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Performance Evaluation of Statistical and Geometrical Algorithms for Spectral Unmixing of Hyperspectral Data

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Abstract

Hyperspectral image processing is an important area of research nowadays. Since the cameras used for capturing the hyperspectral images is having low spatial resolution, the spectra of observed pixels will be the mixtures of various present in the scene. Thus spectral unmixing aims at estimating the no. of endmembers(reference materials), their spectral signatures and corresponding abundance maps in the captured hyperspectral data. This paper presents performance evaluation and comparative study of statistical and geometrical approaches used for spectral unmixing. The algorithms evaluated are ICA(independent Component Analysis), AVMAX (Alternating volume maximization), SVMAX (Successive volume maximization) and ADVMM (Alternating decoupled volume max-min).The algorithms are implemented and validated on real hyperspectral dataset AVIRIS cuprite data collected over Nevada, U.S in 1997.

***Keywords:* ADVMM, AVMAX, ICA,Spectral signature Spectral unmixing, SDVMM,SVMAX**

1. Introduction

Hyperspectral imaging and analysis of hyperspectral data is an interesting research area in present days. The technique of hyperspectral imaging employs the technique of analysing the electromagnetic scattering patterns of various materials in the captured scene[1].It employs not only the visible region of the electromagnetic spectrum, but also the near infrared and mid infrared regions(0.3-2.5μm)[2]. Thus it captures the information in hundreds of narrow contiguous bands, and thus it can provide more spectral information compared to the images which have been taken using only visible region of electromagnetic spectrum [2].This points towards an efficient way for the identification of various materials present over the observed scene. This is being used in various fields as planetary remote sensing, agricultural monitoring, environmental monitoring, mineral identification, oil spill detection etc[3].

Hyperspectral sensors used for acquiring is having limited spatial resolution and thus the spectra of pixels in the observed image will be a complex mixture of various materials present over the scene. This makes the further analysis of captured image more difficult.Fig1 [4] explains the concept of hyperspectral imaging. In this scenario, the spectral unmixing comes to the screen. Spectral unmixing solves this problem of mixing of various spectra, by the decomposition of measured spectrum of the captured scene in to a collection of reference materials (endmembers),their spectral signatures and their corresponding abundance maps ,which helps to identify various materials in the scene[5],[6].

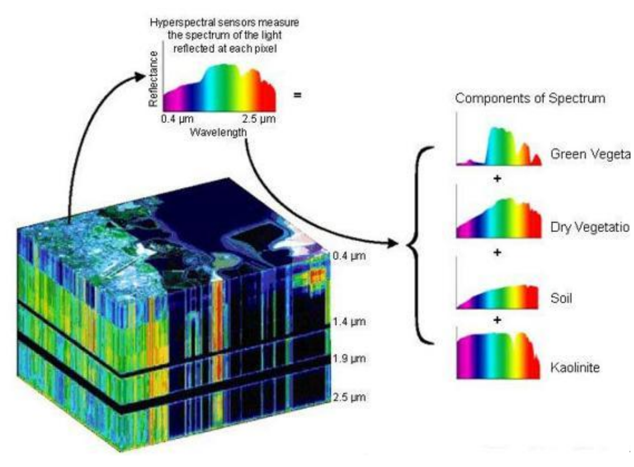


Fig1.Concept of hyper spectral imaging.

1.1SPECTRAL UNMIXING-OVERVIEW

Hyperspectral unmixing is an important problem which is being subject to many investigative researches for past many years. This is an important technique for hyperspectral data exploitation. As stated earlier, spectral unmixing infers a group of pure spectral signatures and their corresponding abundance maps from an observed scene. Since the hyperspectral image contain many sources which combine in a linear or non-linear fashion and statistically dependent, this makes spectral unmixing to be placed in a higher level in the set of source separation problems.

Spectral unmixing can be classified mainly in to 2 models namely Linear [7] and Non linear [8] . In this linear model is the most popular model used for unmixing, whereas nonlinear model is not being used commonly since it's more complicated when compared with linear models. Fig 2 & 3 [6] gives the pictorial explanation of linear and nonlinear models respectively.

When looking in to the case of linear models, the scale of mixing is macroscopic and only single scattering takes place. i.e., the light falls interacts, with one material only. This type of mixing occurs due to the low spatial resolution of hyperspectral sensor used.

The brief overview of mathematical explanation of this model is given as follows [9],[10].

Linear models have an assumption that, the pixels of the observed scene will be a linear combination of all endmembers present over the observed surface. This can be shown mathematically as follows.

$$y = M\alpha + n \qquad (1)$$

Where y is an Lx1 column vector ,M is an Lxq matrix containing q endmembers(pure reference materials) and α is a qx1 vector containing the fractional abundances of the endmembers in the pixel and n is another Lx1 vector indicating the errors which affect the measurements at each pixel[10]. In this modelling both M and α have to be found by unmixing. Here ANC(abundance non-negativity constraint),where i=1,2….q and ASC(abundance sum to one constraint), $1^T\alpha = 1$are imposed to this model. This takes another fact into consideration as α_i,for i=1,2…q ,represent the fractions or proportions of the pure materials or endmembers present in the scene. In this $Y \equiv \{y_i \in R^L, i = 1,...n\}$ of n no. of observed spectral vectors with dimension L.

When considering the case of Nonlinear models, from fig 3,it can be seen that multiple scattering occurs when light falls on the surface. Thus interaction between the light scattered by multiple materials occurs and this interaction can be at microscopic level. This makes the model more complicated and difficult to use.

In this paper we consider linear models for unmixing of hyperspectral data because of it is reduced computational complexity and ease of use.

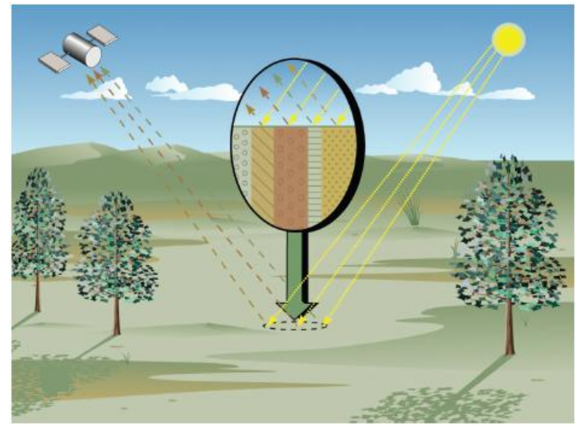


Fig2.Linear model with single scattering

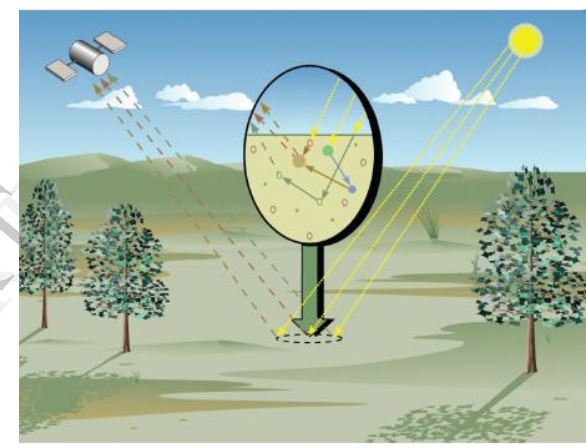


Fig3.Non linear model with multiple scattering

1.2 SPECTRAL UNMIXING ALGORITHMS-OVERVIEW

Spectral unmixing algorithms are broadly classified in to 3 types namely statistical, geometrical and sparsity based unmixing approaches. A brief introduction to these approaches is given as follows.

Statistical approaches[11] are formed on a Bayesian approach. Since the computational complexity of statistical approaches is high when compared with other unmixing approaches ,it's rarely used. Statistical dependence of sources are taken in to consideration in this case, and ICA[12](independent component analysis),DECA[13](Dependent component analysis) etc are examples of some popular algorithms coming under this category.

Geometrical approaches[11] is the another category of spectral unmixing algorithms. In this the basic assumption is that when under linear mixing model, the spectral vectors comes under a simplex, whose vertices correspond to pure reference materials (endmembers) .

These are most popularly used spectral unmixing algorithms. In this again comes 2 sub classes. These are pure pixel based approaches and minimum volume based approaches.

Pure pixel based algorithms assume the presence of at least one pure pixel per endmember. Examples are VCA[14](vertex component analysis), AVMAX[15], SVMAX[15],ADVMM[16],SDVMM[16],N-finder[17] etc. When pure pixels are not available in the observed scene, then we go for minimum volume based algorithms. The examples are MVSA[18],MVES[19], and SISAL[20].

The 3rd type of algorithms comes under sparse based approaches[11].In this unmixing problem is formulated in a semisupervised fashion. In this it's assumed that the observed spectral signatures can be expressed as a linear combination of known spectral signatures from a library. The most popular algorithms under this category are SUNSAL[21],OMP[21],ISMA[21],and SUNSAL-TV[9].

In this paper the performance evaluation and comparative study the following algorithms namely AVMAX,SVMA,ADVMM and ICA is done after applying on real hyperspectral data CUPRITE.

The rest of the paper is organized as follows. section 2 gives a brief overview of algorithms employed, section 3 gives the experimental results and performance evaluation. Section 4 gives the conclusion of the work done followed by references.

2. Algorithms employed-Overview

2.1 AVMAX (Alternating volume maximization)

Alternating volume maximization algorithm[15] is based on winter problem described in [22].In winter's work he proposed that the ground-truth endmembers can be located by finding a collection of pixel vectors whose simplex volume is the largest. The optimization formulation of winters problem is as follows.,[22],[10].

$$\max_{\nu_1,\ldots,\nu_N \in R^{N-1}} \quad vol(\nu_1,\ldots\ldots,\nu_N)$$
$$s.t \quad \nu_i \in conv\{x[1],\ldots\ldots,x[L]\}, \quad i=1,\ldots.,N \quad (2)$$

Where according to winters work each endmember estimate ν_i is restricted to be any vector in $\{x[1],\ldots\ldots,x[L]\}$ .when alternating volume maximization is applied to this it maximizes in a cycic fashion ,the volume of the simplex defined by the pure members(endmembers) but with respect to only one endmember at a time.This is explained as follows in[22].

The starting point is taken as $(\nu_1,\ldots..,\nu_N)$ .The following alternating cycle is repeated as for j=1…N solve the problem

$$\max_{\nu_j \in F} \ \det(\Delta(\nu_1,\ldots\nu_{j-1},\nu_j,\nu_{j+1}..,\nu_N)) \quad (3)$$

And update ν_j as the solution of (3).we have to continue this until the stopping criterion is satisfied. The algorithm is explained in detail in [15].AVMAX is somewhat similar to SC-N-FINDR which is a modified version of N-FINDR described in[23].

2.2 SVMAX (Successive volume maximization)

Successive volume maximization [15] is another strategy of optimization for the winter's problem shown in(2).This requires the winter's problem to be written in a modified fashion as follows in[22],[10].

$$\max_{w_1,\ldots\ldots,w_N \in R^N} \quad |\det(w)|$$
$$s.t \quad w_i \in \overline{F}, \quad i=1,\ldots.,N \qquad (4)$$

Where

$$\overline{F}=\{w \in R^N \mid w=[\nu^T \ 1]^T, \nu \in F\}$$

Then according to rules |det (w)| can be written as follows.

$$|\det(w)| = \sqrt{|\det(w^T w)|} \qquad (5)$$

Thus (4) is modified as follows

$$\max_{w_1,\ldots\ldots,w_N \in R^N} \ f_1(w_1) f_2(w_1,w_2)\ldots. f_N(w_1,\ldots.,w_N) \quad (6)$$
$$s.t \qquad w_i \in \overline{F}, \quad i=1,\ldots\ldots..,N.$$

Where

$$f_1(w_1) = \|w_1\|_2$$

$$f_j(w_1,\ldots..w_j) = \left\| P \perp_{W_{1:(j-1)}w_j} \right\|_2, \quad j=2,\ldots.,N$$

Thus the following procedure is followed.
For j=1:N solve the problem

$$W_j = \arg\max_{w_j \in \overline{F}} \ f_j(w_{1,\ldots\ldots} w_{j-1}, w_j) \qquad (7)$$

At last we will get $(w_1,\ldots\ldots\ldots.,w_N)$ as the approximate solution of(6).It's similar to VCA[14] in some aspects. But unlike VCA algorithm SVMAX considers the whole subspace when the data is projected orthogonally whereas VCA takes random direction in subspace.

2.3 ADVMM (Alternating decoupled volume max-min)

In this winter's problem shown in(2)is formulated as a max-min problem and alternating optimization [16]is used to solve it. This winter's worst case problem is given as a max-min problem as follows in[24],[10].

$$\max_{\substack{v_i \in R^{N-1},\\ i=1,\ldots,N}} \{\min_{\substack{\|u_i\| \le r,\\ i=1,\ldots,N}} \left|\det(\Delta(v_1 - u_{1,\ldots\ldots,} v_N - u_N))\right| \qquad (8)$$

$$s.t\ v_i \in conv\{y[1],\ldots., y[L]\}, i = 1,\ldots.., N$$

Where y[1],…y[L] is the data cloud inside which maximum volume simplex is situated .From the vertices of this simplex endmembers are to be found out.

By taking $v_i = Y\theta_{i,}$ & $Y = [y[1],\ldots., y[L]] \in R^{(N-1)\times L}$

and for any permutation matrix p, $\det(P\Delta) = \pm\det(\Delta)$ we can write the problem in (8) as

$$\max_{\substack{\theta_i \in s\\ i=1,\ldots,N}}\{\min_{\substack{\|u_i\| \le r,\\ i=1,\ldots,N}} \det(\square(Y\theta_1 - u_1,\ldots\ldots, Y\theta_N - u_N))\} \qquad (9)$$

Then by doing the cofactor expansion and simplification of (9) as in[24] it is reduced to

$$\max_{\theta_j \in s}\{\min_{\|u_j\| \le r,} k_j^{\ T}(\square(Y\theta_j - u_j))\} \qquad (10)$$

The above problem can be solved by solving the 2 decoupled problems shown below.

$$u_j = \arg\max_{\|u_j\| \le r} k_j^T u_j = rk_j / \|k_j\| \qquad (11)$$

$$\theta_j = \arg\max_{\theta_j \in s} k_j^T Y\theta_j = e_l, l = \arg\max_{n=1,\ldots.,L} k_j^T y[n] (12)$$

Thus ADVMM solves the max-min problem of spectral unmixing.

2.4 ICA(INDEPENDENT COMPONENT ANALYSIS)

All the 3 algorithms discussed above are geometrical algorithms. But ICA(Independent Component Analysis) comes under statistical approaches. A brief explanation of this algorithm is given as follows and it's explained in detail in[12].

This algorithm mainly works on two assumptions as follows1)The observed spectrum vector is a linear mixture of the constituent spectra (endmember spectra) weighted by the correspondent abundance fractions 2)Sources are statistically independent.In hyperspectral data,[31] the first assumption is valid whenever the multiple scattering among distinct constituent substances (endmembers) is negligible, and the surface is partitioned according to the fractional abundances.The second assumption, is violated, since the sum of abundance fractions associated to each pixel is constant due to physical constraints in the data acquisition process.

ICA consists in finding a linear decomposition of observed data into statistically independent components[31].It is based on the assumption of mutually independent sources, which is not the case of hyperspectral data, since the sum of the abundance fractions is constant, implying dependence among abundances .Hyperspectral data is immersed in noise, which degrades the ICA performance.This method is based on mutual information minimization.It considers behaviour of mutual information as a function of unmixing matrix.The unmixing matrix minimizing the mutual information might be very far from the true one. Nevertheless, some abundance fractions might be well separated, mainly in the presence of strong signature variability, large number of endmembers, and a high signal-to-noise ratio (SNR).

Let "r" be an βx1 observation column vector,such as

r=Ms (13)

Where M is an unknown βxp mixing matrix and s=[s1 s2……….Sp]T is an unknown random data vectorof mutually independent sources having unknown distributions. ICA finds a pXβ seperating matrix W,such that

y=Wr=PCs (14)

Where y is a vector of independent components.This is explained mathematically in detail in [12].Thus ICA solves the problem of spectral unmixing.

3. Experimental results & performance evaluation

In this section the experimental results of the 4 mentioned algorithms AVMAX,SVMAX,ADVMM and ICA on the real hyperspectral dataset cuprite data taken over Nevada, U.S in 1997[25] is evaluated and performance analysis is also done..The three geometrical algorithms AVMAX,SVMAX and ADVMM and statistical approach ICA are applied to this dataset for performing spectral unmixing. We consider only a sub image of the hyperspectral data as a region of interest for reducing the computational complexity, which is of size 250x191 pixels(L=47750).This contains 224 bands over the wavelength region of 0.4µm to 2.5 µm. As the next step, we should have a knowledge about, how many endmembers(pure reference materials) are located in this region of interest of 250x191 pixels. For this we have applied Hyperspectral subspace identification by minimum error (HySime) [26] and thus it was obtained that the number of endmembers existing in this region is N=18.In this dataset, the bands 1-2,104-113,148-167 and 221-224 were already removed from this , due to low SNR effect which occurs due to the effect of water vapour and atmospheric effects. Thus the now dataset contains 188 bands instead of 224. The abundance maps corresponding to each mineral was obtained using fully constrained least square (FCLS) [27] method. The minerals obtained by the unmixing process was identified by doing the visual comparison of the abundance maps from the output with the abundance maps shown in [14],[15],[16]and[28].

As the result of spectral unmixing we will get spectral signatures of endmembers and also their corresponding abundance maps as outputs. Here in this paper the metric used for the evaluation of results is Spectral Angle Mapper (SAM)[29]. It's measured between the original library spectra which we will get from U.S.G.S library [30], and the spectra obtained as the output by the unmixing process. The basic equation for the spectral angle is given as follows in [29].

$$\theta(x,y) = \arccos\left(\frac{(<x,y>)}{\|x\|\|y\|}\right) \qquad (14)$$

Where, x is the reference library spectra and y is the spectra obtained from spectral unmixing. As the value of spectral angle (SA) decreases, the result becomes more precise and when we get a high value for SA we can conclude that the performance of algorithm is poor[10],[29]. The values of SA for all the estimated endmembers obtained by all the four algorithms mentioned is shown in Table1.The numbers which are kept in parantheses denote the value of SA for the estimated endmember which is repeated. Due to the space limit here we have shown the estimated endmember signatures and abundance maps of SVMAX algorithm only. In this repeated mineral maps and signatures are not shown due to space limit. The abundance maps and spectral signatures are shown below.

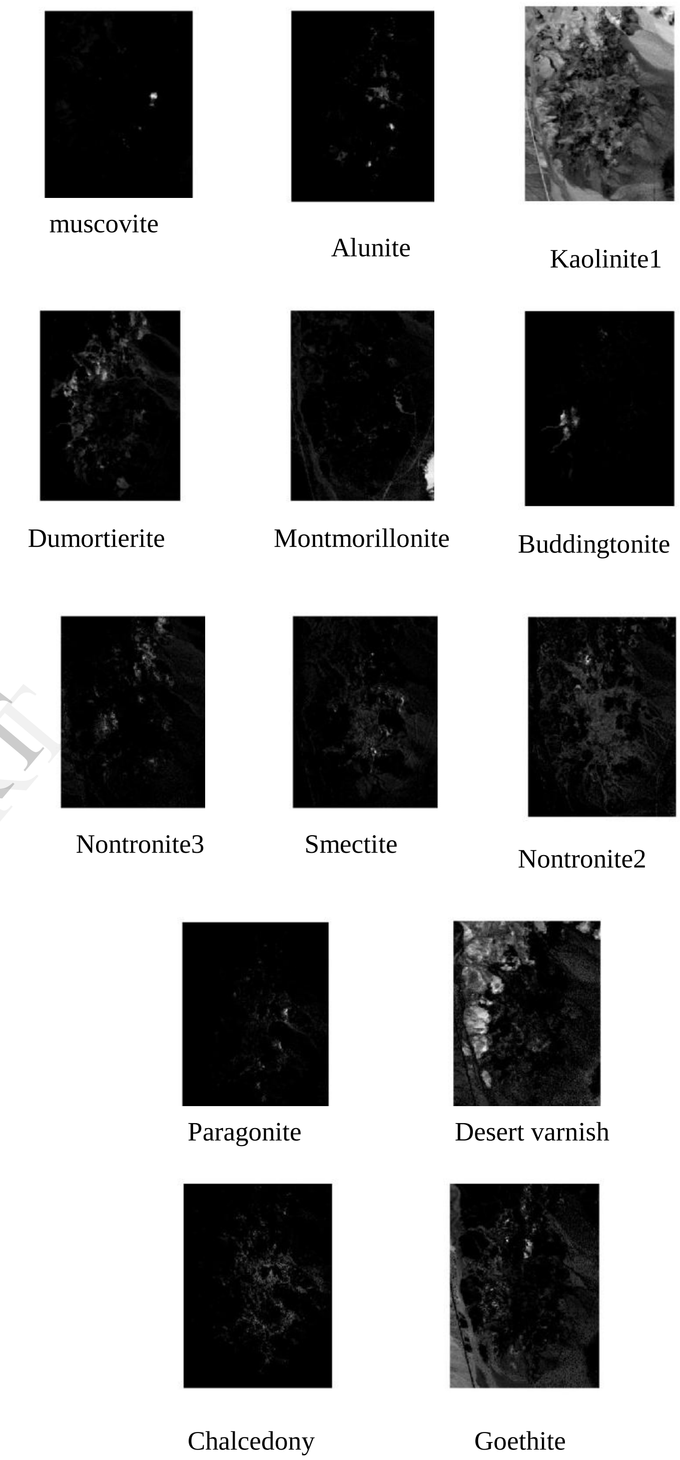
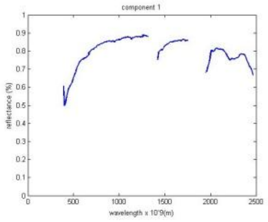
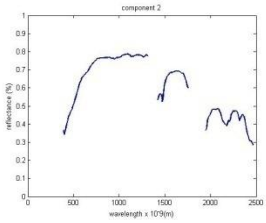


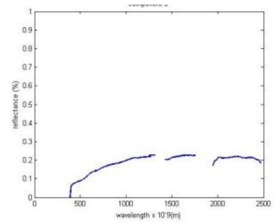
Fig 4 Abundance maps obtained by SVMAX algorithm



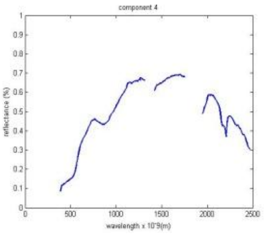
Muscovite



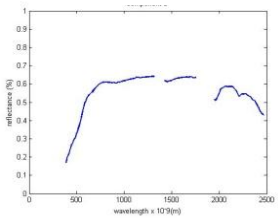
Alunite



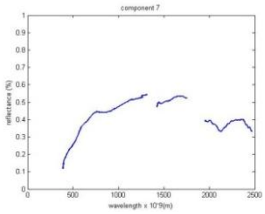
Kaolinite 1



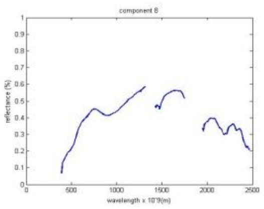
Dumortierite



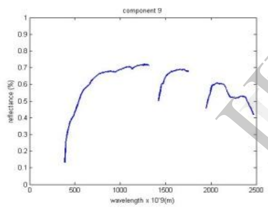
Montmorillonite



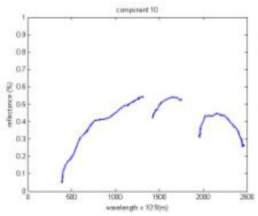
Buddingtonite



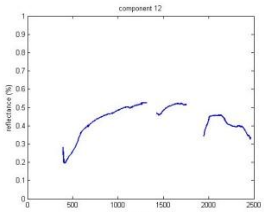
Nontronite 3



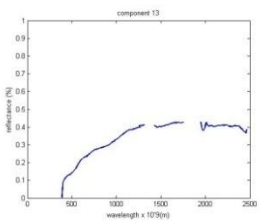
Smectite



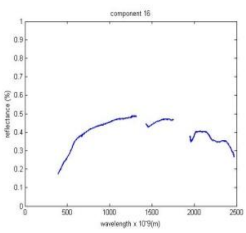
Nontronite 2



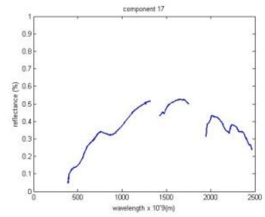
Paragonite



Desert varnish



Chalcedony



Goethite

Fig 5 Spectral signatures estimated by SVMAX algorithm.

The figures shown above shows the spectral signatures and abundance maps of minerals identified by SVMAX(Successive volume maximization).Table 1 shows the SA values measured for all the 4 algorithms as AVMAX,SVMAX,ADVMM and ICA. The SA values are measured with reference to U.S.G.S spectral library. When going through the table it can be seen that, out of 4 algorithms, Average SA is very high for ICA, which is a statistical approach. This gains a value of Average SA as 22.23.This points towards poor performance of ICA algorithm in spectral unmixing. In the group of 4 algorithms ,SVMAX gives better performance compared to all the three algorithms with an average SA of 8.00.Then ADVMM(Alternating decoupled volume max-min) comes after SVMAX with an average SA of 8.37. Then AVMAX (Alternating volume maximization) comes to the third place with an average SA of 9.23. (All these SA values are measured in degrees.)

When looking as a whole it is very clearly seen that in the case of spectral unmixing geometrical approaches based on pure pixel assumption gives better performance compared to statistical approach as ICA. This is because the computational complexity for geometrical approaches is very low when compared with statistical approaches, and these algorithms have a high state of accuracy when compared to statistical approaches.

Thus in this set 4 algorithms, SVMAX a pure pixel based algorithm gives good performance compared to all other algorithms, and this solves winter's problem using successive volume maximization in an efficient way and thus solves the problem of spectral unmixing.

ICA(Independent Component Analysis) gives poor result because it is working based on independent components derived from the given hyperspectral data[31].Moreover it is based on the assumption of mutually independent sources, which is not the case of hyperspectral data, since the sum of the abundance fractions is constant, implying dependence among abundances .In addition to that hyperspectral data is immersed in noise, which also degrades the ICA performance.

Table 1.SAM values for 4 algorithms ICA, AVMAX, SVMAX & ADVMM
(In degrees)

MINERALS	ICA	AVMAX	SVMAX	ADVMM
1)Buddingtonite	25.96	4.37	3.93	4.37
2)Nontronite2	12.69(11.22)	4.06	3.66(4.98)	3.66(4.93)
3)Nontronite3	_	8.56(13.94)	8.90(6.76)	_
4)Kaolinite1	27.50 (29.51) (27.54)	8.45	20.62(8.45)	11.99(21.22)
5)Kaolinite2	12.51	_	_	_
6)Alunite	27.66(21.75)	14.20(11.63)	4.55(12.54)	7.46(8.31)
7)Desert varnish	25.20	11.95	6.33	6.12
8)Dumortierite	_	9.91(6.17)	9.91(6.56)	6.22(6.82)
9)Chalcedony	26.31(29.90) (19.24)	5.85	4.82	8.94
10)Andradite	24.84(25.13)	6.95	_	8.27
11)montmorillonite1	8.35	7.54(8.45)	7.32	7.26
12)Smectite	_	4.73	5.98	3.20
13)Paragonite	_	_	6.35	6.33(6.58)
14)muscovite	12.56	7.51(8.11)	7.94	8.11
15)Goethite	_		14.48	20.93
16)Pyrope1	32.40	_	_	_
17)Nontronite1	_	23.89	_	_
AVERAGE SA	22.23	9.23	8.00	8.37

4. Conclusion

In this paper ,performance evaluation and comparative of 3 pure pixel based geometrical algorithms and one statistical approach is done. All the 4 algorithms are applied on the real hyperspectral dataset CUPRITE taken over NEVADA,U.S in 1997. The metric used for the validation of all the algorithms is SAM(Spectral angle maaper).This is measured in degrees between the original library spectra from U.S.G.S library and the spectra obtained by spectral unmixing. Thus the performance analysis of all 4 algorithms is done.

From the comparative study it was found that SVMAX(Successive volume maximization) gives the better performance compared to other 3 algorithms.ICA (Independent component analysis) a statistical approach is the one which gives much lower performance in spectral unmixing compared to other algorithms.

As a work in future more spectral unmixing algorithms coming under sparse and geometrical approaches can be included in the research studies and more comparative studies can be done .

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