Performance Analysis Of Space Time Block Codes Over Rayleigh Fading Channel

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Abstract

To overcome the effect of multi-path fading of the channel and to achieve full diversity, the multiple antennas seems to be an efficient solution. STBC provides a new concept of transmission over Rayleigh channel using multiple transmit and receive antenna. This paper presents a detailed study of STBC scheme which includes the Alamouti's STBC for two transmitting antennas as well as orthogonal space time codes (OSTBC) for three and four transmitting antennas. Bit Error Rate (BER) performance is simulated and analyzed for different constellation schemes as BPSK, QPSK ,8-PSK and 16-QAM using MATLAB.

Keywords – Space Time Block Code (STBC), Alamouti codes, Orthogonal Space Time Block Code (OSTBC)

1. Introduction

The wireless channel suffers from time-varying impairments like multi-path fading, interference and noise. The channel statistic is usually Rayleigh which makes it difficult for the receiver to decide the exact transmitted signal unless some less attenuated replica of transmitted signal is provided to the receiver. Thus applying multiple transmitters on base stations and multiple receivers at receiving stations is a reliable solution to achieve antenna diversity.

Space Time Block Codes (STBC) were introduced to obtain coded diversity for communication systems with multiple antennas [3], [4]. The very first well-known STBC is the Alamouti code [1], which was a complex orthogonal space-time code particularly for the case of two transmit antennas. After that, Tarokh applied the theory of orthogonal designs, analogous to Alamouti scheme, to present Orthogonal Space Time Block Codes (OSTBC) [2]. This feature of orthogonality makes the detection of the received signal linear. So these codes not only provide the maximum diversity order but also a less complex decoding.

In section II, the mathematical model for the wireless communication system under consideration is represented. Section III and IV includes Alamouti code and OSTBC code along with their encoding and decoding procedures. The mathematical analysis presented by Alamouti [1] and Tarokh [5] has been reproduced in these sections of paper for better understanding. This section covers detailed expressions for both Alamouti's STBC as well as OSTBC. Simulation results and conclusion are discussed in section V and section VI respectively.

2. System Model

Consider a wireless communication system with n transmitting and *m* receiving antennas. At each time slot t, signals c_t^i , i=1,2,3………,n are transmitted from *n* transmit antennas. Assume the channel to be flat fading and path gains from transmitter antenna *i* to receiver antenna *j* is defined to be $h_{i,j}$. The path gains are modeled as samples of independent complex Gaussian random variable with variance 0.5 per real dimension [2]. The channel coefficient $h_{i,j}$ (i=1,2,3…..,n; j=1,2,3…,m) is constant as the quasi-static Reyleigh fading channel is assumed.

At time t the received signal r_t^j , at antenna *j* is given by

$$r_t^j = \sum_{i=1}^{n} h_{i,j}\, c_t^i + n_t^j \qquad (1)$$

Where n_t^j are independent noise samples of zero-mean complex Gaussian random variables. For all code words $c = c_1^1 c_1^2 \dots c_1^n c_2^1 c_2^2 \dots . c_2^n \dots \dots . c_l^1 c_l^2 \dots \dots c_l^n$ the receiver computes the decision metric

$$\sum_{t=1}^{l}\sum_{j=1}^{m}\left| r_t^j - \sum_{i=1}^{n} h_{i,j}\, c_t^i \right|^2 \qquad (2)$$

in favor of the codeword that minimizes this sum and completes the ML (Maximum Likelihood) decoding.

3. Alamouti's STBC

Alamouti's [1], [8] scheme was the first STBC. The Alamouti STBC scheme uses two transmit antennas and N_r receive antennas and can achieve a maximum diversity order of $2N_r$. In addition, Alamouti scheme has full rate since it transmits 2 symbols every 2 time intervals. The Alamouti scheme encoding operation is given by (3).

$$G_2 = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \qquad (3)$$

At a time t, symbol s_1 and s_2 are transmitted from antenna 1 and 2 respectively. Assuming that each symbol has duration T, then at time t + T, the symbols $-s_2^*$ and s_1^* , where (*) is the complex conjugate, are transmitted from antenna 1 and 2 respectively.

In case of 1 Receive Antenna:

The reception and decoding of signal depends on the number of receive antennas. For the case of one receive antenna, the received signals are (4)

$$r_1^{(1)} = r_1(t) = h_{1,1}s_1 + h_{1,2}s_2 + n_1^{(1)} \qquad (4)$$

$$r_1^{(2)} = r_1(t+T) = -h_{1,1}s_2^* + h_{1,2}s_1^* + n_1^{(2)} \qquad (5)$$

where r_1 is the received signal at antenna 1, $h_{i,j}$ is the channel transfer function from the j^{th} transmit antenna and the i^{th} receive antenna n_1 is complex random variable representing noise at antenna 1.

These signals are sent to the decoder and are combined as follows [1]

$$\tilde{s_1} = h_{1,1}^* r_1^{(1)} + h_{1,2} r_1^{*(2)} \qquad (6)$$

$$\tilde{s_2} = h_{1,2}^* r_1^{(1)} + h_{1,1} r_1^{*(2)} \qquad (7)$$

And substituting (4) in (6) and (5) in (7) yields

$$\tilde{s_1} = (\alpha_{1,1,}^2 + \alpha_{1,2}^2)s_1 + h_{1,1}^* n_1^{(1)} + h_{1,2} n_1^{*(2)} \qquad (8)$$

$$\tilde{s_2} = (\alpha_{1,1,}^2 + \alpha_{1,2}^2)s_2 - h_{1,2}^* n_1^{(1)} + h_{1,1} n_1^{*(2)} \qquad (9)$$

where $\alpha_{i,j}^2$ is the squared magnitude of the channel transfer function $h_{i,j}$. The calculated $\tilde{s_1}$ and $\tilde{s_2}$ are then sent to a Maximum Likelihood (ML) decoder to estimate the transmitted symbols s_1 and s_2 respectively.

In case of 2 Receive Antenna:

For the case of two receive antennas, received symbols are

$$r_1^{(1)} = h_{1,1}s_1 + h_{1,2}s_2 + n_1^{(1)} \quad (10)$$

$$r_2^{(2)} = -h_{1,1}s_2^* + h_{1,2}s_2^* + n_1^{(2)} \quad (11)$$

$$r_2^{(1)} = h_{2,1}s_1 + h_{2,2}s_2 + n_2^{(1)} \quad (12)$$

$$r_2^{(2)} = -h_{2,1}s_2^* + h_{2,2}s_2^* + n_2^{(2)} \quad (13)$$

And combined signals are

$$\tilde{s_1} = h_{1,1}^* r_1^{(1)} + h_{1,2}r_1^{*(2)} + h_{2,1}^* r_2^{(1)} + h_{2,2}r_2^{*(2)} \quad (14)$$

$$\tilde{s_2} = h_{1,1}^* r_1^{(1)} + h_{1,2}r_1^{*(2)} + h_{2,1}^* r_2^{(1)} + h_{2,2}r_2^{*(2)} \quad (15)$$

which, after substituting equations (10)-(13), equation (14) and (15) becomes:

$$\tilde{s_1} = (\alpha_{1,1}^2 + \alpha_{1,2}^2 + \alpha_{1,2}^2 + \alpha_{2,1}^2)s_1 + h_{1,1}^* n_1^{(1)} + h_{1,2}n_1^{*(2)} + h_{2,1}^* n_2^{(1)} + h_{2,2}n_2^{*(2)} \quad (16)$$

$$\tilde{s_2} = (\alpha_{1,1}^2 + \alpha_{1,2}^2 + \alpha_{1,2}^2 + \alpha_{2,1}^2)s_2 - h_{1,1}^* n_1^{(1)} + h_{1,2}n_1^{*(2)} - h_{2,1}^* n + h_{2,2}n_2^{*(2)} \quad (17)$$

The ML decoder decision statistic decodes in favor of s_1 and s_2 over all possible values of s_1 and s_2 such that (18) and (19) are minimized where φ is given by (20) for N_t = 2 [2], [5].

$$\left\| \left[\sum_{i=1}^{N_r} (r_i^{(1)} h_{i,1}^* + r_i^{*(2)} h_{i,2} \right] - s_1 \right\|^2 + \varphi |s_1|^2 \quad (18)$$

$$\left\| \left[\sum_{i=1}^{N_r} (r_i^{(1)} h_{i,2}^* - r_i^{*(2)} h_{i,1} \right] - s_2 \right\|^2 + \varphi |s_2|^2 \quad (19)$$

$$\varphi = \left(-1 + \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |h_{i,j}|^2 \right) \quad (20)$$

4. Orthogonal Space Time Block Code

The Alamouti code is the first known Orthogonal Space-Time Block Codes (OSTBCs). Tarokh [2] apply the mathematical framework of orthogonal designs to construct both real and complex orthogonal codes that achieved full diversity. For the case of real orthogonal codes, the 2x2 design is

$$\begin{pmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{pmatrix} \quad (21)$$

The 4x4 design

$$\begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_1 & s_1 \end{pmatrix} \quad (22)$$

Similarly design for 8x8 can be constructed. Further complex orthogonal codes can also be constructed. OSTBC, [5]-[7] for the case of 3 transmitting antennas, block codes can be constructed for both ½ and ¾ coding rate .The G_3 represents the ½ and H_3 represents the ¾ rate codes. The ½ rate code G_3 is given by (23)

$$G_3 = \begin{pmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & s_3^* & s_2^* \end{pmatrix} \quad (23)$$

The decision metric minimized by the decoder for detecting s_1, s_2, s_3, s_4 is given by (24), (25), (26), (27) respectively where

$$\xi = \left(-1 + 2 \sum_{i=1}^{Nr} \sum_{j=1}^{N_t} |h_{i,j}|^2 \right)$$

$$\left\| \left[\sum_{i=1}^{N_r} (r_i^{(1)} h_{i,1}^* + r_i^{*(2)} h_{i,2} + r_i^{(3)} h_{i,3}^* + r_i^{*(5)} h_{i,1} + r_i^{*(6)} h_{i,2} + r_i^{*(7)} h_{i,3}) \right] - s_1 \right\|^2 + \xi |s_1|^2 \quad (24)$$

$$\left\| \left[\sum_{i=1}^{N_r} (r_i^{(1)} h_{i,2}^* - r_i^{*(2)} h_{i,1} + r_i^{(4)} h_{i,3}^* + r_i^{*(5)} h_{i,2} - r_i^{*(6)} h_{i,1} + r_i^{*(8)} h_{i,3}) \right] - s_2 \right\|^2 + \xi |s_2|^2 \quad (25)$$

$$\left\| \left[\sum_{i=1}^{N_r} (r_i^{(1)} h_{i,3}^* - r_i^{*(3)} h_{i,1} - r_i^{(4)} h_{i,2}^* + r_i^{*(5)} h_{i,3} - r_i^{*(7)} h_{i,3} - r_i^{*(7)} h_{i,1}) \right] - s_3 \right\|^2 + \xi |s_3|^2 \quad (26)$$

$$\left\| \left[\sum_{i=1}^{N_r} (-r_i^{(2)} h_{i,3}^* + r_i^{*(3)} h_{i,2} - r_i^{(4)} h_{i,1}^* - r_i^{*(6)} h_{i,3} + r_i^{*(7)} h_{i,2} - r_i^{*(8)} h_{i,1}) \right] - s_4 \right\|^2 + \xi |s_4|^2 \quad (27)$$

And the 3/4 rate code H_3 is given by (28)

$$H_3 = \begin{pmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1 - s_1^* + s_2 - s_2^*)}{2} \\ \frac{s_3^*}{\sqrt{2}} & \frac{-s_3^*}{\sqrt{2}} & \frac{(s_2 + s_2^* + s_1 - s_1^*)}{2} \end{pmatrix} \quad (28)$$

The decision statistics to minimize s_1, s_2, s_3 given by (29), (30), (31) respectively

$$\left\| \left[\sum_{i=1}^{N_r} \left(r_i^{(1)} h_{i,1}^{*(2)} + r_i^{*(2)} h_{i,2} + \frac{\left((r_i^{(4)} - r_i^{(3)}) h_{i,3}^* \right)}{2} - \frac{\left((r_i^{(3)} + r_i^{(4)})^* h_{i,3} \right)}{2} \right) \right] - s_1 \right\|^2 + \varphi |s_1|^2 \quad (29)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,2}^* - r_i^{*(2)}h_{i,1} + \frac{\left(\left(r_i^{(4)}+r_i^{(3)}\right)h_{i,3}^*\right)}{2} + \frac{\left(\left(-r_i^{(3)}+r_i^{(4)}\right)^* h_{i,3}\right)}{2}\right)\right] - s_2\right|^2 + \varphi|s_2|^2 \quad (30)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(\frac{\left(\left(r_i^{(1)}+r_i^{(2)}\right)h_{i,3}^*\right)}{\sqrt{2}} + \frac{\left(r_i^{*(3)}\left(h_{i,1}+h_{i,2}\right)\right)}{\sqrt{2}} + \frac{\left(r_i^{*(4)}\left(h_{i,1}-h_{i,2}\right)\right)}{\sqrt{2}}\right)\right] - s_3\right|^2 + \varphi|s_3|^2 \quad (31)$$

Similarly for the 4 transmit antennas block codes can be constructed for both ½ and ¾ coding rate .The G_4 represents the ½ and H_4 represents the ¾ rate codes. The G_4 can be represented by (32)

$$G_4 = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix} \quad (32)$$

The decision metric to minimize by the ML decoder for detecting s_1, s_2, s_3, s_4 given by (33), (34), (35), (36)

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,1}^* + r_i^{(2)}h_{i,2}^* + r_i^{(3)}h_{i,3}^* + r_i^{(4)}h_{i,4}^* + r_i^{*(5)}h_{i,1} + r_i^{*(6)}h_{i,2} + r_i^{*(7)}h_{i,3} + r_i^{*(8)}h_{i,4}\right)\right] - s_1\right|^2 + \xi|s_1|^2 \quad (33)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,2}^* - r_i^{(2)}h_{i,1}^* - r_i^{(3)}h_{i,4}^* + r_i^{(4)}h_{i,3}^* + r_i^{*(5)}h_{i,2} - r_i^{*(6)}h_{i,1} - r_i^{*(7)}h_{i,4} + r_i^{*(8)}h_{i,3}\right)\right] - s_2\right|^2 + \xi|s_2|^2 \quad (34)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,3}^* + r_i^{(2)}h_{i,4}^* - r_i^{(3)}h_{i,1}^* - r_i^{(4)}h_{i,2}^* + r_i^{*(5)}h_{i,3} + r_i^{*(6)}h_{i,4} - r_i^{*(7)}h_{i,1} - r_i^{*(8)}h_{i,2}\right)\right] - s_3\right|^2 + \xi|s_3|^2 \quad (35)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,4}^* - r_i^{(2)}h_{i,3}^* + r_i^{(3)}h_{i,2}^* - r_i^{(4)}h_{i,1}^* + r_i^{*(5)}h_{i,4} - r_i^{*(6)}h_{i,3} + r_i^{*(7)}h_{i,2} - r_i^{*(8)}h_{i,1}\right)\right] - s_4\right|^2 + \xi|s_4|^2 \quad (36)$$

The decoding decision metric (36) is taken from the paper by Luis Miguel et al [9] as Tarokh [5] had probably mistaken in that metric.

And 4 transmit antenna block code with rate ¾ is given by (37)

$$H_4 = \begin{pmatrix} s_1 & -s_2^* & \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} \\ s_2 & s_1^* & \frac{s_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & \frac{(-s_1-s_1^*+s_2-s_2^*)}{2} & \frac{(s_2+s_2^*+s_1-sx_1^*)}{2} \\ \frac{s_3}{\sqrt{2}} & \frac{-s_3}{\sqrt{2}} & \frac{(-s_2-s_2^*+s_1-s_1^*)}{2} & \frac{-(s_1+s_1^*+s_2-s_2^*)}{2} \end{pmatrix} \quad (37)$$

The decision statistics to minimize s_1, s_2, s_3 is given by (38), (39), (40) respectively

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,1}^* + r_i^{*(2)}h_{i,2} + \frac{\left(\left(r_i^{(4)}-r_i^{(3)}\right)\left(h_{i,3}^*-h_{i,4}^*\right)\right)}{2} - \frac{\left(\left(r_i^{(3)}+r_i^{(4)}\right)^*\left(h_{i,3}+h_{i,4}\right)\right)}{2}\right)\right] - s_1\right|^2 + \varphi|s_1|^2 \quad (38)$$

$$\left\|\left[\sum_{i=1}^{N_r}\left(r_i^{(1)}h_{i,2}^* - r_i^{*(2)}h_{i,1} + \frac{\left(\left(r_i^{(4)}+r_i^{(3)}\right)\left(h_{i,3}^*-h_{i,4}^*\right)\right)}{2} + \frac{\left(\left(-r_i^{(3)}+r_i^{(4)}\right)^*\left(h_{i,3}+h_{i,4}\right)\right)}{2}\right)\right] - s_2\right|^2 + \varphi|s_2|^2 \quad (39)$$

$$\left\|\left[\sum_{i=1}^{Nr}\left(\frac{\left(\left(r_i^{(1)}+r_i^{(2)}\right)h_{i,3}^*\right)}{\sqrt{2}} + \frac{\left(\left(r_i^{(1)}-r_i^{(2)}\right)h_{i,4}\right)}{\sqrt{2}} + \frac{\left(\left(r_i^{*(3)}\left(h_{i,1}+h_{i,2}\right)\right)\right)}{\sqrt{2}} + \frac{\left(\left(r_i^{*}\left(h_{i,1}+h_{i,2}\right)\right)\right)}{\sqrt{2}}\right)\right] - s_3\right|^2 + \varphi|s_3|^2 \quad (40)$$

5. Simulation Results

The mathematical expressions covered in previous section were simulated through MATLAB programming for analyzing the performance of STBC for different modulation schemes, different number of transmitting antennas and code rate. BER results obtained by simulation are shown in figures 1-4 respectively.

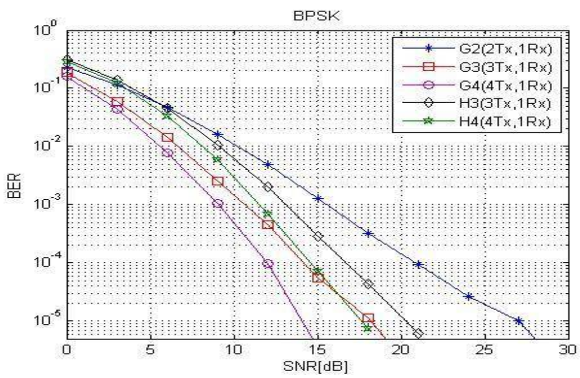

Figure 1: STBC performance in BPSK

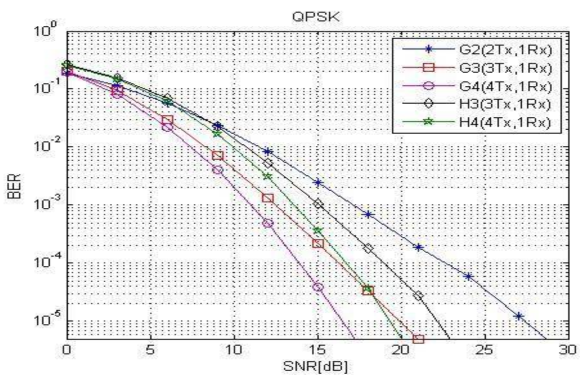

Figure 2: STBC performance in QPSK

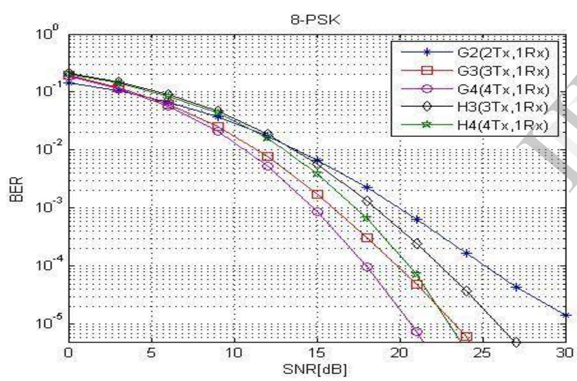

Figure 3: STBC performance in 8-PSK

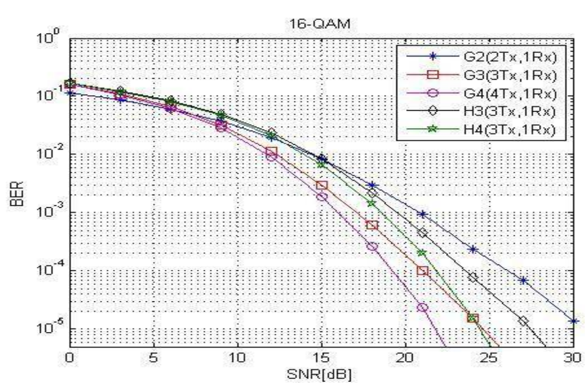

Figure 4: STBC performance in 16-QAM

The modulation schemes considered for simulation include BPSK, QPSK, 8-PSK and 16-QAM. Further, the performance was analyzed by varying the number of transmitting antennas and code rate. From figure 1, SNR requirement for 4-T_x antenna (G_4) is 13 dB lower than using 2-T_x antenna (G_2) where as it is lower 9dB lower in 3-T_x antenna (G_3) at the BER value of 10^{-5} as compared to G_2. At the same BER value, 3-T_x antenna system with higher code rate (H_3) i.e. ¾ have the SNR value 20 dB which is 7dB lower than G_2 and 2dB more than its corresponding lower rate system G_3. Although H_3 and H_4 have higher rate than G_3 and G_4, the performance of G_3 and G_4 is better.

By comparing the figure 1, 2, 3 and 4 for the same SNR, BER performance of STBC degrades with increase in the order of the constellation i.e. BPSK has the best performance followed by QPSK, 8-PSK and 16-QAM.

6. Conclusion

This paper provides a detailed mathematical analysis of Space Time Block Codes. An introduction to Space-Time Coding has been provided by Alamouti's scheme. Then block codes schemes with different code rates for three and four transmitting antennas have been discussed. The encoding and decoding procedure have also been presented. BER performance Alamouti STBC and OSTBC have been analyzed by simulation carried out in MATLAB. Finally, it can be concluded that BPSK modulation technique provides the best results among all modulation techniques. Also, four transmitting antenna system i.e. G_4 provides best BER performance among different number of transmitting antenna system with half rate codes.

7. References

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