Performance Analysis of Wireless MIMO System using Precoded OSTBC

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***Abstract* – The demand for mobile communication systems with high data rates and improved link quality for a variety of applications has dramatically increased in current years. New concepts and techniques are necessary in order to cover this huge demand. MIMO is one of the prime technologies that can achieve the goal of high speed demand. As MIMO systems utilizes multiple no of antennas both at the transmitter and receiver Therefore the necessary requirement to reduce the fading resulting from signal fluctuations in the channel is Diversity. STBC is a MIMO transmit strategy which exploits transmit diversity and high reliability. Orthogonal space-time block codes (OSTBC) achieve full diversity when a linear receiver, such as, zero-forcing (ZF) or minimum mean square (MMSE), is used. This paper involves a transmitted signal consists of a precode followed by an orthogonal space-time block code (OSTBC). A new design criterion and a corresponding design method of precoders are proposed which shows comparison of bit error rate (BER) performance of OSTBC and Precoded OSTBC in Zero Forcing and MMSE equalization techniques.**

***Keywords*– OSTBC, Precoding, BER, ZF, MMSE.**

I. INTRODUCTION

Multiple-Input multiple-output (MIMO) wireless channels have considerably higher capacities than traditional channels. Fading makes it extremely difficult for the receiver to recover the transmitted signal unless the receiver is provided with some form of diversity, i.e. replicas of the same transmitted signal with uncorrelated attenuation. In fact, diversity combining technology has been one of the most important contributors to reliable wireless communications. Consider transmit diversity by deploying multiple antennas at the base station. Moreover, in economic terms, the cost of multiple transmit antennas at the base station can be amortized over numerous mobile users. Hence transmit diversity has been identified as one of the key contributing technologies to the downlinks of 3G wireless systems such as W-CDMA and CDMA2000.We have analyzed the full diversity condition in MIMO to achieve the better BER performance. We take OSTBC (with 2, 4, and 8) transmit antenna to show that BER performance will improve when antenna size improved. But the cost of device improve make it practically impossible and power consumption will more for mobile device, so alternate is using same MIMO with turbo code so same will give better result, with two antenna.

A. Transmit Diversity

We now investigate a different goal, using the multiple antennas to achieve reliability. Here we focus on transmit diversity. So far, we have developed the capacity of MIMO systems in the case of the channel being known at the transmitter and receiver and in the more practical case of the channel known at the receiver only. This answers the question, “How fast can data be transmitted?” i.e., what is the theoretical maximum data rate that can be achieved in a MIMO system. When there is more transmitter than receiver antennas, this is called TX diversity. The simplest scenario uses two TX and one RX antenna. In figure 1, the same data is transmitted redundantly over two antennas. This technique has the advantage that the multiple antennas and redundancy coding is moved from the mobile unit to the base station, where these technologies are easier and cheaper to implement [1].

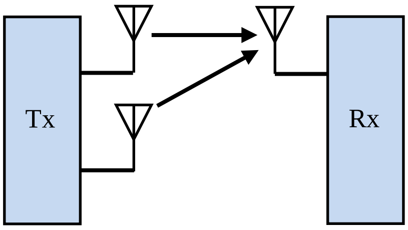


Figure 1: Transmit diversity

B. Receive Diversity

Receiver diversity uses more antennas on the receiver side than on the transmitter side. The easiest scenario consists of two RX and one TX antenna. Figure 2 shows the antenna configurations of two RX and one TX antenna. Receive diversity are widely used in wireless communication systems; it can be achieved by receiving redundant copies of the same signal. The idea behind receive diversity is that each antenna at the receive end can observe an independent copy of the same signal [1].

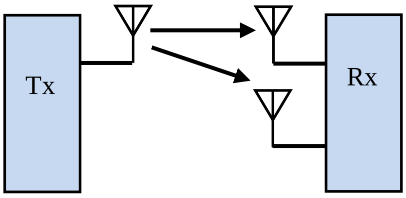


Figure 2: Receive diversity

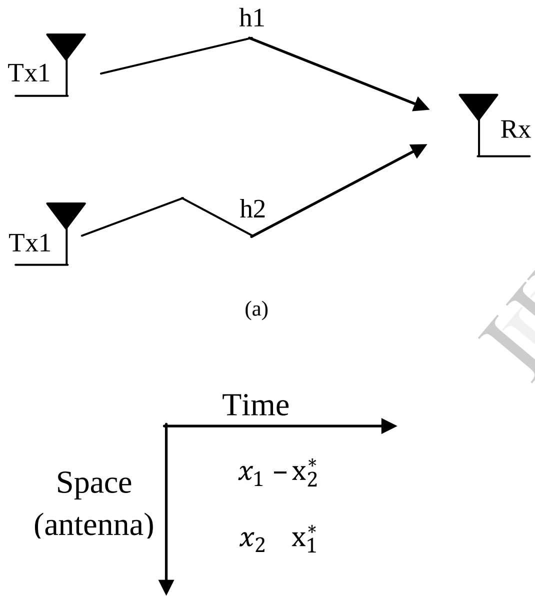
II. SPACE TIME BLOCK CODES

A. *Alamouti STBC*

It is simple method for achieving spatial diversity with two transmit antennas. The scheme is as follows: Consider that we have a transmission sequence, for example

$$\{x_1, x_2, x_3, \ldots\ldots, x_n\}$$

In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.



(a)

(b)

Figure 3: (a) Transmit, (b) Receive Alamouti STBC coding

However, Alamouti suggested that we group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot send $-x_2^*$ and x_1^* from the first and second antenna. In the third time slot send x_3 and x_4 from the first and second antenna. In fourth time slot, send $-x_4^*$ and x_3^* from the first and second antenna and so on. Notice that though we are grouping two symbols, we still need two time slots to send two symbols. Hence, there is no change in the data rate. This forms the simple explanation of the transmission scheme with Alamouti Space Time Block coding. Alamouti scheme is an example of a full-rate full-diversity complex space–time block code [3].

B. *Orthogonal Space-Time Block Codes (OSTBC)*

The transmit diversity scheme designed by Alamouti can be used only in a system with two transmit antennas. It turns out that this technique belongs to a general class of codes named Space–Time Block Codes or, more precisely, Orthogonal STBCs, since they are based on the theory of orthogonal designs. The authors of [2] introduced the theory of generalized orthogonal designs in order to create codes for an arbitrary number of transmit antennas. The general idea behind STBCs construction is based on finding coding matrices X that can satisfy the following condition:

$$X.X^H = p.\left(\sum_{i=0}^{n} |x_i|^2\right).I_{M_T} \tag{1}$$

In this equation, X^H is the Hermitian of X, p is a constant, I_{M_T} is the identitymatrix of size $M_T \times M_T$, M_T represents the number of transmit antennas, and$n$ is the number of symbols x_i transmitted per transmission block in X. The generalized theory of orthogonal design is exploited to provide codes that satisfy the above equation.

The orthgonality property of STBCs is reflected in the fact that all rows of X are orthogonal to each other. In other words, the sequences transmitted from two different antenna elements are orthogonal to each other for each transmission block. For real signal, it is possible to reach full rate. However, it has been proven in [2] that this statement is false for two-dimensional constellations, i.e., complex signals. The encoding and decoding approaches follow the pattern described in Alamouti's scheme. For complex signals, the theory of orthogonal designs can be used to generate coding matrices that achieve a transmission rate of 1/2 for the cases of 3 and 4 transmission antennas:

$$X_{1/2} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix},$$

$$X_{1/2} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}$$

Using the theory of orthogonal design to construct STBCs is not necessarily the optimal approach. There exist some sporadic STBCs that can provide a transmission rate of 3/4 for schemes of either 3 or 4 transmit antennas [4].

$$X_{3/4} = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & -x_3^* \\ x_3 & 0 & -x_1^* & x_2^* \end{bmatrix},$$

$$X_{3/4} = \begin{bmatrix} x_1 & 0 & x_2 & -x_3 \\ 0 & x_1 & x_3^* & x_2^* \\ -x_2^* & -x_3 & x_1^* & 0 \\ x_3^* & -x_2 & 0 & x_1^* \end{bmatrix}$$

It is important to notice that the channel coefficients must remain constant during the transmission of a block of coded symbols X [5].

C. Precoded OSTBC

Consider the MISO system with N_T antennas, that is $\mathrm{h} \in \mathrm{C}^{1\times \mathrm{N_T}}$. Let $\mathrm{C} \in \mathrm{C}^{\mathrm{M}\times \mathrm{T}}$denote a space-time codeword with a length of M, which is represented as

$$\mathrm{C} = [\mathrm{c}_1 \mathrm{c}_2 \dots \dots \mathrm{c}_\mathrm{T}] \tag{2}$$

Where

$$\mathrm{c}_\mathrm{k} = [\mathrm{c}_{\mathrm{k},1} \mathrm{c}_{\mathrm{k},2} \dots \dots \mathrm{c}_{\mathrm{k,M}}]^\mathrm{T},\ \mathrm{k} = 1,2,\dots\dots,\mathrm{T} \text{ and } \mathrm{M} \le \mathrm{N_T} \tag{3}$$

In the precoded OSTBC systems, the space-time codeword C is multiplied by a precoding matrix$\mathrm{W} \in \mathrm{C}^{\mathrm{N_T}\times \mathrm{M}}$, which is chosen from the codebook.

$$\mathrm{F} = \{\mathrm{W}_1, \mathrm{W}_2, \mathrm{W}_3 \dots, \mathrm{W}_\mathrm{L}\} \tag{4}$$

The objective is to choose an appropriate codeword that improves the overall system performance such as channel capacity or error performance. Assuming that $\mathrm{N_T}$ channels remain static over T, the received signal $\mathrm{y} \in \mathrm{C}^{1\times \mathrm{T}}$ can be expressed as,

$$\mathrm{y} = \sqrt{\frac{\mathrm{E_X}}{\mathrm{N_T}}}\mathrm{hWC} + \mathrm{z} \tag{5}$$

In above equation the length of each vector is $M \le \mathrm{N_T}$. The probability of codeword error can be derived as follows: For a given channel h and precoding matrix W, we consider the pair wise codeword error probability$\mathrm{P_r}(\mathrm{C_i} \rightarrow \mathrm{C_j}|\mathrm{H})$. The upper bound of the pair wise error probability is given as

$$\mathrm{P_r}\left(\mathrm{C_i} \rightarrow \mathrm{C_j}\middle|\mathrm{H}\right) = \mathrm{Q}\left(\sqrt{\frac{\rho||HWE_{i,j}||_F^2}{2\mathrm{N_T}}}\right)$$
$$\le exp\left(-\frac{\rho||HWE_{i,j}||_F^2}{4\mathrm{N_T}}\right) \tag{6}$$

Where ρ is the signal-to-noise ratio (SNR), given as $\rho = E_x/N_0$ and$E_{i,j}$is the error matrix between the codewords $C_i$and $C_j$which is defined as $E_{i,j} = C_i - C_j$for a given STBC scheme. From equation above we see that $||HWE_{i,j}||_F^2$ needs to be maximized in order to minimize the pairwise error probability. This leads us to the following codeword selection criterion:

$$W_{opt} = \underbrace{\arg max}_{W\in F, i\ne j} ||HWE_{i,j}||_F^2$$
$$= \underbrace{\arg max}_{W\in F, i\ne j} Tr(HWE_{i,j}E_{i,j}^H W^H H^H)$$
$$= \underbrace{\arg max}_{W\in F} Tr(HWW^H H^H)$$
$$= \underbrace{\arg max}_{W\in F} ||HW||_F^2 \tag{7}$$

In the course of deriving equation (7), we have used the fact that the error matrix of OSTBC has the property of $E_{i,j}E_{i,j}^H = aI$ with constant a. When the constraint $W\epsilon\ F$ is not imposed, the above optimum solution W_{opt} is not unique, because $||HW_{opt}||_F^2 = ||HW_{opt}Z||_F^2$.Where Z is a unitary matrix. The unconstrained optimum solution of equation (7) can be obtained by singular value decomposition (SVD) of channel $H = U\Sigma V^H$, where the diagonal entry of S is in descending order. It is shown that the optimum solution of above equation is given by the leftmost M columns of V, that is,

$$W_{opt} = [v_1 v_1 \dots\dots v_M] \triangleq \bar{V} \tag{8}$$

Since $\bar{V}$ is unitary, $\lambda_i(W_{opt}) = 1, i = 1,2,\dots, M$ where $\lambda_i\ (A)$denotes the i^{th}largest eigenvalue of the matrix A. In case that a channel is not deterministic, the following criterion is used for the codebook design:

$$E\left\{\underbrace{min}_{W\in F}\left(||HW_{opt}||_F^2 - ||HW||_F^2\right)\right\} \tag{9}$$

Where the expectation is with regards to the random channel H. W_{opt} in equation (9) follows from equation (8) for the given channel H.

The above expected value in equation (9) is upper-bounded as

$$E\left\{\underbrace{min}_{W\in F}\left(\left||HW_{opt}|\right|_F^2 - \left||HW|\right|_F^2\right)\right\}$$
$$\le E\{\lambda_1^2\{H\}\}E\left\{\underbrace{min}_{W\in F}\frac{1}{2}\left||\bar{V}\bar{V}^H - WW^H|\right|_F^2\right\} \tag{10}$$

Since $\lambda_1^2\{H\}$is given, the codebook must be designed so as to minimize $E\left\{\underbrace{min}_{W\in F}\frac{1}{2}\left||\bar{V}\bar{V}^H - WW^H|\right|_F^2\right\}$in equation (10).

III. EQUALIZATION TECHNIQUES

A. Zero Forcing Equalizer

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems; it inverts the frequency response of the channel. The name Zero forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero in a noise free case. This will be useful when ISI is more predominant when comparing to the noise.

Consider a 2x 2 MIMO channel, the received signal on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 = [h_{11}h_{12}]\begin{bmatrix}x_1\\x_2\end{bmatrix} + n_1 \tag{11}$$

The received signal on the Second receive antenna is,

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 = [h_{21}h_{22}]\begin{bmatrix}x_1\\x_2\end{bmatrix} + n_2 \tag{12}$$

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix}y_1\\y_2\end{pmatrix} = \begin{pmatrix}h_{11} & h_{12}\\h_{21} & h_{22}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}n_1\\n_2\end{pmatrix} \tag{13}$$

Equivalently, Y = HX + N

Where, Y = Received Symbol Matrix. H = Channel matrix. X = Transmitted symbol Matrix. N = Noise Matrix.

To solve for x, we need to find a matrix W which satisfies WH = I. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \tag{14}$$

Where

W - Equalization Matrix and H - Channel Matrix

This matrix is known as the Pseudo inverse for a general m x n matrix where

$$H^H H = \begin{pmatrix} h_{11}{}^* h_{21}{}^* \\ h_{12}{}^* h_{22}{}^* \end{pmatrix} \begin{pmatrix} h_{11} h_{21} \\ h_{12} h_{22} \end{pmatrix}$$

$$= \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}{}^* h_{12} + h_{21}{}^* h_{22} \\ h_{12}{}^* h_{11} + h_{22}{}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix} \tag{15}$$

Zero forcing equalizer tries to null out the interfering terms when performing the equalization while doing so, there can be amplification of noise. Hence Zero forcing equalizer is not the best possible equalizer.

B. MMSE Equalizer

Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix using the MMSE performance-based criterion.

The MMSE approach tries to find a coefficient W which minimize the criterion,

$$E\left\{[W_{y-x}][W_{y-x}]^H\right\} \tag{16}$$

On solving

$$W = (H^H H + N_0 I)^{-1} H^H \tag{17}$$

When comparing to the equalization, apart from the $N_0 I$ team both the equations are comparable. When the noise term is zero, the MMSE equalization reduced to ZF equalizer.

IV. SIMULATION AND RESULTS

The proposed methodology is implemented in MATLAB software.

Simulation Parameters:

- Modulation scheme M = 2 (BPSK Modulation)
- Simulation SNR ranges (SNR_db) = 0:3:21
- Sampling rate of message sequence (Fd) =1
- Sampling rate of the modulated signal (Fs) =1
- Number of packets of data (N) =100
- Number of bits or samples in each data packet (Ns) =200

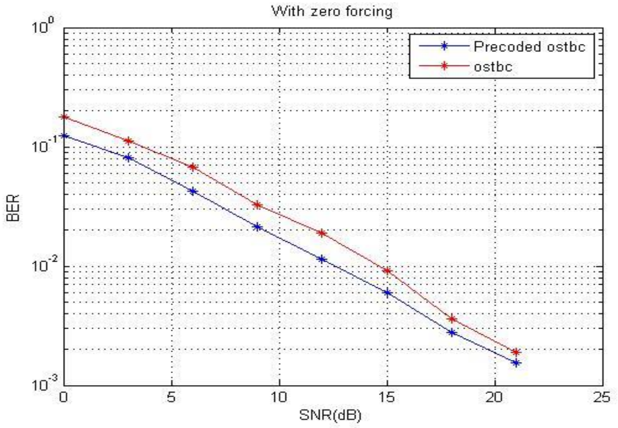


Figure 4: Comparison of BER performance of OSTBC without precoding and with precoding in ZF

Figure 4 shows the performance of OSTBC and Precoded OSTBC system with Zero Forcing equalization technique. At SNR=15dB, BER performance of Precoded OSTBC is $10^{-2.4}$ while without precoding it is $10^{-2.1}$. It is clear that precoded OSTBC performs better than normal OSTBC system.

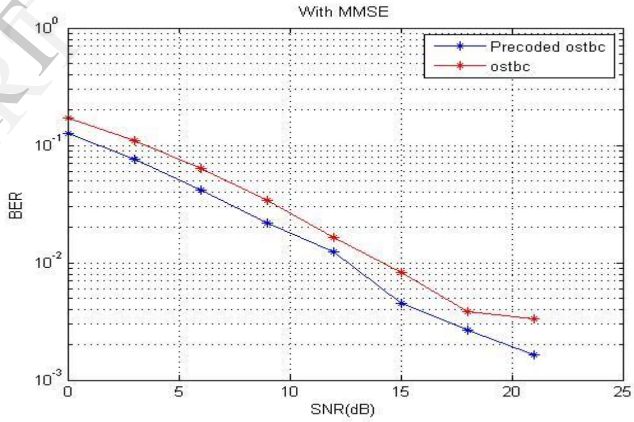


Figure 5: Comparison of BER performance of OSTBC without precoding and with precoding in MMSE

Figure 5 shows the performance of OSTBC and Precoded OSTBC system with MMSE equalization technique. At SNR=15dB, BER performance of Precoded OSTBC is $10^{-2.6}$ while without precoding it is $10^{-2.2}$. It is clear that Precoded OSTBC performs better than normal OSTBC system.

V. CONCLUSION

The most prominent space-time block codes are OSTBCs. The approach in this paper has been to isolate the analysis of precoded MIMO systems. The proposed precoded OSTBC have a layered structure, which is implemented in the simplest Grassmannian precoding. Simulation results shows comparison of BER performance between OSTBC and Precoded OSTBC for Zero Forcing and MMSE equalization techniques. And finally it is found that the Precoded OSTBC shows better BER.

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