

Performance Comparison of Different Multipliers using Booth Algorithm

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Abstract— Low power consumption and smaller area are some of the most important criteria for the fabrication of DSP systems and high performance systems. Optimizing the speed and area of the multiplier is a major design issue. However, area and speed are usually conflicting constraints so that improving speed results mostly in larger areas. In this paper we try to determine the best solution to this problem by comparing a few multipliers. The parallel multipliers like radix 2 and radix 4 modified booth multiplier does the computations using lesser adders and lesser iterative steps. As a result of which they occupy less space as compared to the serial multiplier. This is a very important criteria because in the fabrication of chips and high performance system requires components which are as small as possible.

Keywords: multiplier, radix-R, booth algorithm.

I. INTRODUCTION

Multipliers play an important role in today’s digital signal processing and various other applications. With advancements in technology, many researchers have already tried and are still trying to design multipliers which provides either greater speed, less power consumption, regularity of layout and hence small area or even combination of them in one multiplier which makes them suitable for various increased speed, minimized power and compact VLSI implementation. The usual multiplication method is “add and shift” algorithm. In parallel multipliers number of partial products that needs to be added is the main parameter that defines the performance of the multiplier. In order to minimize the number of partial products to be added, Booth algorithm and Modified Booth algorithm is one of the most popular algorithms [1].

II. MULTIPLIERS

A Binary multiplier is an electronic hardware circuit that used in digital electronics or a computer or other electronic devices to perform rapid multiplication of two numbers in binary representation. It is obtained using binary adders. The rules for binary multiplication are as follows

1. If the multiplier digit is a 1, then the product will be same as multiplicand and simply it will be copied down.

2. If the multiplier digit is a 0 the product is also 0. The multiplication algorithm for an N bit multiplicand by N bit multiplier is shown in fig1.

$$Y = Y_{n-1} Y_{n-2} \dots Y_2 Y_1 Y_0 \text{ Multiplicand}$$

$$X = X_{n-1} X_{n-2} \dots X_2 X_1 X_0 \text{ Multiplier}$$

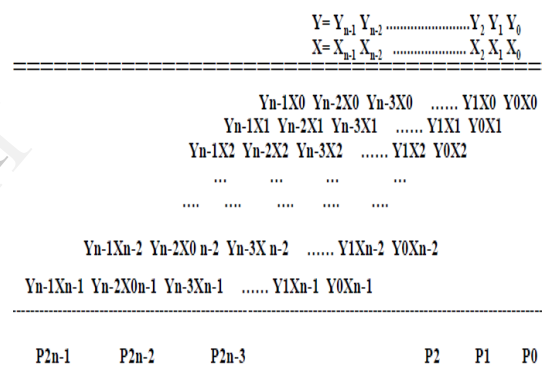


Figure 1. Multiplication algorithm for N*N bit

III. TYPES OF MULTIPLIER

Basically there are three types of multipliers. They are as follows.

a. Serial Multiplier

Serial multiplier generates partial products sequentially and adds each newly generated product to previously accumulated partial product.

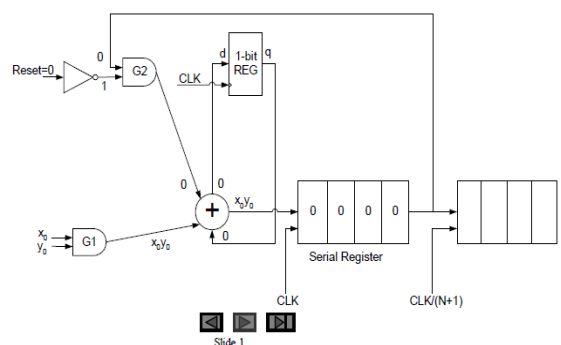


Figure 2. Serial Multiplier

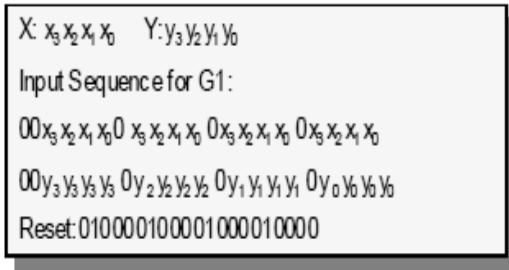


Figure 3

Serial multiplier is used where area and power is important, & delay can be tolerated. Circuit uses one adder to add the $m \cdot n$ partial products. The circuit is shown for $m=n=4$. Multiplicand and Multiplier inputs have to be arranged in a special manner synchronized with circuit behavior as shown in the fig 2. The inputs could be presented at different rates depending on the length of the multiplicand and the multiplier. Two clocks are used, one to clock the data & one for the reset. A first order approximation of the delay is $O(m,n)$. With this circuit arrangement the delay is given as $D=[(m+1)n+1] t_{fa}$. As shown in fig 3 the individual PP is formed. The addition of the PPs are performed as the intermediate values of PPs, addition are stored in the D Flip Flop, circulated and then added together with the newly formed PP [2].

Disadvantage

This approach is not suitable for larger values of M & N.

b. Parallel multiplier

Generates partial products in parallel, accumulates using a fast multi-operand adder.

Serial/Parallel Multiplier

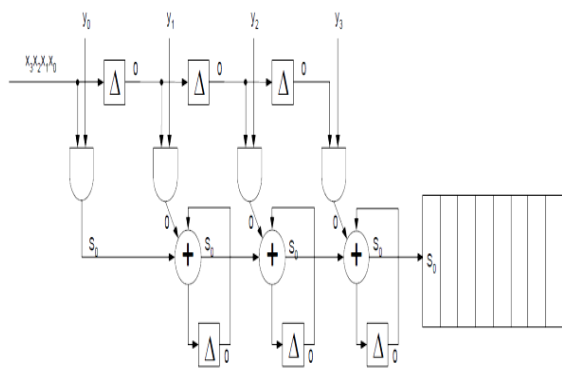


Figure 4. Serial/Parallel Multiplier

One operand is fed to the circuit in parallel while the other is in serial. N partial products are formed for each cycle. On successive cycles, each cycle does the addition of one column of the multiplication table of $M \cdot N$ PPs. The final

results are then stored in the output register after completing $N+M$ cycles.

Disadvantage

Area required is $N-1$ for $M=N$.

		A3	A2	A1	A0				
	x	B3	B2	B1	B0	Inputs			
	C	$B0 \times A3$	$B0 \times A2$	$B0 \times A1$	$B0 \times A0$				
+		$B1 \times A3$	$B1 \times A2$	$B1 \times A1$	$B1 \times A0$				
C	sum	sum	sum	sum		Internal Signals			
+		$B2 \times A3$	$B2 \times A2$	$B2 \times A1$	$B2 \times A0$				
C	sum	sum	sum	sum					
+		$B3 \times A3$	$B3 \times A2$	$B3 \times A1$	$B3 \times A0$				
C	sum	sum	sum	sum					
	Y7	Y6	Y5	Y4	Y3	Y2	Y1	Y0	Outputs

Figure 5. Generation of individual PP and their addition

c. Array Multiplier

Array of identical cells generating new partial products and accumulating them simultaneously is as shown in figure 6. No separate circuits for generation and accumulation is required. This implementation reduces execution time but increases hardware complexity.

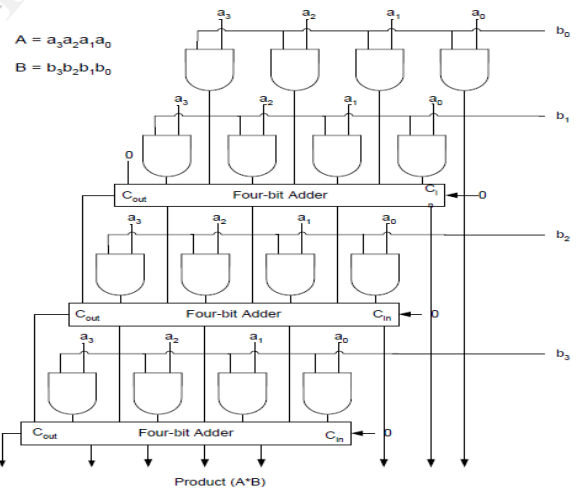


Figure 6. Array Multiplier

Array multiplier is well known due to its regular structure. Multiplier is based on add and shift algorithm. Each and every partial product is generated by the multiplication of the multiplicand with one multiplier bit. The partial product are shifted according to their bit orders and then added. The addition can be performed with normal carry propagate adder. $N-1$ adders are required where N is the multiplier length. Although the method is simple as it can be seen from this example, the addition is done serially as well as in parallel.

Disadvantage

Hardware complexity increases with $N \times M$. Now as both multiplicand and multiplier may be positive or negative, 2's complement number system is used to represent them. If the multiplier operand is positive then essentially the same technique can be used but care must be taken for sign bit extension. The reason for dealing with signed number incorrectly is the absence of sign bit expansion in this multiplier.

IV. MULTIPLICATION ALGORITHM

A circuit that multiplies two unsigned n bit binary numbers, uses a 2 dimensional array of identical subcircuits. Each of which contains a full adder and an AND gate. For large number of bits this approach may not be appropriate because of the large number of gates needed. Another approach is to use shift register in combination with an adder to implement the traditional method of multiplication.

```
[1]
P=0;
For i=0 to n-1 do
If bi=1 then
    P=P+A;
End if;
Left shift A;
End for;
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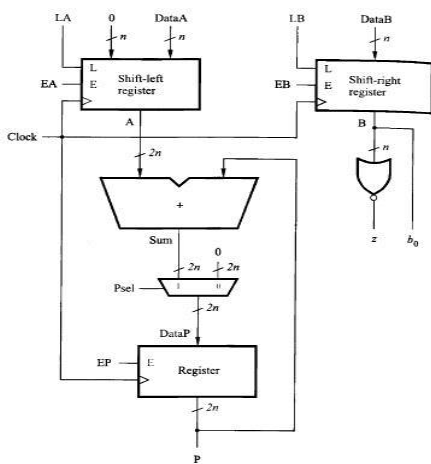


Figure 7. Data circuit of multiplier

V. BOOTH MULTIPLIERS

This algorithm was invented by Andrew Donald Booth in 1950 while doing study on crystallography. Booth used reception desk calculators that shifts faster than adding and formed the algorithm to increasing the speed. Booth's algorithm is important in the study of computer architecture.[3]. It is a powerful algorithm for signed-number multiplication, which considers both positive and negative numbers uniformly [4]. Booth's Algorithm is a smart move for multiplying signed numbers. It starts with the ability to both add and subtract.[5] An algorithm that uses two's complement notation of signed binary numbers for multiplication.[6]

VI. MODIFIED BOOTH'S ALGORITHM

Modified Booth's is two times faster than Booth's algorithm. Modified Booth encoding algorithm is an efficient way to reduce the number of partial products by grouping consecutive bits in one of the two operands to form the signed multiples. The operand that is Booth encoded is called the multiplier and the other operand is called the multiplicand. [7]

1. Radix-2

Booth algorithm gives a procedure for multiplying binary integers in signed -2 's complement representation. [3] Illustration of the booth algorithm with example:

Example, $2^{10} \times (-4)^{10}$
 $0010^{two} \times 1100^{two}$

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Step 1: Making the Booth table [3]

I. From the above two numbers, pick the number with the smallest difference between a series of consecutive numbers, and make it a multiplier.

Therefore, multiplication of $2 \times (-4)$, where 2^{10} (0010^{two}) is the multiplicand and $(-4)^{10}$ (1100^{two}) is the multiplier.

Table 1

U	V	X	X-1
0000	0000	1100	0

Load the value
 1st cycle
 2nd cycle
 3rd Cycle
 4th Cycle

Let $X = 1100$ (multiplier)
 Let $Y = 0010$ (multiplicand)
 2's complement of Y ; $-Y = 1110$

Load the X value in the table.
 Load 0 for $X-1$ value it should be the previous first least significant bit of X .
 Load 0 in U and V rows which will have the product of X and Y at the end of operation.
 Make four rows for each cycle; this is because we are multiplying four bits numbers.

Step 2: Booth Algorithm

Booth algorithm requires examination of the multiplier bits, and shifting of the partial product. Prior to the shifting, the multiplicand may be added to partial product, subtracted from the partial product, or left unchanged according to the following rules:

Table 2.

U	V	X	X-1
0000	0000	1100	0
0000	0000	0110	0

Look at the first least significant bits of the multiplier “X”, and the previous least significant bits of the multiplier “X - 1”.

- 0 0 Shift only
- 1 1 Shift only.
- 0 1 Add Y to U, and shift
- 1 0 Subtract Y from U, and shift or add (-Y) to U and shift.

Take U & V together and shift arithmetic right shift which preserves the sign bit of 2’s complement number. Thus a positive number remains positive, and a negative number remains negative. Shift X circular right shift because this will prevent us from using two registers for the X value. Repeat the same steps until the four cycles are completed. [8]

Table 3.

U	V	X	X-1
0000	0000	1100	0
0000	0000	0110	0
0000	0000	0011	0

2. Radix-4

Radix-4 Booth algorithm scans strings of 3 bits with the algorithm given below Append a 0 to the right side of the LSB of the multiplier consider the bits in groups of three, in a way that each group overlaps with the previous group by one bit. Grouping starts from the LSB and the first group only uses 2 bits of the multiplier. According to the value of each vector, Partial Product will be 0, +Y, -Y, +2Y,-2Y. The negative values of y are considered by taking the 2’s complement to the Booth recode the multiplier term, we have to consider the bits in groups of three, in a way that each group overlaps with the previous group by one bit. Grouping starts from the LSB and the first group only uses 2

bits of the multiplier. [7] Multiplier is equal to 0 1 0 1 1 10 then a 0 is placed to the right most bit which gives 0 1 0 1 1 10 0 the 3 digits are selected at a time with overlapping left most bit as follows:

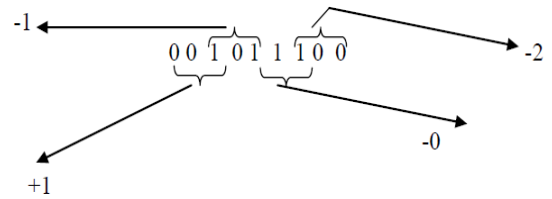
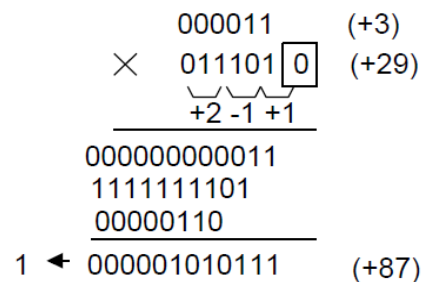


Figure 8 Grouping of three bits

Table 4. Encoding of Radix-4 Booth Multiplier[9] [10] [11]

Groups	Partial Product
000	0
001	1*multiPLICand
010	1*multiPLICand
011	2*multiPLICand
100	-2*multiPLICand
101	-1*multiPLICand
110	-1*multiPLICand
111	0



VII. CONCLUSION

We found that the parallel multipliers are much faster than the serial multiplier. In case of parallel multipliers, the total area is much less than that of serial multipliers. Hence the power consumption is also less. This speeds up the calculation and makes the system faster. While comparing the radix 2 and the radix 4 booth multipliers we found that radix 4 Consumes lesser power than that of radix 2. This is because it uses almost half number of iteration and adders when compared to radix 2. When all the multipliers were compared we found that array multipliers are most power consuming and have the maximum area. This is because it uses a large number of adders. As a result it slows down the system because now the system has to do a lot of calculation. Multipliers are one the most important component of many systems. So we always need to find a better solution in case of multipliers. Our multipliers should

always consume less power and cover less area. In the end we determine that radix 4 modified booth algorithm works the best.

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