

Performance Evaluation of Recursive DFT as Phasor Estimator in PMUs under Power Quality Disturbances

Munukutla Naga Chaitanya
PG Student, Department of Electrical Engineering
Andhra University
Visakhapatnam-530003, India

Dr. G. V. Siva Krishna Rao
Professor, Department of Electrical Engineering
Andhra University
Visakhapatnam-530003, India

Abstract—Due to the increased penetration of distributed generators into the distribution system as well as due to various power electronic devices in the power network, there exists strong disturbances in electrical waveforms i.e., in amplitude, phase and frequency. These fluctuations need to be supervised and monitored for efficient energy management, safety and also protection purposes. Nowadays, this task is performed by Phasor Measurement Units (PMUs), which measure the phasor of voltage and current waveforms on a common timescale synchronized to the Coordinated Universal Time (UTC). Phasor Measurement Units (PMUs) are also expected to quickly measure fundamental frequencies and rate of change of such frequencies (ROCOF) by accurate parameter estimation algorithms. In this paper the commonly used recursive DFT algorithm for phasor estimation in PMUs was simulated in MATLAB platform. Then this algorithm is tested with various power quality disturbance waveforms and its performance is investigated.

Index Terms—Distributed generators, energy management, phasor measurement unit (PMU), coordinated universal time (UTC), recursive DFT algorithm.

I. INTRODUCTION

Synchronized Phasor Measurement Units (PMUs), since their introduction in 1980s gave impetus to large-scale implementation of wide-area measurement systems (WAMS) using PMUs and phasor data concentrators (PDCs) in a hierarchical structure. Initially PMU technology applications are mostly concerned with the validation of system models, accurate postmortem analysis. With widespread application, they are able to perform linear state estimation and track dynamic phenomena in real time. Due to the utilization of time synchronized sampling for PMUs located over the entire power system, we are able to obtain simultaneously the individual phasor from individual PMUs located at remote areas also at particular instants of time. This lead to the PMUs utility to improve protection and control functions [12].

In Power Systems, high voltage transmission and distribution lines are important links from the generating units to the end users. Protection of these lines using relays plays a vital role from the view point of security, economics and quality of power supplied. Hence correct action of relays is important in power systems. Due to DC offset present in fault current waveforms the line relays tends to overreach. So, these

DC offset components has to be removed from these waveforms [13].

As per [2] DFT based phasor estimation is the most commonly used technique in currently available PMUs. Two classifications of power system transients are electromagnetic transients and electromechanical transients. Electromechanical transients are characterized by magnitude and phase angle modulation of power system voltages and current with low frequency signals corresponding to the movement of rotors of large electric machines around the synchronous speed. Electromechanical transients are characterized by step changes in the magnitude and phase angles of the waveform. They contain sustained harmonics and non harmonic content. Disturbances in phasor estimations due to harmonics can be eliminated by DFT type phasor estimators.

With the advent of advanced microcontrollers and digital signal processors (DSPs) in relay implementation, DFT filter has gained importance for measuring fundamental and harmonic content of a waveform. DFT filter can be implemented in recursive and non recursive forms. Recursive DFT is commonly used algorithm. Errors are introduced during phase and magnitude estimation using DFT filter due to sampled signal dynamics and frequency deviations [5].

A practical PQ event data recorded at an industrial site consists of interruption, sag, swell, harmonics, transients, system imbalance, frequency deviations and combinations of these disturbances [4].

In this paper, performance evaluation of recursive Discrete Fourier Transform for phasor estimation under various power disturbance waveforms is analyzed. This is carried to validate the performance of recursive DFT as a phasor estimator in a PMU. Basic concept of phasor and phasor measurement are covered in introductory section II. Recursive DFT mathematics is reviewed in section III. In Section IV performance evaluation of PMU considered will be carried out. Finally Section V concludes up with the key points analyzed in this paper.

II. BASICS OF PHASOR, PHASOR TECHNOLOGY AND ITS APPLICATIONS

A. Concept of Phasor

A sinusoidal signal can be formulated as

$$x(t) = X_m \cos(2\pi f_0 t + \phi) \quad (2a)$$

Where X_m the amplitude of the signal, f_0 is the nominal system frequency and ϕ is the angular position with respect to an arbitrary time reference.

A phasor is the representation of a sinusoidal signal by its magnitude and phase angle at a given frequency. The phasor representation \mathbf{X} of the sinusoid of (2a) is given by

$$\mathbf{X} = \left(\frac{X_m}{\sqrt{2}} \right) e^{j\phi} \quad (2b)$$

The magnitude of the phasor is the rms (root mean square) value of the sinusoid. As in steady state the frequency of phasor does not change, it is easier to represent signal and carry out computations in phasor form rather than in instantaneous form. But, during a disturbance in power system, frequency is not constant but time varying. Hence phasor concept cannot be applied under transient conditions. A recently published IEEE standard C37.118.1 [17] provides necessary guidelines on steady state and dynamic characteristics of phasor measurement.

B. Phasor Measurement Technology & its Applications

Phasor Measurement Units are devices used to measure input signal fundamental phasor, frequency and rate of change

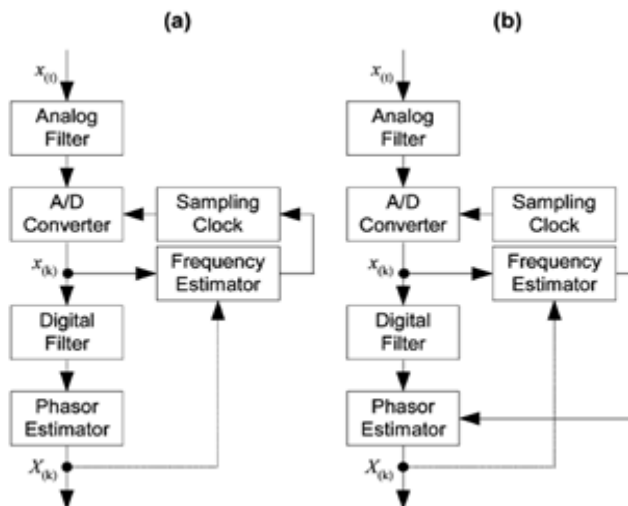


Fig.1. Basic phasor estimation architectures (a) Frequency tracking (b) Frequency compensation

of frequency (ROCOF). Two possible architectures used in PMUs for real time phasor estimation are shown in Fig.1 (a) and (b) [2].

During a power disturbance, the voltage and current signals are affected by decaying DC & high frequency components.

Fundamental component of a signal is of greater importance for most power system applications and sometimes 2nd, 3rd, 5th harmonics for some specific applications. Higher order frequencies if present leads to the corruption of digital signal. So, at first the input signal is band limited to half the sampling frequency (Nyquist Criterion) by passing it through low pass analog anti-aliasing filter. Then the signal is passed through analogue to digital (A/D) converter where the signal is sampled at a frequency based on power system nominal frequency. Now, a digital filter may be used to filter unwanted frequency components that may cause errors for a specific application. The phasor representation of the signal is determined by applying the signal samples received to phasor estimator. Power system frequency is time varying as the loads are not constant. Frequency estimator is used to track the changes in input signal frequency for accurate phasor measurement.

The Discrete Fourier Transform (DFT) is widely used method for phasor estimation and is given by

$$\mathbf{X}_{(k)} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x(k+n) \cdot e^{-j(2\pi n/N)} \quad (2c)$$

Here k is the first sample in the data window and N is the number of samples in a cycle of the fundamental frequency component. Power system frequency is time varying as the loads are not constant. Frequency estimator is used to track the changes in input signal frequency for accurate phasor measurement.

Some of the major applications of PMUs include:

- Real time monitoring of power system.
- Wide area measurement, protection and control of power system.

III. RECURSIVE DFT ALGORITHM

In this paper, we consider recursive DFT [3] as an algorithm for phasor estimation in phasor estimator block. Consider an input signal of the form (2a) at the nominal frequency of the power system f_0 . The signal is sampled at a frequency given by N times per cycle of 50 Hz waveform to produce the sample set $\{x_n\}$. The sampled angle is given by $\theta = \frac{2\pi}{N}$. The phasor estimation process begins with the data samples. The N data samples of this input $x_n: \{n=0, 1, 2, 3, \dots, N-1\}$

$$x_n = X_m \cos(n\theta + \phi) \quad (3a)$$

Now, upon application of DFT on the sample set $\{x_n\}$ we obtain the spectrum of various frequency components present in the signal. But we are mainly concerned with the computation of fundamental frequency component i.e., at power system nominal frequency. So, following equations drive us to obtain the fundamental phasor. The real part of the

fundamental phasor containing cosine terms is computed as given below

$$X_c^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \cos(n\theta) \quad (3b)$$

$$X_c^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} X_m \cos(n\theta + \phi) \cos(n\theta)$$

$$X_c^{N-1} = \left(\frac{X_m}{\sqrt{2}} \right) \cos\phi \quad (3c)$$

The imaginary part containing the sine terms is

$$X_s^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \sin(n\theta) \quad (3d)$$

$$X_s^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} X_m \cos(n\theta + \phi) \sin(n\theta)$$

$$X_s^{N-1} = - \left(\frac{X_m}{\sqrt{2}} \right) \sin\phi \quad (3e)$$

Now substituting (3c) and (3e) in (3f) we obtain

$$X^{N-1} = X_c^{N-1} - jX_s^{N-1} \quad (3f)$$

$$X^{N-1} = \left(\frac{X_m}{\sqrt{2}} \right) \cos\phi + \left(\frac{X_m}{\sqrt{2}} \right) \sin\phi$$

$$X^{N-1} = \left(\frac{X_m}{\sqrt{2}} \right) e^{j\phi} \quad (3g)$$

As in real time, new samples keep on streaming into the digital device for phasor estimation; considerable amount of computational time can be saved by relating the DFT calculated in one data window to the new data window. A new data window is formed by considering the newest sample and removing the oldest sample from the old data window. Hence after modifying the DFT for continuous fashion and also after phase adjustment the recursive phasor estimate is given by

$$\mathbf{X}^{N+r} = \left\{ e^{-j\theta} \mathbf{X}^{N+r-1} + \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} (x_{N+r} - x_r) e^{-jr\theta} \right\}$$

$$\mathbf{X}^{N+r} = \left\{ \mathbf{X}^{N+r-1} + \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} (x_{N+r} - x_r) e^{-jr\theta} \right\} \quad (3h)$$

From [17], the way to measure the frequency and ROCOF for a sinusoidal signal given as

$$x(t) = X_m \cos(\theta(t)) \quad (3i)$$

The frequency is defined by

$$f(t) = \frac{1}{2\pi} \left(\frac{d\theta(t)}{dt} \right) \quad (3j)$$

The rate of change of frequency (ROCOF) is given by

$$\text{ROCOF}(t) = \frac{d\left(\frac{1}{2\pi} \left(\frac{d\theta(t)}{dt} \right)\right)}{dt} = \frac{df(t)}{dt} \quad (3k)$$

We consider Total Vector Error (TVE) as the performance criteria for the given algorithm. It is an index which shows us the vector error between the theoretically computed phasor and the estimated one at a particular instant of time. It is defined as the square root of the squared sum of error between the real and imaginary parts of the estimated phasor to the actual phasor, which is given by the following equation:

$$\text{TVE} = \sqrt{\frac{(X_r(n) - X_r)^2 + (X_i(n) - X_i)^2}{X_r^2 + X_i^2}} \quad (3i)$$

Here $X_r(n)$ is the real part of the estimated phasor, X_r is the real part of the actual phasor, $X_i(n)$ is the imaginary part of the estimated phasor, X_i is the imaginary part of the actual phasor at a particular instant n.

IV. SIMULATION OF RDFT ALGORITHM

A. Parameter Selection:

Recursive DFT algorithm was programmed by assuming the fundamental frequency of the signal equal to 50 Hz in MATLAB programming environment. The following design parameters were chosen

- nominal system frequency f_0 is 50 Hz
- window length of 24 samples per cycle
- a sampling rate of 1200 Hz
- number of cycles of voltage waveform is 100

Analysis was conducted using a disturbance waveform generated simulated in software. The data were samples, taken at 1200 Hz, of the voltage waveform. In order to study the performance of the algorithm in section III, 34% third harmonic, 20.45 % fifth harmonic, 0.23 % inter harmonic

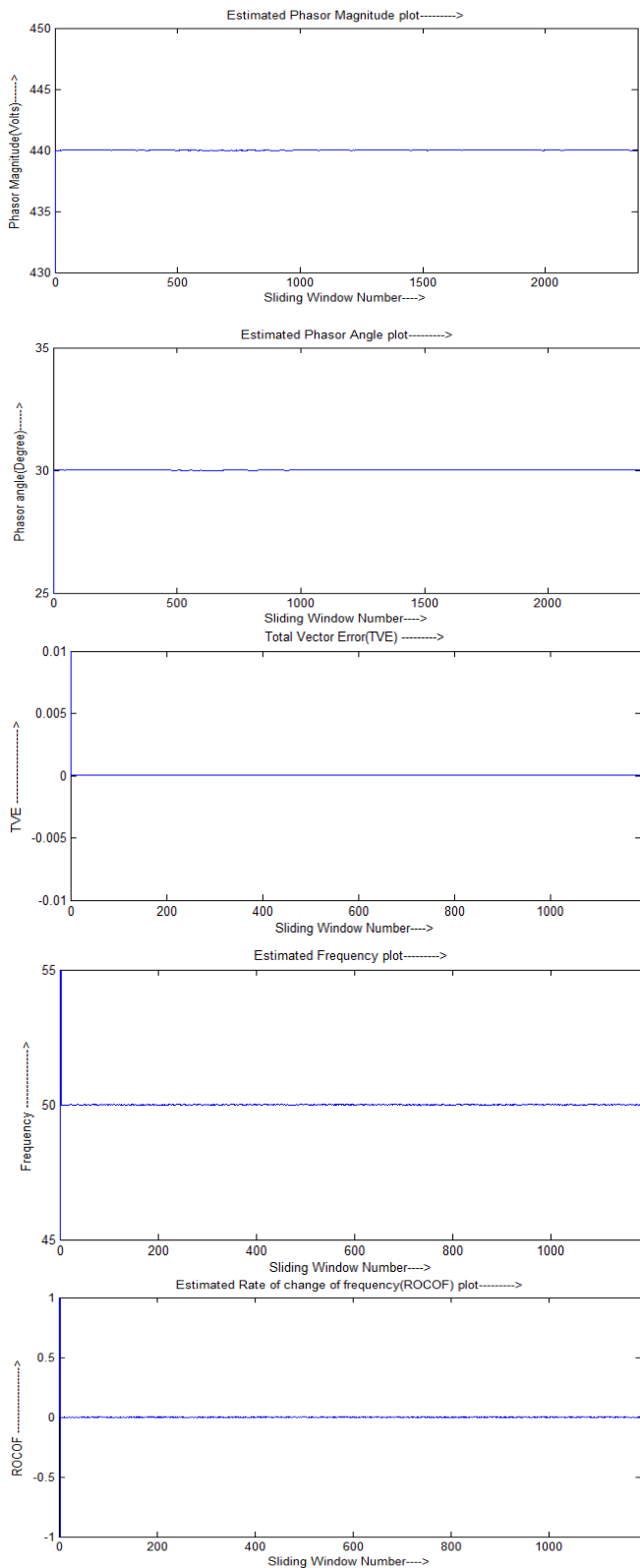


Fig.2.Measured phasor magnitude, phase angle, TVE, frequency, ROCOF computed under case1.

and 1 % random noise of fundamental was added. Effects of these disturbances on recursive DFT were studied in preceding section.

B. Performance Evaluation:

The performance of the algorithm considered will be evaluated by four different cases where different types of disturbances will be introduced [4]. For each case we are going to obtain phasor magnitude, phasor angle, total vector error (TVE), frequency and rate of change of frequency(ROCOF). The details of four simulated cases are as follows

Case 1: The test signal used for testing the effect of integer harmonic components in a voltage signal for phasor estimation is given below:

$$v_1(t) = 440\sqrt{2} \sin(2\pi f_0 t + \pi/6) + 150\sqrt{2} \sin(6\pi f_0 t + \pi/3) + 90\sqrt{2} \sin(10\pi f_0 t + 5\pi/6)$$

Case 2: The test signal used for testing power imbalance i.e., when the system frequency is reduced by 1 Hz is shown below:

$$v_2(t) = 440\sqrt{2} \sin(2\pi f_1 t + \pi/6) + 150\sqrt{2} \sin(6\pi f_1 t + \pi/3) + 90\sqrt{2} \sin(10\pi f_1 t + 5\pi/6)$$

Where f_1 is 49 Hz

Case 3: The test signal used for testing the effect of decaying DC component and integer harmonic components in a voltage waveform is given below:

$$v_3(t) = 399.0e^{-t/0.02} + 440\sqrt{2} \sin(2\pi f_0 t + \pi/6) + 150\sqrt{2} \sin(6\pi f_0 t + \pi/3) + 90\sqrt{2} \sin(10\pi f_0 t + 5\pi/6)$$

Case 4: In case 4, the voltage waveform is superposed with decaying D.C component, integer harmonic components, inter harmonic component and random noise is shown below:

$$v_4(t) = 399.0e^{-t/0.02} + 440\sqrt{2} \sin(2\pi f_0 t + \pi/6) + 150\sqrt{2} \sin(6\pi f_0 t + \pi/3) + 90\sqrt{2} \sin(10\pi f_0 t + 5\pi/6) + \sin(2\pi f_h t) + e(t)$$

Where f_h is 275.5 Hz and $e(t)$ is the random noise.

In the Fig.2, TVE is 0 indicating that the measured fundamental component is same as that of the actual value to be measured when simulated by case 1. In case 2, when the signal frequency is varied from nominal frequency i.e., 50 Hz. Now harmonic components present in them can no longer be integer multiples of fundamental frequency f_0 . This results in spectral leakage and errors are introduced in the phasor estimation.

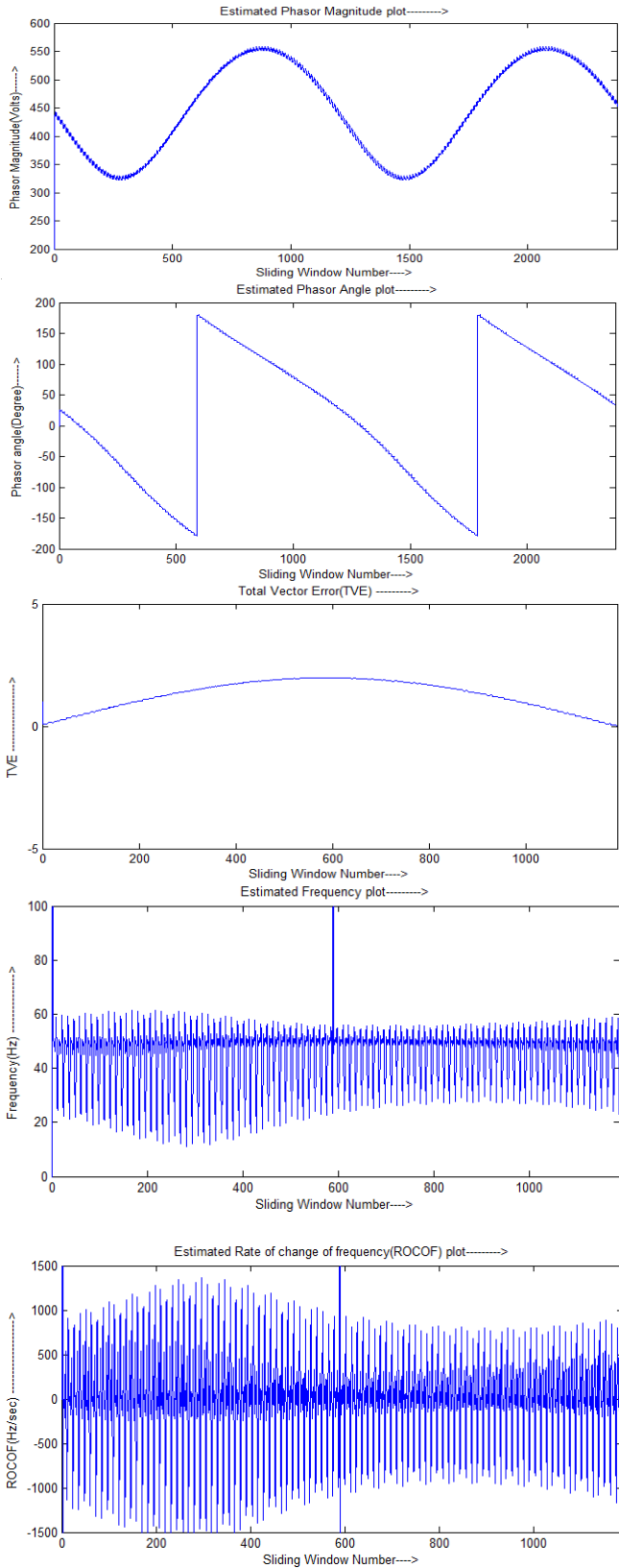


Fig.3.Measured phasor magnitude, phase angle, TVE, frequency, ROCOF computed under case2.

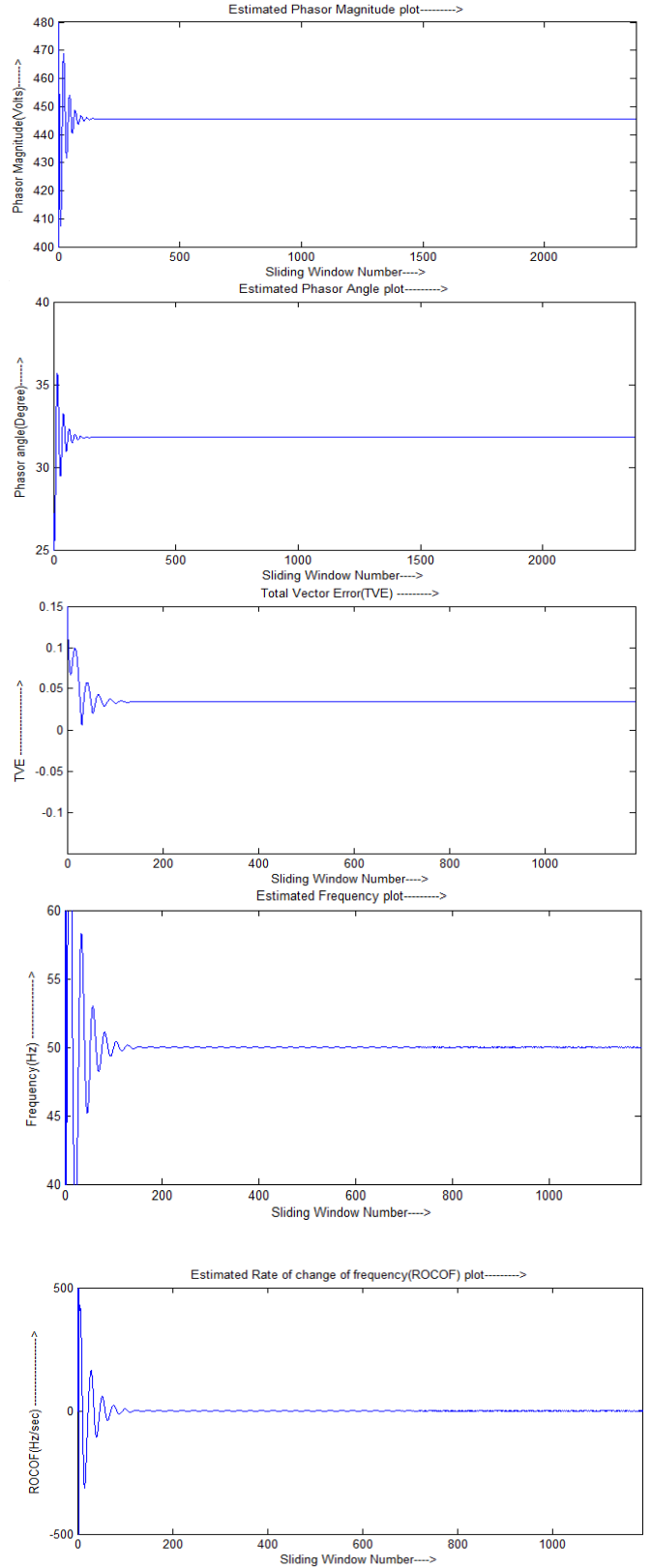


Fig.4.Measured phasor magnitude, phase angle, TVE, frequency, ROCOF computed under case3.

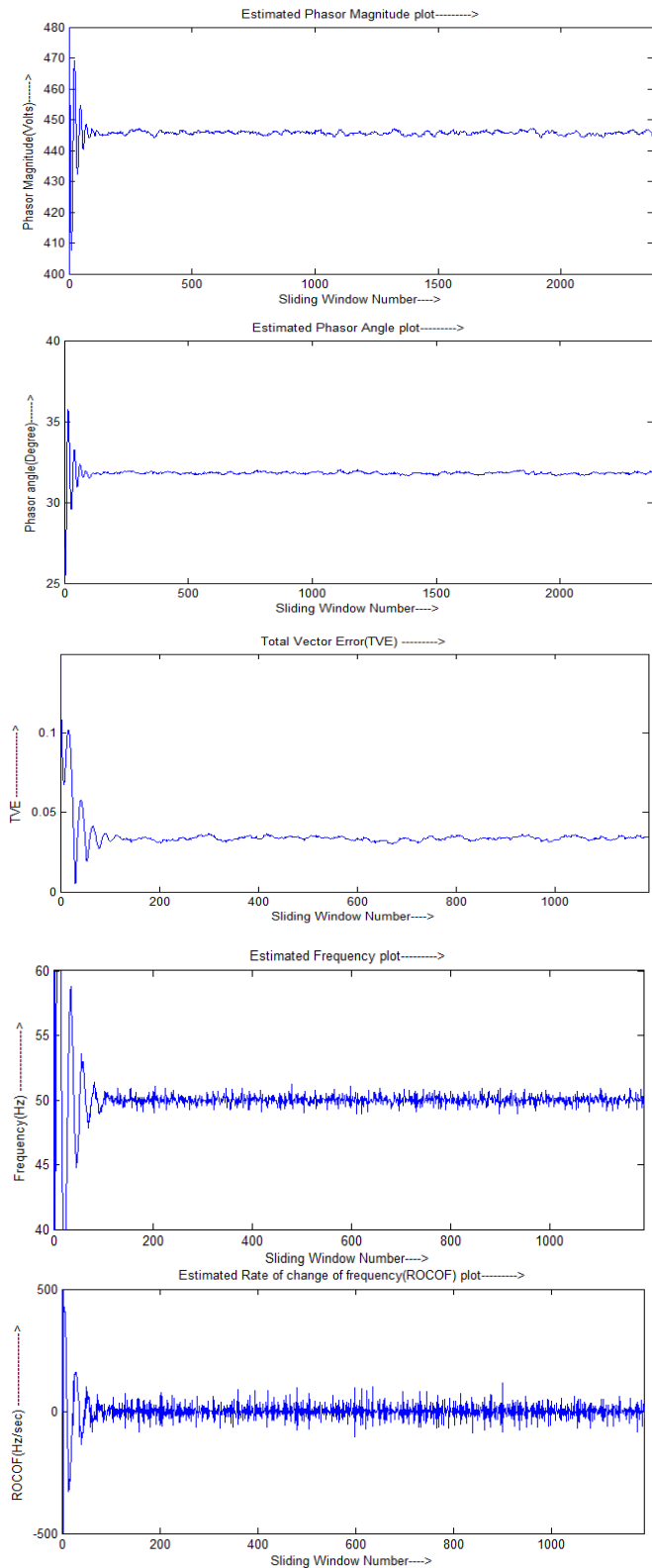


Fig.5.Measured phasor magnitude, phase angle, TVE, frequency, ROCOF computed under case4.

In fig.3, we can observe severe drifts in phasor magnitude and angle. Maximum value of TVE reaches to 1.9883. Here frequency and rate of change of frequency are also affected by the frequency variation. Now, in case 3, although the signal frequency is f_0 but due to the presence of exponential

decaying component errors are introduced in both phasor magnitude and phase angle. Here after reaching a steady condition TVE is 0.0341. It can also be observed from fig.4 that oscillations are introduced in measurement of all the parameters just after the introduction of disturbance. Here frequency and rate of change of frequency values settles down to the correct values in spite of the error in phasor measurement. From fig.5, when inter harmonic component and random noise is introduced to the waveform it can be observed that a steady state values cannot be observed in their measurement. Here performance evaluation index TVE varying over a mean value that is observed in case 3.

As per IEEE standard, TVE should not exceed 1% for phasor measurement. In this analysis TVE is more than 3% under case 2, 3, 4 but it is zero during case 1.

V. CONCLUSION

In this paper recursive DFT algorithm has been subjected to various real time power quality disturbance waveforms. The simulation results showed the possible ways where the considered algorithm produces errors in phasor measurement.

From the analysis, the factors affecting the measurement process are observed to be:

- Off-nominal frequency variations due to power imbalance in the system.
- Decaying DC offsets which occur during switching actions or post fault conditions due to predominant inductiveness of power system.
- Inter-harmonic components and high frequency noise introduced due to sudden change in amplitude of the parameter under measurement.

Errors due to off-nominal frequency variations can be reduced by tracking the system frequency by any of the frequency tracking methods and changing the nominal frequency for DFT calculation accordingly. Errors due to decaying DC offsets can be reduced by passing the samples through DC rejection filters before computing the DFT. If the samples are passed through band pass digital filter which extract only fundamental components the errors due to DC offsets, inter-harmonic components and high frequency noise may be reduced to greater extent.

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