2, s)$ is obtained as

$$K(t_1, t_2, s) = \frac{A}{T} + \frac{cQ}{T} + \frac{h}{T} \left[\int_0^\gamma h(t) dt + \int_\gamma^{t_1} h(t) dt + \int_{t_1}^{t_2} h(t) dt \right] + \frac{\pi}{T} \int_{t_2}^{t_3} I(t) dt \quad (3.15)$$

using Taylor's series expansion for the exponential function for small values of $\alpha(t-r)^\beta$, the power series expansion and ignoring the higher order terms we get,

$$\begin{aligned} K(t_1, t_2, s) &= \frac{A}{T} + \frac{ck}{T} (t_1 + T - t_3) + \frac{h}{T} (k - \lambda(s, t)) \left[\int_0^\gamma dt \right] \\ &+ \frac{h}{T} [k - \lambda(s, t)] \left\{ \int_\gamma^{t_1} [1 - p\alpha_1(t-\gamma)^{\beta_1} - (1-p)\alpha_2(t-\gamma)^{\beta_2}] \left[\int_\gamma^t (1 + p\alpha_1(u-\gamma)^{\beta_1} - (1-p)\alpha_2(u-\gamma)^{\beta_2}) du + \gamma \right] dt \right\} \\ &+ \frac{h}{T} \lambda(s, t) [1 - \alpha_1(t-\gamma)^{\beta_1} - (1-p)\alpha_2(t-\gamma)^{\beta_2}] \left\{ \int_{t_1}^{t_2} (1 + p\alpha_1(t-\gamma)^{\beta_1} + (1-p)\alpha_2(t-\gamma)^{\beta_2}) dt \right. \\ &\left. - \int_{t_1}^t (1 + p\alpha_1(t-\gamma)^{\beta_1} + (1-p)\alpha_2(t-\gamma)^{\beta_2}) dt \right\} + \frac{\pi}{T} \lambda(s, t) \int_{t_2}^{t_3} (t_2 - t) dt \end{aligned} \quad (3.16)$$

Profit rate function P (t_1, t_2, s) is

$$P(t_1, t_2, s) = s\lambda(s) - K(t_1, t_2, s)$$

Hence,

$$\begin{aligned} P(t_1, t_2, s) &= s\lambda(s) - \left[\frac{A}{T} + \frac{kc}{2T} (2t_1 + T - t_2) \right] + \frac{hk}{T} \left[\frac{t_1^2}{2} + p \frac{\alpha_1}{(\beta_1+1)(\beta_2+2)} (t_1 - \gamma)^{\beta_1+2} \right. \\ &+ (1-p) \frac{\alpha_2}{(\beta_2+1)(\beta_2+2)} (t_1 - \gamma)^{\beta_2+2} - p \frac{\alpha_1}{\beta_2+1} t_1 (t_1 - \gamma)^{\beta_1+1} + \frac{p\alpha_1 (t_1 - \gamma)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)} \\ &\left. - \frac{(1-p)\alpha_2 t_1 (t_1 - \gamma)^{\beta_2+4}}{(\beta_2+1)} + \frac{(1-p)\alpha_2 (t_1 - \gamma)^{\beta_2+2}}{(\beta_2+1)(\beta_2+2)} \right] \\ &+ \frac{h\lambda(s, t)}{T} \left[\frac{t_2^2}{2} - t_1 t_2 - p \frac{\alpha_1}{\beta_1+1} t_1 (t_2 - \gamma)^{\beta_1+1} - p \frac{\alpha_1}{(\beta_1+1)(\beta_1+2)} (t_2 - \gamma)^{\beta_1+2} \right. \\ &\left. - (1-p) \frac{\alpha_2}{\beta_2+1} t_1 (t_2 - \gamma)^{\beta_2+1} - \frac{(1-p)\alpha_2}{(\beta_2+1)(\beta_2+2)} (t_2 - \gamma)^{\beta_2+2} \right] \end{aligned}$$

$$\begin{aligned}
& + p \frac{\alpha_1}{\beta_1+1} t_2 (t_1 - \gamma)^{\beta_1+1} + p \frac{\alpha_1}{\beta_1+1} t_2 (t_2 - \gamma)^{\beta_1+1} \\
& - \frac{p\alpha_1 (t_2 - r)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)} + \frac{(1-p)\alpha_2 t_2 (t_2 - r)^{\beta_1+1}}{\beta_2+1} - \frac{(1-p)\alpha_2 (t_2 - \gamma)^{\beta_2+2}}{(\beta_2+1)(\beta_2+2)} \Big] \\
& + \frac{\pi \lambda(s,t)}{8} (2Tt_2 - t_2^2 - T^2) \quad] \quad (3.17)
\end{aligned}$$

4. OPTIMAL POLICIES OF THE MODEL

In this section the optimal pricing and ordering policies of the inventory system by taking a specific form from the demand is discussed. Let the demand rate be of the form

$$\lambda(s,t) = \frac{srt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \quad (4.1)$$

which indicates that the demand as an increasing function of time and selling price. This situation prevails in oil exploration industry.

Hence,

$$\begin{aligned}
K(t_1, t_2, s) &= \frac{A}{T} + \frac{ck}{T} (t_1 + T - t_3) + \frac{h}{T} \int_0^\gamma \left[kt - \frac{srt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right] dt \\
& + \frac{h}{T} \left\{ \int_\gamma^{t_1} [1 - p\alpha_1 (t - \gamma)^{\beta_1} - (1-p)\alpha_2 (t - \gamma)^{\beta_2}] \right. \\
& \left. \left[\int_\gamma^t (1 + p\alpha_1 (u - \gamma)^{\beta_1} - (1-p)\alpha_2 (u - \gamma)^{\beta_2}) \left[k - \frac{sru^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right] du - k\gamma - \frac{sr\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}} \right] \right\} \\
& + \frac{hsr}{nT^{\frac{1}{n}+1}} \int_{t_1}^{t_2} \left\{ [1 - \alpha_1 (t - \gamma)^{\beta_1} - (1-p)\alpha_2 (t - \gamma)^{\beta_2}] \right. \\
& \left. \int_{t_1}^{t_2} t^{\frac{1}{n}-1} (1 + p\alpha_1 (t - \gamma)^{\beta_1} + (1-p)\alpha_2 (t - \gamma)^{\beta_2}) dt \right\}
\end{aligned}$$

$$\left. - \int_{t_1}^t u^{\frac{1}{n}-1} (1 + p\alpha_1(u-\gamma)^{\beta_1} + (1-p)\alpha_2(u-\gamma)^{\beta_2}) du \right\} + \frac{\pi sr}{T^{\frac{1}{n}+1}} \int_{t_2}^{t_3} (t^{\frac{1}{n_2}} - t^{\frac{1}{n}}) dt$$

Profit rate function $P(t_1, t_2, s)$ is

$$\begin{aligned} P(t_1, t_2, s) &= \frac{s}{T} \int_0^T \frac{s.r.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}} dt - K(t_1, t_2, s) \\ &= \frac{s^2 r}{T} - \left\{ \frac{A}{T} + \frac{kc}{2T} (2t_1 + T - t_2) \right. \\ &\quad + \frac{hk}{T} \left[\frac{t_1^2}{2} + \frac{2p\alpha_1}{(\beta_1+1)(\beta_1+2)} (t_1 - \gamma)^{\beta_1+2} + 2(1-p) \frac{\alpha_2}{(\beta_2+1)(\beta_2+2)} (t_1 - \gamma)^{\beta_2+2} \right. \\ &\quad \left. \left. - p \frac{\alpha_1}{\beta_1+1} (t_1 - \gamma)^{\beta_1+1} - (1-p) \frac{\alpha_2}{\beta_2+1} (t_1 - \gamma)^{\beta_2+1} \cdot t_1 \right] \right. \\ &\quad \left. - \frac{h.s.r}{T^{\frac{1}{n}+1}} p \frac{\alpha_1}{n\beta_1+1} \left[\gamma^{\frac{1}{n}+\beta_1} - \frac{\gamma^{\frac{1}{n}+\beta_1-1}}{\frac{1}{n}+\beta_1-1} - \gamma^{\frac{1}{n}+\beta_1} \cdot t_1 \right] \right. \\ &\quad \left. - \frac{p\alpha_1\beta_1\gamma}{n\beta_1-n+1} \left[\gamma^{\frac{1}{n}+\beta_1} - \frac{\gamma^{\frac{1}{n}+\beta_1}}{\frac{1}{n}+\beta_1-1} - \gamma^{\frac{1}{n}+\beta_1-1} \cdot t_1 \right] + (1-p) \frac{\alpha_2}{n\beta_2+1} \left[\gamma^{\frac{1}{n}+\beta_2-1} - \frac{\gamma^{\frac{1}{n}+\beta_2+1}}{\frac{1}{n}+\beta_2+1} - \gamma^{\frac{1}{n}+\beta_2} \cdot t_1 \right] \right. \\ &\quad \left. - (1-p) \frac{\alpha_2\beta_2\gamma}{n\beta_2-n+1} \left[\gamma^{\frac{1}{n}+\beta_2} - \frac{\gamma^{\frac{1}{n}+\beta_2}}{\frac{1}{n}+\beta_2} - \gamma^{\frac{1}{n}+\beta_2-1} - t_1 \right] + p \frac{\alpha_1}{\frac{1}{n}+\beta_1+1} \gamma^{\frac{1}{n}+\beta_1+1} - \frac{p\alpha_1\beta_1\gamma^{\frac{1}{n}+\beta_1+1}}{\frac{1}{n}\beta_1} \right. \\ &\quad \left. + (1-p) \frac{\alpha_2}{\frac{1}{n}+\beta_2+1} \gamma^{\frac{1}{n}+\beta_2+1} - (1-p) \frac{\alpha_2\beta_2\gamma^{\frac{1}{n}+\beta_2+1}}{\frac{1}{n}+\beta_2} \right\} \\ &\quad + \frac{h.s.r}{T^{\frac{1}{n}+1}} \left\{ t_2^{\frac{1}{n}+1} - \frac{t_2^{\frac{1}{n}+1}}{\frac{1}{n}+1} - t_1 t_2^{\frac{1}{n}} + \frac{p\alpha_1}{n\beta_1+1} \left[t_2^{\frac{1}{n}+\beta_1+1} - t_1 t_2^{\frac{1}{n}+\beta_1} - \frac{t_2^{\frac{1}{n}+\beta_1+1}}{\frac{1}{n}+\beta_1+1} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& -\frac{p\alpha_1\beta_1\gamma}{n\beta_1-n+1} \left[t_2^{\frac{1}{n}+\beta_1} - \frac{t_2^{\frac{1}{n}+\beta_1}}{\frac{1}{n}+\beta_1} - t_2^{\frac{1}{n}+\beta_1-1} \cdot t_1 \right] + \frac{(1-p)\alpha_2}{n\beta_2+1} \left[t_2^{\frac{1}{n}+\beta_1+1} - \frac{t_2^{\frac{1}{n}+\beta_1-1}}{\frac{1}{n}+\beta_2+1} - t_2^{\frac{1}{n}+\beta_1} \cdot t_1 \right] \\
& - (1-p) \frac{\alpha_2\beta_2\gamma}{n\beta_2-n+1} \left[t_2^{\frac{1}{n}+\beta_2} - \frac{t_2^{\frac{1}{n}+\beta_2}}{\frac{1}{n}+\beta_2} - t_2^{\frac{1}{n}+\beta_2-1} \cdot t_1 \right] + \frac{p\alpha_2}{\beta_1+1} \left[t_2^{\frac{1}{n}}(t_2-\gamma)^{\beta_1+1} - t_2^{\frac{1}{n}}(t_1-\gamma)^{\beta_1+1} \right] \\
& + p \frac{\alpha_1 t_2^{\frac{1}{n}+\beta_1+1}}{\frac{1}{n}+\beta_1+1} - p \frac{\alpha_1\beta_1\gamma \cdot t_2^{\frac{1}{n}+\beta_1}}{\frac{1}{n}+\beta_1} - (1-p) \frac{\alpha_2}{\beta_2+1} t_2^{\frac{1}{n}} \left[(t_2-\gamma)^{\beta_1+1} - t_2^{\frac{1}{n}}(t_1-\gamma)^{\beta_2+1} \right] \\
& + (1-p) \frac{\alpha_2}{\frac{1}{n}+\beta_2+1} \cdot t_2^{\frac{1}{n}+\beta_2+1} - (1-p) \frac{\alpha_2\beta_2\gamma}{\frac{1}{n}+\beta_2} t_2^{\frac{1}{n}+\beta_2} \left. \right\} + \frac{\pi s r}{T^{\frac{1}{n}+1}} \left[\frac{T}{2} \cdot t_2^{\frac{1}{n}} + \frac{n-1}{2(n+1)} \cdot t_2^{\frac{1}{n}+1} - \frac{n}{n+1} \left(\frac{T+t_2}{2} \right)^{\frac{1}{n}+1} \right] \} (4.2)
\end{aligned}$$

To find the optimal values of t_1 , t_2 , s equate the first order partial derivatives of $P(t_1, t_2, s)$ with respect to t_1 , t_2 and s to zero.

By differentiating $P(t_1, t_2, s)$ given in equation (4.2) with respect to t_1 and equating to zero, we get

$$\begin{aligned}
& + \frac{ck}{T} + \frac{hk}{T} \left[t_1 + p \frac{\alpha_1}{\beta_1+1} (t_1-\gamma)^{\beta_1+1} + (1-p) \frac{\alpha_2}{\beta_2+1} (t_1-\gamma)^{\beta_2+1} - p \cdot t_1 \cdot \alpha_1 (t_1-\gamma)^{\beta_1} - (1-p) t_2 \alpha_2 (t_1-\gamma)^{\beta_2} \right] \\
& - \frac{h.s.r}{T^{\frac{1}{n}+1}} \left[-p \frac{\alpha_1}{n\beta_1+1} \gamma^{\frac{1}{n}+\beta_2} + p \frac{\alpha_1\beta_1}{n\beta_2-n+1} \gamma^{\frac{1}{n}+\beta_1} - (1-p) \frac{\alpha_2}{n\beta_2+1} \gamma^{\frac{1}{n}+\beta_2} + (1-p) \frac{\alpha_2\beta_2}{n\beta_2-n+1} \cdot \gamma^{\frac{1}{n}+\beta_2} \right] \\
& - \frac{h.s.r}{T^{\frac{1}{n}+1}} \left(-t_2^{\frac{1}{n}} - p \frac{\alpha_1}{n\beta_1+1} \cdot t_2^{\frac{1}{n}+\beta_1} + p \frac{\alpha_1\beta_1\gamma}{n\beta_1-n+1} t_2^{\frac{1}{n}+\beta_1-1} - (1-p) \frac{\alpha_2}{n\beta_2+1} t_2^{\frac{1}{n}+\beta_2} + (1-p) \left[\frac{\alpha_2\beta_2\gamma}{n\beta_2-n+1} \cdot t_2^{\frac{1}{n}+\beta_2-1} \right] \right. \\
& \left. - p \alpha_1 t_2^{\frac{1}{n}} (t_1-\gamma)^{\beta_1} + (1-p) \alpha_2 t_2^{\frac{1}{n}} (t_1-\gamma)^{\beta_2} \right] = 0 \quad (4.3)
\end{aligned}$$

By differentiating P (t₁, t₂, s) with reference to t₂ and equating to zero, one can get

$$\begin{aligned}
 & -\frac{kc}{2T} + \frac{h.s.r}{T^{\frac{1}{n}+1}} \left\{ \frac{1}{n} t_2^{\frac{1}{n}} - \frac{1}{n} t_1 t_2^{\frac{1}{n}-1} + p \cdot \frac{\alpha_1}{n\beta_1+1} \left[\left(\frac{1}{n} + \beta_1 + 1 \right) t_2^{\frac{1}{n}+\beta_1} - \left(\frac{1}{n} + \beta_1 \right) t_1 t_2^{\frac{1}{n}+\beta_1-1} - t_2^{\frac{1}{n}+\beta_1} \right] \right. \\
 & - p \frac{\alpha_1 \beta_1 \gamma}{n\beta_1 - n + 1} \left[\left(\frac{1}{n} + \beta_1 \right) t_2^{\frac{1}{n}+\beta_1-1} - t_2^{\frac{1}{n}+\beta_1-1} - \left(\frac{1}{n} + \beta_1 - 1 \right) t_1 t_2^{\frac{1}{n}+\beta_1} \right] \\
 & + (1-p) \frac{\alpha_2 \beta_2 \gamma}{n\beta_2 + 1} \left[\left(\frac{1}{n} + \beta_2 + 1 \right) t_2^{\frac{1}{n}+\beta_2} - t_2^{\frac{1}{n}+\beta_2} - \left(\frac{1}{n} + \beta_2 \right) t_1 t_2^{\frac{1}{n}+\beta_2-1} \right] \\
 & - (1-p) \frac{\alpha_2 \beta_2 \gamma}{n\beta_2 - n + 1} \left[\left(\frac{1}{n} + \beta_2 \right) t_2^{\frac{1}{n}+\beta_2-1} - t_2^{\frac{1}{n}+\beta_2-1} - \left(\frac{1}{n} + \beta_2 - 1 \right) t_2^{\frac{1}{n}+\beta_2-2} \cdot t_1 \right] \\
 & - p \frac{\alpha_1}{\beta_1 + 1} \left[t_2^{\frac{1}{n}} (\beta_1 + 1) (t_2 - \gamma)^{\beta_1} + (t_2 - \gamma)^{\beta_1+1} \cdot \frac{1}{n} t_2^{\frac{1}{n}-1} - \frac{1}{n} t_2^{\frac{1}{n}-1} (t_1 - \gamma)^{\beta_1+1} \right] \\
 & + p \alpha_1 t_2^{\frac{1}{n}+\beta_1} - p \alpha_1 \beta_1 \gamma t_2^{\frac{1}{n}+\beta_1-1} - (1-p) \frac{\alpha_2}{\beta_2 + 1} \left[t_2^{\frac{1}{n}} (\beta_2 + 1) (t_2 - \gamma)^{\beta_2} + (t_2 - \gamma)^{\beta_2+1} \frac{1}{n} t_2^{\frac{1}{n}-1} - (t_1 - \gamma)^{\beta_2+1} \frac{1}{n} t_2^{\frac{1}{n}-1} \right] \\
 & \left. + (1-p) \alpha_2 t_2^{\frac{1}{n}+\beta_2} - (1-p) \alpha_2 \beta_2 \gamma t_2^{\frac{1}{n}+\beta_2-1} \right\} + \frac{\pi s.r}{2T^{\frac{1}{n}+1}} \left[\frac{T}{n} \cdot t_2^{\frac{1}{n}-1} + \frac{n-1}{n} \cdot t_2^{\frac{1}{n}} - \left(\frac{T+t_2}{2} \right)^{\frac{1}{n}} \right] = 0 \quad (4.4)
 \end{aligned}$$

By differentiating P (t₁, t₂, s) with reference to s and equating to zero, we get

$$\begin{aligned}
 & \frac{2rs}{T} + \frac{h.s.r}{T^{\frac{1}{n}+1}} \left(p \frac{\alpha_1}{n\beta_1+1} \left[\gamma^{\frac{1}{n}+\beta_1-1} - \frac{\gamma^{\frac{1}{n}+\beta_1+1}}{\frac{1}{n} + \beta_1 - 1} - \gamma^{\frac{1}{n}+\beta_1} \cdot t_1 \right] \right. \\
 & - \frac{p \alpha_1 \beta_1 \gamma}{n\beta_1 - n + 1} \left[\gamma^{\frac{1}{n}+\beta_1} - \frac{\gamma^{\frac{1}{n}+\beta_1}}{\frac{1}{n} + \beta_1 - 1} - \gamma^{\frac{1}{n}+\beta_1-1} \cdot t_1 \right] + (1-p) \frac{\alpha_2}{n\beta_2+1} \left[\gamma^{\frac{1}{n}+\beta_2-1} - \frac{\gamma^{\frac{1}{n}+\beta_2+1}}{\frac{1}{n} + \beta_2 + 1} - \gamma^{\frac{1}{n}+\beta_2} \cdot t_1 \right] \\
 & \left. - (1-p) \frac{\alpha_2 \beta_2 \gamma}{n\beta_2 - n + 1} \left[\gamma^{\frac{1}{n}+\beta_2} - \frac{\gamma^{\frac{1}{n}+\beta_2}}{\frac{1}{n} + \beta_2} - \gamma^{\frac{1}{n}+\beta_2-1} \cdot t_1 \right] + p \frac{\alpha_1}{\frac{1}{n} + \beta_1 + 1} \gamma^{\frac{1}{n}+\beta_1+1} - \frac{p \alpha_1 \beta_1 \gamma^{\frac{1}{n}+\beta_1+1}}{\frac{1}{n} \beta_1} \right)
 \end{aligned}$$

$$\begin{aligned}
& + (1-p) \left. \frac{\alpha_2}{\frac{1}{n} + \beta_2 + 1} \gamma^{\frac{1}{n} + \beta_2 + 1} + (1-p) \frac{\alpha_2 \beta_2 \gamma^{\frac{1}{n} + \beta_2 + 1}}{\frac{1}{n} + \beta_2} \right\} \\
& - \frac{h \cdot r}{T^{\frac{1}{n} + 1}} \left\{ t_2^{\frac{1}{n} + 1} - \frac{t_2^{\frac{1}{n} + 1}}{\frac{1}{n} + 1} - t_1 t_2^{\frac{1}{n}} + \frac{p \alpha_1}{n \beta_1 + 1} \left[t_2^{\frac{1}{n} + \beta_1 + 1} - t_1 t_2^{\frac{1}{n} + \beta_1} - \frac{t_2^{\frac{1}{n} + \beta_1 + 1}}{\frac{1}{n} + \beta_1 + 1} \right] \right. \\
& - \frac{p \alpha_1 \beta_1 \gamma}{n \beta_1 - n + 1} \left[t_2^{\frac{1}{n} + \beta_1} - \frac{t_2^{\frac{1}{n} + \beta_1}}{\frac{1}{n} + \beta_1} - t_2^{\frac{1}{n} + \beta_1 - 1} \cdot t_1 \right] + \frac{(1-p) \alpha_2}{n \beta_2 + 1} \left[t_2^{\frac{1}{n} + \beta_2 + 1} - \frac{t_2^{\frac{1}{n} + \beta_2 - 1}}{\frac{1}{n} + \beta_2 + 1} - t_2^{\frac{1}{n} + \beta_1} \cdot t_1 \right] \\
& - (1-p) \frac{\alpha_2 \beta_2 \gamma}{n \beta_2 - n + 1} \left[t_2^{\frac{1}{n} + \beta_2} - \frac{t_2^{\frac{1}{n} + \beta_2}}{\frac{1}{n} + \beta_2} - t_2^{\frac{1}{n} + \beta_2 - 1} \cdot t_1 \right] + \frac{p \alpha_2}{\beta_1 + 1} \left[t_2^{\frac{1}{n}} (t_2 - \gamma)^{\beta_1 + 1} - t_2^{\frac{1}{n}} (t_1 - \gamma)^{\beta_1 + 1} \right] \\
& + p \frac{\alpha_1 t_2^{\frac{1}{n} + \beta_1 + 1}}{\frac{1}{n} + \beta_1 + 1} - p \frac{\alpha_1 \beta_1 \gamma \cdot t_2^{\frac{1}{n} + \beta_1}}{\frac{1}{n} + \beta_1} - (1-p) \frac{\alpha_2}{\beta_2 + 1} t_2^{\frac{1}{n}} \left[(t_2 - \gamma)^{\beta_1 + 1} - t_2^{\frac{1}{n}} (t_1 - \gamma)^{\beta_2 + 1} \right] \\
& \left. + (1-p) \frac{\alpha_2}{\frac{1}{n} + \beta_2 + 1} \cdot t_2^{\frac{1}{n} + \beta_2 + 1} - (1-p) \frac{\alpha_2 \beta_2 \gamma}{\frac{1}{n} + \beta_2} t_2^{\frac{1}{n} + \beta_2} \right\} + \frac{\pi \cdot r}{T^{\frac{1}{n} + 1}} \left[\frac{T}{2} \cdot t_2^{\frac{1}{n}} + \frac{n-1}{2(n+1)} \cdot t_2^{\frac{1}{n} + 1} - \frac{n}{n+1} \left(\frac{T+t_2}{2} \right)^{\frac{1}{n} + 1} \right] \} = 0 \quad (4.5)
\end{aligned}$$

Solving the equations (4.3), 4.4) and (4.5) numerically for given values of T , α_1, β_1 , $\alpha_2, \beta_2, \gamma, p$, r , n , A , c , h and π one can obtain the optimal values of t_1^* of t_1 , t_2^* of t_2 , and s^* of s . also obtain the optimal expected total cost per unit time. Even though T is fixed the expected total cost is function of T . So the expected cost function is effected by the various values of T . Hence the optimal cycle length can also be obtained by using search methods after substituting the optimal value of t_1^* , t_2^* , s^* and Q^* as initial values.

5 NUMERICAL ILLUSTRATION

For various values of α_1, β_1 , $\alpha_2, \beta_2, \gamma, p$ and T the optimal values of t_1^*, t_2^*, s^* and Q^* are computed by solving the equations (4.3), (4.4) and (4.5) and presented on table 1 . From Table 1 , it is observed that the decision variables of the system namely the unit selling price, the ordering quantity and the time of stopping the production are influenced by the production rate 'K'. As 'K' increases the unit selling price is decreasing, and the optimal time period of production is increasing for fixed values of the

parameters. That is if the capacity utilization of the production process is maximum then the selling price and the ordering quantity are optimal. From Table 1 it is

Table – 1

Optimum values of production time, time at which shortages occur, selling price, profit and ordering quantity

C	h	R	N	π	K	T	α_1	α_2	β_1	β_2	γ	p	A	.t1	.t2	S	P	Q
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
3														3.628	6.886	4.140	13.614	11.421
4														3.524	6.884	4.091	12.921	11.328
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
	.2													3.748	6.888	4.141	14.715	11.568
	.3													3.641	6.721	4.121	13.123	11.564
	.4													3.746	6.692	4.040	12.529	11.419
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
		.8												3.521	6.642	3.921	15.123	11.612
		.9												3.641	6.428	4.040	14.816	11.592
		1												3.748	6.888	4.141	14.715	11.568
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
			1											3.748	6.888	4.141	14.715	11.568
			2											3.691	6.921	4.416	14.521	11.521
			3											3.920	6.020	4.230	14.328	11.409
				1										4.112	6.910	4.516	14.816	11.582
				2										3.748	6.888	4.141	14.715	11.568
				3										3.752	7.12	4.014	14.529	11.468
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
					2									3.748	6.888	4.141	14.714	11.568
					3									3.690	6.914	4.140	14.961	11.614
					4									3.640	7.210	4.091	15.381	11.628
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
						8								3.841	6.889	4.121	14.691	11.231
						9								3.792	6.901	4.092	14.522	11.489
						10								3.748	6.888	4.141	14.714	11.568
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
							2.0							3.748	6.888	4.141	14.714	11.568
							2.1							3.546	6.894	4.040	14.813	11.612
							2.2							3.249	6.859	3.921	15.029	11.642
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
								0.2						3.748	6.888	4.141	14.714	11.568
								0.3						3.612	6.841	3.916	14.913	11.619
								0.4						3.332	6.571	3.812	15.106	11.712

2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
									2.2					3.748	6.888	4.141	14.714	11.568
									2.3					3.541	6.688	4.146	14.512	11.734
									2.4					3.256	6.821	4.218	14.216	11.928
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
										0.2				3.748	6.888	4.141	14.714	11.568
										0.3				3.720	6.928	4.140	14.816	11.612
										0.4				3.560	6.967	4.062	15.924	11.813
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
											.1			3.942	6.788	4.216	14.719	11.841
											.2			4.106	6.421	4.314	14.816	11.928
2	.2	1	1	2	2	10	2	.2	2.2	.2	0	.5	5	3.748	6.888	4.141	14.714	11.568
												.25		3.751	6.889	4.148	14.926	11.716
												.5		3.748	6.888	4.141	14.714	11.568
												.75		3.812	6.905	4.206	15.381	11.948

Observed that the optimal ordering quantity, the time at Which shortage occur and the expected total cost per unit time are much influenced by the parameters, various costs and cycle length. It is observed that as cycle length increases, the optimal ordering quantity and the time at which shortages occur are increasing when the other parameters and cost are fixed. The optimal ordering quantity and the time at which shortages occur are also influenced by the mean lifetime of the commodity. The optimal ordering quantity and the time at which shortages occur are very sensitive to the penalty cost, when other parameters are fixed. When the penalty cost is increasing, the optimal ordering quantity is increasing. The same phenomenon can be visualized in case of the time at which shortages occur. This is very close to the practical situations where one wishes to minimize the shortages when the penalty is more. So by ordering the optimal ordering quantity one can reduce the cost due to shortages and hence can increase the profits. However if the increase in the penalty cost is in proportion to the increase in the cost per unit, then the optimal ordering quantity is an increasing function to the increase in the cost. If the cost per unit is much higher than the penalty cost, then the optimal ordering quantity is a decreasing function of the cost per unit.. The optimal ordering quantity is a decreasing function of the holding cost. However even if the holding cost is much less compared to the penalty cost then optimal ordering quantity is decreasing function of the holding cost. So by properly choosing the penalty cost one can have the optimal inventory management. It is also observed that the parameters of the model influence the mean lifetime of the commodity and hence the rate of deterioration, which results in the change of the optimal ordering quantity and time at which shortages occur. The optimal ordering quantity is a decreasing function of the location parameter, γ increases, the time at which shortages occur is increasing As a result of the decrease in the optimal ordering quantity and increase in the time at which shortages occur, the total expected cost per unit time is decreasing. Since the location and shape parameters of the lifetime of the commodity have a significant influence on the rate of deterioration, there is a need to analyze the influence of these two parameters on the optimal values of their inventory models with respect to their inter-relationship Even though T , the cycle length is considered to be a known value in this inventory model, it is also interesting to note that the cycle

length has a vital influence on the optimal ordering quantity and expected total cost per unit time. For deriving the optimal ordering quantity one has to properly estimate the parameters involved in the lifetime of the commodity.

6. SENSITIVITY ANALYSIS

In order to study how the parameters affect the optimal solution sensitivity analysis is carried out taking the values $c=2$, $h=0.2$, $r=1$, $n=1$, $\Pi=2$, $k=2$, $T=10$, $\alpha_1=2$, $\alpha_2=0.2$, $\beta_1=2.2$, $\beta_2=0.2$, $\gamma=0$, $p=0.5$ and $A=5$ in appropriate units. Sensitivity analysis is performed by decreasing and increasing these parameter values. First changing the value of one parameter at a time while keeping all the rest at their true values and then changing the values of all the parameters simultaneously. The result of this analysis is given in Table 1.

From Table 1 we observe that the production downtime $t1^*$ is moderately sensitive to $\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma$, and p and slightly sensitive to the changes in other parameter values. The production uptime $t3^*$ is moderately sensitive to h and less sensitive to all other parameters. The optimal production quantity Q^* is highly sensitive to the deterioration distribution parameter α and less sensitive to others.

PARTICULAR CASE In this , the perishable inventory model having deterministic demand as function of both selling price and time and without shortages is discussed as limiting case of the model by taking $\Pi \rightarrow \infty$ and $t_2 \rightarrow T$.

7. CONCLUSIONS

Inventory models play a dominant role in manufacturing and production industries like cement, food processing, petrochemical, and pharmaceutical and paint manufacturing units. In this paper, an inventory model for deteriorating items with constant rate of replenishment, time and selling price dependent demand and mixed Weibull decay has been developed and analyzed in the light of various parameters and costs and with the objective of maximizing the total system profit. The model was illustrated with numerical examples and sensitivity analysis of the model with respect to costs and parameters was also carried out. This model also includes the exponential decay model as a particular case for specific values of the parameters. The proposed model can further be enriched by incorporating salvage of deteriorated units, inflation, quantity discount, and trade credits etc. It can also be extended to a multi-commodity model with constraints on budget, shelf space, etc., These models may also be formulated in fuzzy environments.

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