International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 3, March - 2013

Performance Of MIMO Channel Over SISO

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Abstract

During the last decade, the demand for capacity in wireless local area networks and cellular mobile systems has grown in a literally explosive manner. In particular, compared to the data rates made available by today's technology, the need for wireless Internet access and multimedia applications require an increase in information throughout with order of magnitude. One major technological breakthrough that will make this increase in data rate possible is the use of multiple antennas at the transmitters and receivers in the system. In this paper, a tutorial introduction on the channel capacity of a MIMO channel will be given.

1. Introduction

Due to the ever increasing demand of faster data transmission speed in the recent or future telecommunication systems. In conventional wireless communications, a single antenna is used at the source, and another single antenna is used at the destination. In some cases, this gives rise to problems with multipath effects. When an electromagnetic field (EM field) is met with obstructions such as hills, canyons, buildings, and utility wires, the wave fronts are scattered, and thus they take many paths to reach the destination. The late arrival of scattered portions of the signal causes problems such as fading, cut-out (cliff effect), and intermittent reception (picket fencing). In digital communications systems such as wireless Internet, it can cause a reduction in data speed and an increase in the number of errors. The use of two or more antennas, along with the transmission of multiple signals (one for each antenna) at the source and the destination, eliminates the trouble caused by multipath wave propagation, and can even take advantage of this effect.

2.CHANNEL CAPACITY

At the input of a communication system, discretesource symbols are mapped into a sequence of channel symbols. The channel symbols are then transmitted/ conveyed through a wireless channel that by nature is random. In addition, random noise is added to the channel symbols. In general, it is possible that two different input sequences may give rise to the same output sequence, causing different input sequences to be confusable at the output. To avoid this situation, a non-confusable subset of input sequences must be chosen so that with a high probability, there is only one input sequence causing a particular output. It is then possible to reconstruct all the input sequences at the output with negligible probability of error. A measure of how much information that can be transmitted and received with a negligible probability of error is called the channel capacity. To determine this measure of channel potential, assume that a channel encoder receives a source symbol every *Ts*second. With an optimal source code, the average code length of all source symbols is equal to the entropy rate of the source. If *S* represents the set of all source symbols and the entropy rate of the source is written as $H(S)$, the channel encoder will receive on average$H(S)/T(S)$ informationbits per second.1 Assume that a channel codeword leaves the channel encoder every *Tc*second. In order to be able to transmit all the information from the source, there must be

$$R = \frac{H(S)T_c}{T_s} \qquad (1)$$

information bits per channel symbol. The number *R* is called the *information rate* of the channel encoder.The maximum information rate that can be used causing negligible probability of errors at the output is called the capacity of the channel. By transmittinginformation with rate *R*, the channel is used every*Tc*seconds. The channel capacity is then measuredin bits per channel use. Assuming that the channelhas bandwidth *W*, the input and output can be representedby samples taken$T_s = 1/2W$ seconds apart.With a band-limited channel, the capacity is measured in information bits per second. It is common to represent the channel capacity within a unit bandwithof the channel. The channel capacity is then measured in bits/s/Hz.

It is desirable to design transmission schemes thatexploit the channel capacity as much as possible. Representing the input and output of a memory less wireless channel with the random variables *X* and *Y* respectively, the channel capacity is defined as below.

$$C = \max I(x;y), \qquad (2)$$

whereI(X;Y) represents the mutual information between X and Y . Eq.(2) states that the mutual information is maximized with respect to all possible transmitter statistical distributions p(x). Mutual information is a measure of the amount of information that one random variable contains about another variable. The mutual information between X and Y canalso be written as

$$I\ (X;Y)=H(Y)-H(Y/X), \qquad (3)$$

where*H*(*Y* |*X*) represents the conditional entropy betweenthe random variables *X* and *Y*. The entropy of a random variable can be described as a measure of the amount of information required on average to describe the random variable. It can also described as a measure of the uncertainty of the random variable. Due to (3), mutual information can be described as the reduction in the uncertainty of one random variable due to the knowledge of the other. Note that the mutual information between *X* and *Y* depends on the properties of the channel (through a channel matrix **H**) and the properties of *X* (through the probability distribution of *X*). The channel matrix **H** used in the representation of the input/output relations of a MIMO channel is defined in the next section.

3.MIMO System Model

Consider a Multi-Input communication system consisting with $N_t$transmit antennas, N_r receiver antennas as shown in the figure 1,a narrowband time-invariant wireless channel can be represented by N_t X N_r matrix. Consider a transmitted symbol vector x, which is composed of N_t independent input symbols x1,x2….xNt. Then received signal y can be written in a matrix form as follows:

$$\sqrt{\frac{E_x}{N_t}}H_{X+Z}(4)$$

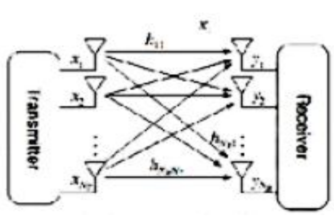

Fig 1. MIMO Communication system Model

Where $Z=(z_1,z_2,\ldots\ldots z_r)^r$ which is assumed to be zero-mean circular symmetric complex Gaussian. ZMCSCG noise with covariance matrix, $E\{nn^H\}=N_0I_{Nr}$. The autocorrelation of transmitted signal vector is defined as $R_{xx}=E\{xx^H\}$ the transmission power for each antenna is assumed to be 1. Then Tr(Rxx)=Nt. A general entry of the channel is denoted by {hij}. This represents the complex gain of the channel between the j^{th} transmitter and i^{th}receiver . With a MIMO system consisting of Nt transmitter and Nr receiver antennas, the channel matrix is written as

$$H=\left[\begin{bmatrix} h_{11} & \cdots & h_{iNt} \\ \vdots & \ddots & \vdots \\ h_{Nr1} & \cdots & h_{NrNt} \end{bmatrix}\right](5)$$

4.SISO CHANNEL CAPACITY

The ergodic (mean) capacity of a random channel with $n_T= n_R = 1$ and an average transmit power constraint P_T can be expressed as [2]

$$C = E_H\{\ \max I(X;Y)\}, \qquad (6)$$

Where P is the average power of a single channel codeword transmitted over the channel and E_H denotes the expectation over all channel realizations. Compared to the definition in (2), the capacity of the channel is now defined as the maximum of the mutual information between the input and the output over all statistical distributions on the input that satisfy the power constraint. If each channel symbol at the transmitter is denoted by s, the average power constraint can be expressed as

$$P = E\ [|s|^2] \leq P_T \qquad (7)$$

5.Deterministic MIMO channel capacity:

The maximum information rate that can used causing negligible probability of errors at the output is called the capacity of the channel. The capacity of a deterministic channel is defined as

$$C = \max I(x;y) \text{ bits/channel use} \quad (8)$$

Where I(x;y) is mutual information of random vector x and y. Eq.(8) states that mutual information is maximized with respect to all possible transmitter statistical distribution f(x). Mutual information is a measure of the amount of information that one random variable contains about another variable. The mutual information of the two continues random vector ,x and y, is given as

$$I(x;y)=H(y)-H(y/x) \quad (9)$$

In which H(y) is the differential entropy of y and H(y/x) is the conditional differential entropy of y when x is given. Using the statistical independence of two random vectors Z and X in equation (4), we can show the following relationship:

$$H(y/x)=H(z)$$

$$I(x;y)=H(y)-H(z) \quad (10)$$

From equation (10), given than H(z) is a constant, we can see that the mutual information is maximized when H(y) is maximizes. Note that the mutual information between x and y depends on the properties of the channel (through channel matrix) and properties of x (through the probability distribution of x).

Using Equation (5), meanwhile, the auto correlation matrix of y is given as

$$R_{yy}= E\{yy^H\} =\left\{\left(\sqrt{\frac{E_x}{N_t}}Hx+z\right)\left(\sqrt{\frac{E_x}{N_t}}x^H H^H+z\right)\right\}$$

$$=E\left\{\left(\sqrt{\frac{E_x}{N_t}}HxxH^H+zz^H\right)\right\}$$

$$=\frac{E_x}{N_t}E\{Hxx^H H^H+zz^H\}$$

$$=\frac{E_x}{N_t}HE\{xx^H\}H^H+E\{zz^H\} \quad (11)$$

Where Ex is the energy of the transmitted signals, and N0 is the power spectral density of the additive noise. The differential entropy H(y) is maximized when y is ZMCSCG, which consequently requires x to be ZMCSCG as well. Then , the mutual information of y and z is respectively given as

$$H(y)=\log_2\{\det\{\pi e R_{yy}\}\}$$

$$H(y)=\log_2\{\det\{\pi e N_o I_{Nr}\}\} \quad (12)$$

Using Equation (12), the mutual information of Equation (10) is expressed as

$$I(x;y)= \log_2\det\left(I_{Nr} + \frac{Ex}{N_t N_o}R_{xx}HH^H\right) \text{ bps/Hz}$$

$$C= \max \log_2\det\left(I_{Nr} + \frac{Ex}{N_t N_o}R_{xx}HH^H\right) \text{ bps/Hz} \quad (13)$$

6.Channel knowledge at receiver:

When the transmitter has no knowledge about the transmitter, it is optimal to spread the energy equally among all transmitter antennas. That is the autocorrelation function of the transmitter signal vector x is given as

$$R_{xx}=I_{Nt} \quad (14)$$

In this case channel capacity is given as

$$C=\log_2\det\left(I_{Nr} + \frac{Ex}{N_t N_o}HH^H\right) \quad (15)$$

Using eigen value decomposition the matrix product is written as

$$HH^H=Q\Lambda Q^H \quad (16)$$

Where Q is the eigen vector matrix with orthogonal columns and Λ is a diagonal matrix with the eigen values on the main diagonal.

$$C=\log_2\det\left(I_{Nr}+\frac{Ex}{N_tN_o}Q\Lambda Q\right)=\log_2\det\left(I_{Nr}+\frac{Ex}{N_tN_o}\Lambda\right)$$

$$=\log_2\det\left(I_{Nr}+\frac{Ex}{N_tN_o}\beta_i\right)$$

It is easier to see that the total capacity of MIMO channel is made up by the sum of parallel AWGN channels. The number of sub channels is determined by the rank of the channel matrix where r denotes the rank matrix , that is r= Nmin ≤ min(Nt,Nr). When CSI is not available at the transmitter and thus, the total power is equally allocated to all transmit antennas.

7.Channel Knowledge available at the transmitter side:

When the channel state information available at the transmitter, modal decomposition can be performed as shown in the figure (8), in which a transmitted signal is preprocessed with V in the transmitter and then, a received signal is post processed with U^H in the receiver.

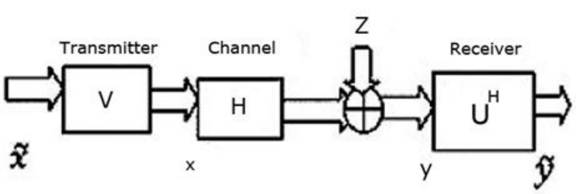

Figure 3. System model when channel knowledge is available at the transmitter side

$$\tilde{y}=U^H\left(\sqrt{\frac{E_x}{N_t}}Hx+z\right)$$

$$\tilde{y}=\left(\sqrt{\frac{E_x}{N_t}}U^HHV\tilde{x}+U^Hz\right) \quad (17)$$

SVD of H yields $H=u\sum V^H$, inserting this into Equation (17) and rearranging the parameters we get,

$$\tilde{y}=\left(\sqrt{\frac{E_x}{N_t}}U^H\sum V^HV\tilde{x}+U^Hz\right)$$

$$\tilde{y}=\left(\sqrt{\frac{E_x}{N_t}}\sum\tilde{x}+U^Hz\right) \quad (18)$$

Which is equivalent to the following r virtual SISO Channel, that is

$$\tilde{y}=\left(\sqrt{\frac{E_x}{N_t}}\sqrt{\lambda_i}\check{x}_i+\check{z}_z,\ i=1,2\ldots.r. \quad (19)$$

If the transmit power for the ith transmit antenna is given by γ_i the capacity of the i th virtual SISO channel is

$$C_i(\gamma_i)=\log_2\left(1+\frac{E\gamma_i}{N_tN_o}\lambda_i\right),\ i=1,2\ldots.r \quad (20)$$

Assume that total available power at the transmitter is limited to

$$E\{xx^H\}=\sum_{i=0}^{N_t}E\{|x_i|^2\}=N_t \quad (21)$$

Where the total power constraint in Equation (21) must be satisfied. The capacity in equation (21) can be maximized by solving the following allocation problem:

$$C=\max\sum_{i=1}^{r}log_2\left(1+\frac{E_x\gamma_i}{N_tN_o}\lambda_i\right) \quad (22)$$

Subjected to $\sum_{i=1}^{r}\gamma_i=N_t$.

The capacity can be increased by resorting to the so-called "water filling principle", by assigning various levels of transmitted power to various transmitting antennas. This power is assigned on the basis that the better the channel gets, the more power it gets and vice versa. This is an optimal energy allocation algorithm.

8.Simulation results:

It is found that, assuming perfect channel knowledge at the receiver side, capacity increases linearly with the min(Nt , Nr).

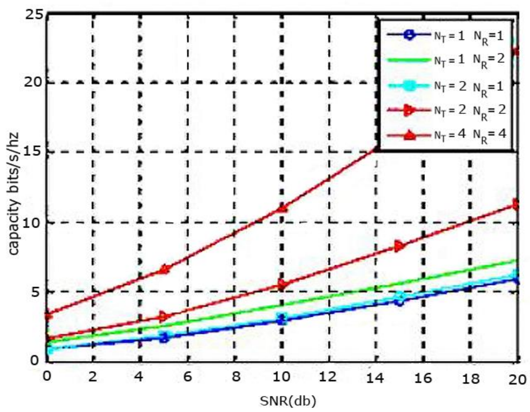

Figure 4.1 MIMO Channel capacity for SISO,SIMO, MISO and MIMO when CSI not available at the transmitter.

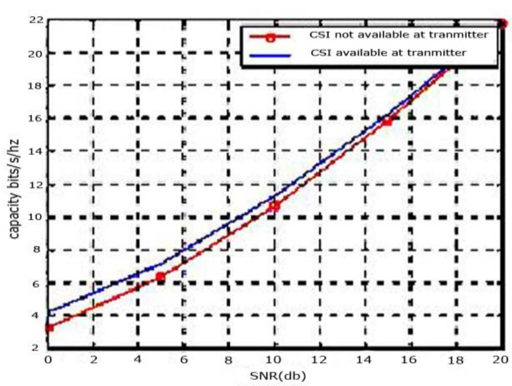

Figure 4.2 Channel capacity comparisons with out and with CSI at transmitter.

Transmitter side channel knowledge has good affect on the capacity if SNR is low. However, if SNR is high channel knowledge at the transmitter side does not have a noticeable effect in the capacity increase. When channel distribution knowledge is available at both sides, increasing the number of antennas in highSNR have only a negligible effect in capacity increase, at moderate SNR the increase in capacity is greater, but now very effective.

Antenna correlation also plays a role in channel capacity. If the SNR is low, and assuming channel knowledge both side, antenna correlation increases capacity, however it decreases channel capacity at high SNR.

Conclusion

This paper describes the capacity calculation of MIMO system. We have presented simulation results comparing channel capacities of SISO, SIMO, MISO and MIMO formula. The simulation results shows that MIMO system with CSI available at the transmitter can greatly improve spectral efficiency over MIMO system without CSI at transmitter.

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