Performance Investigation Of Different Wavelet Families To Optimize Mse Of Digital Image

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Abstract

Denoising is based on incorporating neighbouring wavelet coefficients, with different threshold value for different subband. The choice of the threshold estimation is carried out by analyzing the statistical parameters of the wavelet subband coefficients. To prove the efficiency of this method in image denoising, this method is compared with various conventional wavelet denoising approaches like VisuShrink, and NeighShrink algorithm which is based on neighbouring wavelet coefficients with universal threshold,which gives significant improvement of Mean Square Error (MSE) in both the cases. The observed result shows that, the proposed method yields superior image quality and better MSE.

Keywords: Imagedenoising, wavelet transform, VS, NS, RNS, MSE.

1. Introduction

Image noise is undesired variation in pixel intensity values in a captured or transmitted image. Various wavelet-based methods have been proposed for the purpose of image enhancement and restoration. Basic wavelet image restoration methods are based on thresholding in the sense that each wavelet coefficient of the image is compared to a given threshold; if the coefficient is smaller than the threshold, then it is set to zero, otherwise it is kept or slightly reduced in magnitude. The intuition behind such an approach follows from the fact that the wavelet transform is efficient at energy compaction, thus small wavelet coefficient are more likely due to noise, and large coefficient are generally due to important image features, such as edges. The main idea behind it is that, if the wavelet coefficients estimates are bigger in absolute value of a certain specified threshold then the same value is either retained as such or is diminished by the amount corresponding to the threshold. The smaller coefficients are instead eliminated, hence sparsifying the wavelet expansion. There are two basic approaches to image denoising, spatial domain methods and transform domain methods. The main difference between these two categories is that a transform domain method decomposes the image by a chosen basis before further processing [3] while a spatial domain method processes the observed image data directly.

Transform domain methods have developed rapidly since Donoho's soft thresholding technique [4], which was introduced in 1995. The noise is considered as a high-frequency component in the transform domain for both fast Fourier transform (FFT) and discrete wavelet transform (DWT) and hence thresholding or truncating eliminates noise. The advantage of transform domain methods is that images often have sparse representations in transform domain. Thus dealing with the transform domain is very efficient.

Problem of image denoising can be summarized as follows:Let A(i,j) be the noise-free image and B(i,j) the image corrupted with independent gaussian noise Z(i,j),

$$\mathbf{B_{i,j} = A_{i,j} + \sigma Z_{i,j}} \quad (1.1)$$

Z(I,j) has normal distribution N(0,1).In the wavelet domain the problem can be formulated as

$$\mathbf{Y(I,j)=W(I,j)+N(I,j)} \qquad (1.2)$$

Where Y(I,j) is noisy wavelet coefficient;W(I,j) is true coefficient and N(I,j) is independent Gaussian noise.In this paper the performance of various image – denoising algorithms is evaluated in terms of MSE.

2.Discrtete Wavelet Transform

2.1 Wavelet Thresholding Process

Wavelet thresholding for image denoising attempts to remove the noise present in the signal while preserving most of the signal characteristics, regardless of its frequency content. The complete process of image denoising is shown in figure 2.1 [5] and it involves the following steps:

1. Acquire the noisy digital signal.

2. Compute a linear forward discrete wavelet transform of the noisy signal.

3. Perform a non-linear thresholding operation on the wavelet coefficients of the noisy signal.

4. Compute the linear inverse wavelet transform of the thresholded wavelet coefficients.

This simple four-step process is known as wavelet thresholding or shrinkage.

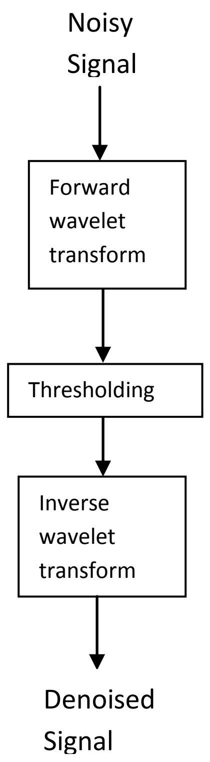

Figure 2.1 Wavelet based image denoising [5]

2.2 Threshold Selection:

As one may observe, threshold determination is an important question when applying the wavelet thresholding scheme. A small threshold may yield a result close to the input, but the result may be still be noisy. A large threshold produces a signal with a large number of zero coefficients. This leads to an overly smooth signal and smoothness generally suppresses the details and edges of the original signal and causes blurring and ringing artifact.

2.3 VisuShrink Algorithm

For VisuShrink algorithm [5], the wavelet coefficients'd' of the noisy signal are obtained first. Then with the universal threshold$\lambda = \sigma\sqrt{2\log n^2}$, ($\sigma$ is the noise level and n is the length of the noisy signal) the coefficients d= $\{d_i\}$, where i = 1, 2. . . n are shrinked according to the soft-shrinkage rule or soft thresholding methodgiven

$$\eta(d) = \begin{cases} \mathrm{sign}(d_i).(|d_i| - \lambda), & |d_i| \geq \lambda \\ 0, & |d_i| < \lambda \end{cases}$$

2.4 NeighShrink Algorithm

NeighShrink algorithm [9] threshold the wavelet coefficients according to the magnitude of the square sum of all the wavelet coefficients within the neighbourhood window.

It is based on the incorporating neighbouring wavelet coefficients with universal threshold. The NeighShrink algorithm is described as follows.

2.4.1Incorporating Neighbouring Wavelet Coefficients

The wavelet transform can be accomplished by applying the low-pass and high-pass filters on the same set of low frequency coefficients recursively. That means wavelet coefficients are correlated in a small neighbourhood. A large wavelet coefficient will probably have large coefficients at its neighbour locations. Therefore, Cai et al. [23] proposed the following wavelet denoising scheme for 1D signal by incorporating neighbouring coefficients into the thresholding process.

Let $d_{j,k}$ is the set of wavelet coefficients of the noisy 1D signal than in equation 2.1

$$\mathbf{s^2}_{(\mathbf{j,k})} = \mathbf{d^2}_{(\mathbf{j,k-1})} + \mathbf{d^2}_{(\mathbf{j,k})} + \mathbf{d^2}_{(\mathbf{j,k+1})} \quad (2.1)$$

If $s^2_{(j,k)}$is less than or equal toλ^2, then set the wavelet coefficient $d_{j,k}$ to zero. Otherwise, these coefficients shrink according to equation 2.2

$$\mathbf{D_{j,k} = d_{j,k}(1\text{-}\lambda^2/\, s^2_{(j,k)})} \qquad (2.2)$$

Where $\lambda = \sigma\sqrt{2\log n}$ and 'n' is the length of the signal. Note that the first (last) term in $s^2_{(j,k)}$ is omit if $d_{j,k}$ is at the left (right) boundary of level j wavelet coefficients. For image denoising, the wavelet coefficients are arranged as a square matrix. For every level of wavelet decomposition, first produce four frequency subbands, namely, LL, LH, HL, and HH. Since the Gaussian noise will be averaged out in the low frequency wavelet coefficients, so keep the small coefficients in these frequencies, only wavelet coefficients in the high frequency levels need to be threshold. That means only the high frequency subbands LH, HL and HH need to be thresholded. For every wavelet coefficient $d_{j,k}$ of our interest, so consider a neighbourhood window $Q_{j,k}$ around it [24] and choose the window by having the same number of pixels above, below, and on the left or right of the pixel to be threshold. That means the neighbourhood window size should be 3 × 3, 5 × 5, 7 × 7, 9 × 9, etc. figure 2.2 illustrates a 3 × 3 neighbourhood window centered at the wavelet coefficient to be thresholded. It should be mentioned in this algorithm that different wavelet coefficient subbands are threshold independently. This means when the small window surrounding the wavelet coefficient to be thresholded touches the coefficients in other subbands, we do not include those coefficients in the calculation. For 2D the square of summation around the window of wavelet coefficients is given by equation 2.3.

$$\mathbf{s^2}_{(\mathbf{j,k})} = \sum\nolimits_{(\mathbf{j,k})\varepsilon \mathbf{Q_{j,k}}} \mathbf{d^2}_{(\mathbf{j,k})} (2.3)$$

Where $d_{j,k}$ is the wavelet coefficient after 2D discrete wavelet transform and $Q_{j,k}$ is the window size centered at the wavelet coefficients to be thresholded as shown in figure 2.2.

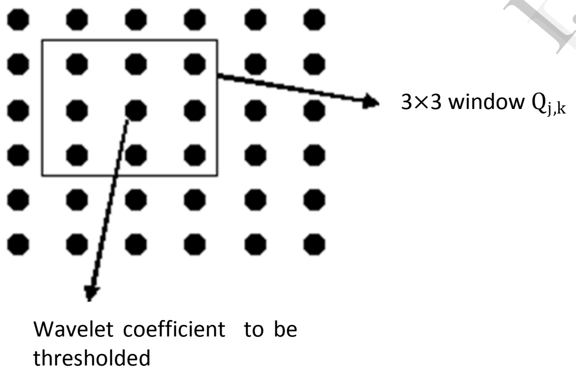

Figure 2.2 An illustration of the neighbourhood window centered at the wavelet coefficient to be thresholded [9].

When the above summation has pixel indices out of the wavelet subband range, the corresponding terms in the summation is omitted.

For the wavelet coefficient to be thresholded [25], it is shrinked according to the following equation 2.4

$$\widehat{\mathbf{d_{j,k}}} = \mathbf{d_{j,k}Q_{j,k}} \ (2.4)$$

Where the shrinkage factor can be defined as equation 2.5

$$\mathbf{Q_{j,k}} = (\mathbf{1} - \boldsymbol{\lambda}^2 / \mathbf{s^2}_{(\mathbf{j,k})})_+ (2.5)$$

Here, the + sign in the formula means it takes nonnegative value, and $\lambda = \sigma\sqrt{2\log n^2}$ is the threshold for the image. This thresholding formula is a modification to the classical soft thresholding scheme developed by Donoho and his coworkers [4]. The neighbourhood window size around the wavelet coefficient to be thresholded has influence on the denoising ability of this algorithm. The larger the window size, the relatively smaller the threshold, If the size of the window around the pixel is too large, a lot of noise will be kept, so an intermediate window size of 3 × 3 or 5 × 5 should be used. The neighbour wavelet image denoising algorithm can be described as follows:

(1) Perform forward 2D wavelet decomposition on the noisy image.

(2) Apply the proposed shrinkage scheme to threshold the wavelet coefficients using a neighbourhood window $Q_{j,k}$ and the universal threshold $\sigma\sqrt{2\log n^2}$

(3) Perform inverse 2D wavelet transform on the thresholded wavelet coefficients.

This algorithm is known as NeighShrink algorithm. Because VisuShrink algorithm kills too many small wavelet coefficients, so this shrinkage schemes gives the better result.

2.4.2 Limitation of NeighShrink Algorithm:

In the above mention that this algorithm is based on soft thresholding technique that is based on kill or shrink rule according to the wavelet coefficients and threshold value but it is use the universal threshold for every subbands. Normally in wavelet subbands, as the level increases the coefficients of the subband becomes smoother [1]. For example the subband HL2 is smoother than the corresponding subband in the first level (HL1) and so the threshold value of HL2 should be smaller than that for HL1. This is the limitation of this method which is use universal threshold for every subbands. This limitation is overcome in our proposed method. In propose proposed method we take the NeighShrink algorithm with different threshold value for different subbands which is based on Generalized Gaussian Distribution (GGD) modeling of subband coefficients.

2.5 Revised NeighdShrinkAlgorithm(proposed method)

In the NeighShrink algorithm different wavelet coefficient subbands are shrinked independently, but the threshold λ keep unchanged in all subbands. The shortcoming of this method is that the threshold λ in all subbands is suboptimal. The optimal λ of every subband should be data-driven and maximize the peak signal to noise ratio (PSNR). We will improve NeighShrink by determining an optimal threshold for every wavelet subbandwhich is based on Generalized Gaussian Distribution (GGD) [1] modeling of subband coefficients. In this proposed method, the choice of the threshold (λ) estimation is carried out by analyzing the statistical parameters of the wavelet subband coefficients like standard deviation, arithmetic mean and geometrical mean as shown in equation 2.6

$$\Lambda = C(\sigma - (|AM-GM|) \quad (2.6)$$

Here σ is the noise variance of the corrupted image [21],[22] .
The term C is depend on number of decomposition level and the level where the subband is available at that time which is given in equation 2.7.

$$C = 2^{(L-k)} \quad (2.7)$$

Where, L is the no. of wavelet decomposition level, k is the level at which the subband is available.
The Arithmetic Mean and Geometric Mean of the subband matrix $d_{(j,k)}$ are given in equation 2.8 and 2.9.

$$\text{Arithmetic Mean} = \frac{\sum_{j=1}^{m} \cdot \sum_{k=1}^{m} d_{(j,k)}}{M^2} \quad (2.8)$$

$$\text{Geometric Mean} = \left[\prod_{j=1}^{m} \cdot \prod_{k=1}^{m} d_{(j,k)}\right]^{\frac{1}{M^2}} \quad (2.9)$$

Steps of Revised NeighShrink algorithm:
The Complete algorithm of proposed wavelet based image denoising technique is explained in the following steps.

(1) Perform the DWT of the noisy image using Mallat algorithm [18] upto L levels to obtain (3L+1) subbands, for L=2 levels subbands are named as HH1, LH1, HL1, HH2, LH2, HL2 and LL2.In figure 2.3 the LL1, LH1, HL1 and HH1 be the four subbands of image after first decomposition step and LL1LL2, LL1LH2, LL1HL2, LL1HH2 are the four subbands of image when LL1 subband is decomposed in second decomposition step. Similarly LH1LL2, LH1LH2, LH1HL2, LH1HH2 be the subbands when LH1 is decomposed in second step. HL1LL2, HL1LH2, HL1HL2, HL1HH2, be the subbands when HL1 is decomposed in second step and HH1LL2, HH1LH2, HH1HL2, HH1HH2 are the subbands when HH1 is decomposed. The total no. of subbands after second decomposition level is 16. After L decompositions, a total of D(L) = subbands are obtained. Where L is the no. of decomposition level.

LL1LL2	LL1HL2	HL1LL2	HL1HL2
LL1LH2	LL1HH2	HL1LH2	HL1HH2
LH1LL2	LH1HL2	HH1LL2	HH1HL2
LH1LH2	LH1HH2	HH1LH2	HH1HH2

Fig 2.3 Subband structure after two level packet decomposition.

(2) Compute the threshold value for each subband, except the approximate coefficients band using equation (2.5) after finding out its' following terms.Obtain the noise variance from equation (2.10) Find the term C for each subband using equation [1] (2.7).Calculate the term |AM-GM| for each subband (except approximate coefficients subband) using equations (2.8) and (2.9).

(3) Put the threshold value in equation [9] (2.5) of all subband coefficients (except approximate coefficients subband) for calculating the shrinkage factor. And then find the noiseless coefficient using equation (2.4)

(4) Perform the inverse DWT to reconstruct the denoised image.The information from the four sub-images is up-sampled and then filtered with the corresponding inverse filters along the columns The two results that belong together are added and then again up-sampled and filtered with the corresponding inverse filters. The result of the last step is added together in order to get the original image again.Hence there is no loss of information when the image is composed at full precision.

3. RESULTS AND DISCUSSION

Table 3.1 MSE of the noisy images and denoised images of standard image testpat1 using db5 wavelet

S.No	Noise levels	MSE of noisy images	MSE of denoised images using different algorithms		
			VS	NGS	RNGS
1.	10	100	190.2	66.8179	53.7066
2.	15	225	288.7	132.3688	108.4250
3.	20	400	384.4	214.8365	176.5279
4.	25	625	478.8	307.0986	255.7125
5.	30	900	577.1	402.2768	342.6551
6.	35	1225	680.6	489.7501	432.3564
7.	40	1600	793	576.6547	524.8454
8.	45	2025	915.1	664.2541	619.3196
9.	50	2500	1046.8	747.9031	717.5827

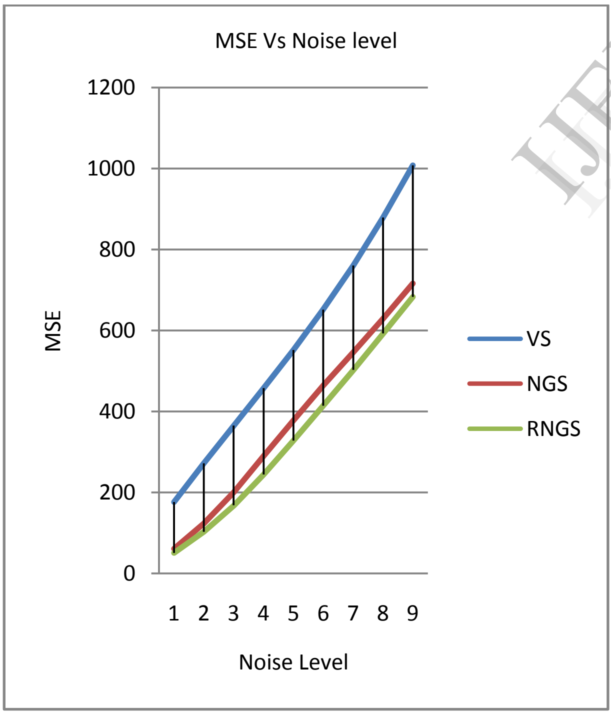

Figure 3.1 MSE of the noisy images and denoised images of standard image testpat1 using db5 wavelet.

Table 3.2 MSE of the noisy images and denoised images of standard image testpat1 using sym5 wavelet

S.No	Noise levels	MSE of noisy images	MSE of denoised images using different algorithms		
			VS	NGS	RNGS
1.	10	100	182.6	65.1276	52.9235
2.	15	225	278.1	130.1315	106.2353
3.	20	400	368.4	212.2812	172.9380
4.	25	625	460.8	302.2990	251.1762
5.	30	900	556.9	394.2354	335.7390
6.	35	1225	658.5	487.5670	425.5164
7.	40	1600	766.8	577.4514	516.2479
8.	45	2025	884.0	667.1109	609.0710
9.	50	2500	1011.6	759.1345	703.8926

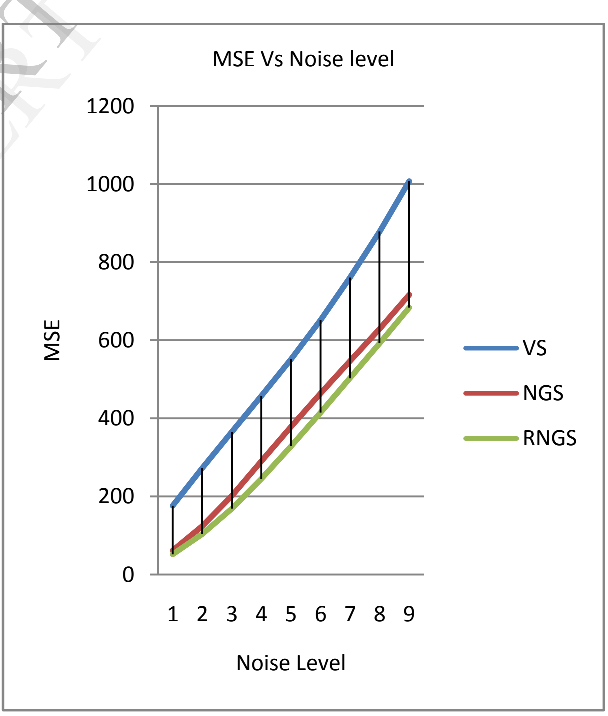

Figure 3.2 MSE of the noisy images and denoised images of standard image testpat1 using sym5 wavelet.

4. Conclusion:

In this paper work, firstly a comparative analysis between the two conventional denoising algorithms i.e. VisuShrink and NeighShrink has been made. Out of these two algorithms NeighShrink gives the better MSEthan the other two algorithms.The conventional NeighShrink algorithm is modified by considering the different threshold value for different subbands that is based on Generalized Gaussian Distribution (GGD) modeling.The results have shown that the denoising of images through the Revised NeighShrink algorithm achieved enhancement in MSE.

5. Future Scope

The field of images processing has been growing at a very fast pace. The day to day emerging technology requires more and more revolution and evolution in the images processing field.

The work proposed in this paper also portrays a small contribution in this regard. The proposed denoising technique can provide a good platform for further research work in this respect.

Future work may be done for improving the Mean Square Error by considering the adaptive window size for every sub band over Proposed algorithm.

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