Performance Analysis of Primary user and Secondary users of a Random Access in OFDM based Cognitive Radio Network

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Abstract : **One of the main application concerns in cognitive radio (CR) systems is spectrum sensing by the secondary receiver because of the uncertainties in propagation channel, hidden primary user (PU) problem, sensing duration and security purpose. In this work an orthogonal frequency-division multiplexing (OFDM)- based CR spectrum sharing system is considered, that assumes random access of primary network subcarriers by secondary users (SUs) and absence of the PU's spectrum utilization information, i.e., no spectrum sensing is appointed to obtain information about the PU's activity or availability of free subcarriers. In the non-attendance of information about the PU's activity, the SUs randomly access the subcarriers of the primary network and collide with the PU's subcarriers with a certain probability. Also, inter-cell collisions among the subcarriers of SUs (belonging to different cells) which can appear due to the inherent nature of random access scheme is being considered. This work leads a stochastic analysis of the number of subcarrier collisions between the SUs' and PU's subcarriers presuming fixed and random number of subcarriers requirements for each user. The performance of the random scheme in terms of capacity and capacity (rate) loss happens because of the subcarrier collisions is caused by assuming an interference power limitation at PUs to protect their operation.**

I. INTRODUCTION

Spectrum measurements around the world have revealed the reality that the obtainable spectrum is under-utilized. Hence, efficient utilization of the spectrum represents a serious issue in the wireless communications field. One of the most remarkable solutions to cope with the under-utilization of spectrum is the concept of cognitive radio (CR). CRs assume that the radio frequency (RF) spectrum can be utilized by secondary users (SUs) in addition to the legacy users also termed primary users (PUs) by complying with some predefined requirements imposed by PUs on SUs. Two of the most popular SU spectrum utilization methods are spectrum sharing and opportunistic access methods,
which are also referred to as underlay and interweave CR networks, respectively. In the spectrum sharing method, a SU can concurrently use the same spectrum with a PU by regulating (adapting) its peak or average transmit power below a PU predefined interference temperature (IT) (power) constraint, so that the quality of service (QoS) requirement of PU is maintained. In the opportunistic access method, a SU can only access the spectrum when it is not occupied by PU. One of the most challenging issues in the implementation of CR networks is the acquisition of information about the spectrum occupancy of PU(s) Deploying an efficient spectrum sensing mechanism is difficult because of the uncertainties present in the propagation channels at device and network-level, the hidden PU problem induced by severe fading conditions and the limited sensing duration. Nonetheless, considering practical systems (multiple secondary networks or cells),
there may exist inter-cell subcarrier collisions not only between SUs and PUs but also among the SUs themselves due to the random access scheme. Therefore, in this paper, two different SU transmitter and receiver pairs belonging to different cells are considered, and the performances in terms of capacity and rate loss due to collisions (interference) between SUs in addition to that of PU are studied. The two SUs case is considered for the sake of analytical tractability. Nonetheless, further insights are provided throughout the paper in order to shed light for the case of multi-cell scenario. The average capacity expressions of target SU's (SU-1) at the *i*th subcarrier are derived for no interference case, and when there is interference from only SU-2, only PU, and both SU-2 and PU. The number of subcarriers required by PU or SUs can also vary based on either PU or SUs rate requirements. The long term average performance of the system is investigated by using a stochastic model for the required number of subcarriers of PU and SUs, which is assumed to be *fixed* in. The statistical analysis of the number of subcarrier collisions between the users is also conducted. The probability mass functions (PMFs) and the average number of subcarrier collisions are derived when there is fixed and random number of subcarriers required by users. Finally, upper bounds for instantaneous and average maximum capacity (rate) loss of SU-1 due to collisions are derived.

II. SYSTEM AND CHANNEL MODELS

The orthogonal frequency-division multiplexing (OFDM)-based CR system is illustrated in Figure1., where a PU and

SUs are assumed to be present in the primary and secondary networks, respectively, where each SU transmitter and receiver pair belongs to separate cells1. The total number of available subcarriers in the primary network is denoted by *F*, and the number of PU's subcarriers is denoted by *FP*. The number of subcarriers utilized by SU-1 and SU-2 are represented by *FS*1 and *FS*2, respectively. SUs randomly access the available subcarriers set, *F*, in the primary network without having access to the PU's channel occupancy information. Subcarrier collisions occur when SUs randomly employ subcarriers, which are in use by PU and/or other SU, and the probabilistic model for the number of subcarrier collisions follows a hypergeometric distribution. Due to the random access (allocation) of subcarriers by SUs in different secondary cells, collisions occur with a certain probability between the subcarriers of SUs and PU. In addition, intercell collisions between the subcarriers of SUs might occur in addition to those that are utilized by PU. This set-up could be considered as the worst case scenario, where the collisions among the SUs subcarriers severely affect the performance due to the overall caused interference. One can observe from Figure 1 that the occurrence of collisions can be classified into different groups such as collisions between PU and SU-1, PU and SU-2, SU-1 and SU-2, and the worst case situation that assumes collisions among PU, SU-1 and SU-2. Notice that under the assumption of orthogonal allocation of subcarriers to the SUs in each secondary cell, subcarrier collision will only severely impact the performance of the SUs located at the cell-edge. It is assumed there is no cooperation between the cells (secondary base stations) or users (SUs)belonging different cells, therefore the collisions between the SUs belonging separate cells is probable due to random access (allocation) scheme and it will degrade the performance of cell-edge SUs. Such a scenario will severely degrade the performance of the PU. It is further assumed that there is no correlation among the subcarriers.

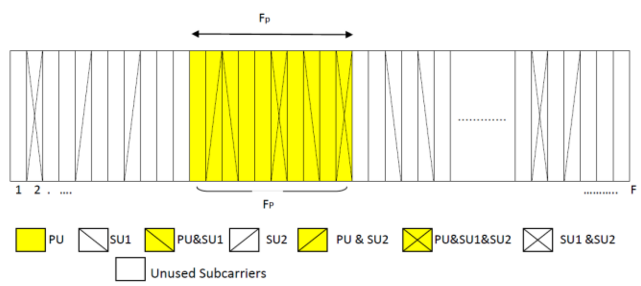

Fig 1. OFDM-based CR system for SUs in different secondary networks (cells) with subcarrier collisions with each other and PU due to the random access method.

In Figure 2, the channel model at the *i*th subcarrier (*i ϵ{*1, . . ., *F}*) is shown. The *channel power gains* from PU-Tx to PU-Rx, SU-Rx-1, and SU-Rx-2 are denoted by *gi*, *gs*1,*i* and *gs*2,*i*, respectively. Similarly, *h*1,*i*, *h*1*p*,*i* and *h*1*s*,*i* represent the channel power gains from SU-Tx-1 to SU-Rx-1, PU-Rx, andSU-Rx-2, respectively. In addition, *h*2,*i*, *h*2*s*,*i* and *h*2*p*,*i* denote the channel power gains for the *i*th subcarrier from SU-Tx-2 toSU-Rx-2, SU-Rx-1 and PU-Rx, respectively. The performance analysis of shaded SU (SU-1) is of interest in this work. To preserve the QoS requirement of PU, the interference power levels caused by the SU-transmitters at the PU-Rx must not be larger than a predefined value for each subcarrier, referred to as the interference temperature (power) constraint.

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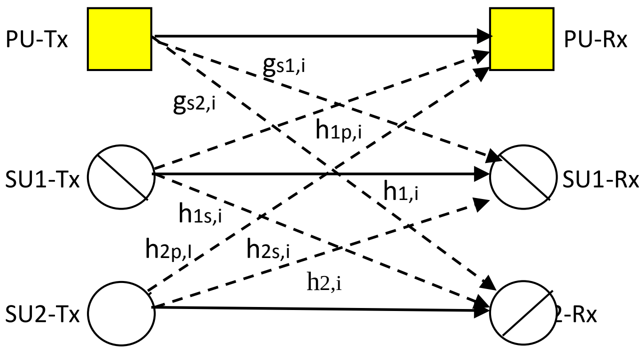

Fig 2. Channel model for the ith subcarrier, i ∈ {1, . . . ,F}, with SUs and PU-transmitter and receiver pairs, the performance of shaded SU pairs (SU-1) is of interest.

III. STATISTICAL ANALYSIS OF THE NUMBER OF SUBCARRIER COLLISIONS

The PMFs and the average number of subcarrier collisions for different cases are derived in this chapter.

A. Fixed Number of Subcarriers

Throughout this chapter, the number of subcarriers required by PU and SUs is assumed fixed. In order to properly assess the effect of the random access scheme on the subcarrier collisions, first only a single SU case (secondary cell) is considered.

1) Single Secondary Cell: Here, only a single SU (SU-1) is assumed to be available in the system. The following result holds.

Proposition 1:
PMF of the Number of Subcarrier Collisions: If SU-1 randomly utilizes (accesses) F_1^S subcarriers from a set of F available subcarriers without replacement, while FP subcarriers are being utilized by PU, then the PMF of the number of subcarrier collisions, k_{p1}, follows the hypergeometric distribution, $k_{p1} \sim HYPG(F_1^S, F^P, F)$, and is expressed as:

$$Pr(Kp1 = kp1) = p(kp1) = \binom{F^P}{k_{p1}} \binom{F-F^P}{F_1^S-k_{p1}} \Big/ \binom{F}{F_1^S}$$

The average number of subcarrier collisions is $E[k_{p1}] = F_1^S F^P / F$, where $E[\cdot]$ denotes the expectation operator, and the support of k_{p1} is $kp1 \in [(F_1^S + F^P - F)^+, \ldots, \min\{F_1^S, F^P\}$ $, where\ (x)^+ = max\{0, x\}$.

2) Two Secondary Cells: In this scenario, due to the random access method, there can be inter-cell collisions between the subcarriers of SUs (belonging to the separate cells) in addition to those that collide with PU subcarriers. There are four possible cases of subcarrier collisions for the target SU (say SU-1):

- Case 1: collisions between SU-1, PU and SU-2 subcarriers: k_{p12}.
- Case 2: collisions only between SU-1 and PU subcarriers: $k_{p1}^o = k_{p1} - k_{p12}$.
- Case 3: collisions only between SU-1 and SU-2 subcarriers: $k_{12}^o = k_{12} - k_{p12}$.
- Case 4: collisions-free subcarriers of SU-1: $k_{f1} = F_1^S - k_{p1}^o - k_{12}^o - k_{p12}$.

Random variable kp1 represents the number of subcarrier collisions between SU-1 and PU in the absence of SU-2. Similarly, k_{12} denotes the number of subcarrier collisions between the SU-1 and SU-2 in the absence of PU. It is evident to observe from Proposition 1 that the PMF of k12 also follows a hypergeometric distribution, $k_{12} \sim HYPG(F_1^S, F_2^S, F)$. The PMFs and expected values of the aforementioned RVs are presented next.

Case 1 (k_{p12}): Let k_{p2} stand for the number of subcarrier collisions between SU-2 and PU in the absence of SU- 1: $k_{p2} \sim HYPG(F_2^S, F^P, F)$. Once the number of subcarrier collisions between SU-2 and PU is given, one can obtain the conditional PMF:

$$p(kp12 | kp2) = \binom{k_{p2}}{k_{p12}} \binom{F-k_{p2}}{F_1^S-k_{p12}} \Big/ \binom{F}{F_1^S}$$

Using the above equation, the PMF is obtained:

$$p(kp12) = \sum_{kp2} p(kp12, kp2) = \sum_{kp2} p(kp12/kp2) p(kp2)$$

$$\sum_{kp2=(F_2^S + F^P - F)^+}^{\min\{F_1^S, F^P\}} \binom{k_{p2}}{k_{p12}} \binom{F-k_{p2}}{F_1^S - k_{p12}} \binom{F^P}{k_{p2}}$$

$$\times \binom{F-F^P}{F_2^S - k_{p2}} / \left[\binom{F}{F_1^S} \binom{F}{F_2^S} \right]$$

The average number of subcarrier collisions, $E[k_{p12}]$, is expressed as given in

$$E[k_{p12}] = \sum_{kp12} kp12\, p(kp12)$$

Case 2 (k_{p1}^o): Following the same approach as in Case 1, the PMF of k_{p1}^o is expressed as:

$$p(\mathring{k}_{p1}) = \sum_{kp12} p(\mathring{k}_{p1}, kp12) = \sum_{kp12} p(\mathring{k}_{p1} | kp12) p(kp12)$$

$$= \left[\sum_{kp12} p(\mathring{k}_{p1}, kp12) \left[\sum_{k_{p2}} p(kp12 | k_{p2}) p(k_{p2}) \right] \right]$$

Plugging $p(K_{p12})$ into the equation above yields the desired PMF. The average number of subcarrier collisions between SU-1 and PU, $E[k_{p12}]$, is given by

$$E[\mathring{k}_{p1}] = E[k_{p1} - k_{p12}] = E[k_{p1}] - E[k_{p12}]$$

$$= \frac{F_1^S F^P}{F} - \frac{F_1^S F_2^S F^P}{F^2} = \frac{F_1^S F^P (F - F_2^S)}{F^2}$$

Case 3 (k_{12}^o): Similar to the Case 2, the PMF of k_{12}^o can be obtained by replacing k_{p1}^o with k_{12}^o in (4), and its expression is omitted for brevity. Nonetheless, the expression for the average number of subcarrier collisions between SU-1 and SU-2, $E[k_{12}^o]$, is expressed as

$$E[\mathring{k}_{12}] = E[k_{12} - k_{p12}] = \frac{F_1^S F_2^S (F - F^P)}{F^2}$$

Case 4 (k_{f1}): Lastly, the average number of collisionsfree subcarriers for SU-1, $E[k_{f1}]$, is given by

$$E[k_{f1}] = E[F_1^S - \mathring{k}_{p1} - \mathring{k}_{12} - k_{p12}]$$

$$= F_1^S - \frac{F_1^S F^P (F - F_2^S)}{F^2} - \frac{F_1^S F_2^S (F - F^P)}{F^2} - \frac{F_1^S F_2^S F^P}{F^2}$$

$$= \frac{F_1^S (F - F_2^S)(F - F^P)}{F^2}.$$

B. Random Number of Subcarriers

In the preceding section, the number of utilized subcarriers by both PU and SUs is assumed to be fixed. However, considering practical scenarios, the number of subcarriers required by PU or SUs can vary. Based on either PU or

SUs rate requirements, the number of subcarriers utilized by users can be different at any time instant. Next the long term average performance of the system is investigated by using a stochastic model for the required number of subcarriers for PU and SUs. Intuitively, the number of subcarriers required by a user can be variable based on its communication needs throughout a period (a day or a week), such as QoS, BER, and rate.

It is assumed that the number of subcarriers utilized by the users follows a binomial distribution. Mathematically, the number of utilized subcarriers by SU-1 is $f_1^s \sim B(T_1^s, q_1^s)$, and its PMF is given by $Pr(F_1^S = f_1^s) = p(f_1^s) = \binom{T_1^s}{f_1^s} (q_1^s) f_1^s (1 - q_1^s) T_1^s - f_1^s$, where T_1^s denotes the number of trials, which can be considered as the maximum number of subcarriers used by SU-1, and $q_1^s \in [0, 1]$ is the probability of success in each trial. Under these assumptions, the average number of required subcarriers is given by $E[f_1^s] = T_1^s q_1^s$. Similarly, the numbers of subcarriers utilized by PU and SU-2 are assumed to be binomially distributed RVs, i.e., $f^p \sim B(T^p, q^p)$ and $f_2^s \sim B(T_2^s, q_2^s)$, respectively. Let $\widehat{k_{p1}}$ denote the number of subcarriers collisions between SU-1 and PU when both of them utilize a random number of subcarriers. When the users' utilized numbers of subcarriers are random, Proposition 1 can be restated in the following form.

Proposition 2: Let f_1^s and f^p be independent but not necessarily identically distributed binomial RVs, representing the utilized number of subcarriers by SU-1 and PU, respectively. If SU-1 randomly accesses f_1^s subcarriers from a set of F available subcarriers without replacement while f^p subcarriers are being used by the PU, then the PMF of the number of subcarrier collisions, $\widehat{k_{p1}}$, is given by:

$$p(\hat{k}_{p1}) = \sum_{f_1^S=0}^{T_1^S} \sum_{f_p^P=0}^{T^P} \left[\binom{f^P}{\hat{k}_{p1}} \binom{F-f^P}{f_1^S-\hat{k}_{p1}} \binom{T_1^S}{f_1^S} \binom{T^P}{f^P} \Big/ \binom{F}{f_1^S} \right]$$

$$\times (q_1^s)^{f_1^S} (1-q_1^s)^{T_1^S - f_1^S} (q_p)^{f^P} (1-q_p)^{T^P - f^P}$$

and the average number of subcarrier collisions is: $E[\widehat{k_{p1}}] = T_1^s T^p q_1^s q^p / F$.

Following a similar reasoning, one can readily obtain the expressions for the PMFs and the expected values of $\hat{k}_{p1}^o$, $\hat{k}_{12}$, $\hat{k}_{12}^o$, $\hat{k}_{p12}$ and $\hat{k}_{f1}$. Briefly, the following important results for the expected values of $\hat{k}_{p12}$, $\hat{k}_{p1}^o$, $\hat{k}_{12}^o$ and $\hat{k}_{f1}$, can be expressed, respectively, as:

$$E[\hat{k}_{p12}] = \frac{T_1^S T_2^S T^p q_1^s q_2^s q_p}{F^2}$$

$$E[\hat{k}_{p1}^o] = \frac{T_1^S T^p q_1^s q_p (F - T_2^S q_2^s)}{F^2}$$

$$E[\hat{k}_{12}^o] = \frac{T_1^S T_2^S T^p q_1^s q_2^s (F - T^p q_p)}{F^2}$$

$$E[\hat{k}_{f1}] = \frac{T_1^S q_1^s (F - T_2^S q_2^s)(q_2^s (F - T^p q_p)}{F^2}$$

CHAPTER IV
PERFORMANCE ANALYSIS OF SECONDARY USER

The performance of the target SU (SU-1) is investigated by using the average capacity as performance measure. In addition, the capacity (rate) loss of SU-1 due to the subcarrier collisions with the subcarriers of PU and SU- 2 is studied. The sets of subcarriers are defined as follows. Let $\mathcal{K}_{p1}^o$ be the set of collided subcarriers only between the SU-1 and PU, and $k_{p1}^o = \mathcal{K}_{p1}^o$ (fixed case) or $\hat{k}_{p1}^o = \mathcal{K}_{p1}^o$ (random case), the cardinality of the set $\mathcal{K}_{p1}^o$. Similarly, the same reasoning can be applied for the sets $\mathcal{K}_{12}^o, \mathcal{K}_{p12}, \mathcal{K}_{f1}, F^P, F_1^S$, and F_2^S with their cardinalities given, respectively, as $\hat{k}_{12}^o = |K_{12}^o|$, $\hat{k}_{p12} = |K_{p12}|$, $\hat{k}_{f1} = |K_{f1}|$, $f^p = F^P$, $f_1^s = F_1^S$, and $f_2^s = F_2^S$.

6.1 Average Capacity of SU

Next the expressions for the instantaneous and average capacity of SU-1 over an arbitrary channel fading model with a random access scheme are presented.

Theorem 1: Let $S_{p1,i}^o, S_{12,i}^o$ and $S_{12,i}^p$ denote the signal-to-interference plus noise ratio (SINR) levels for the ith subcarrier of SU-1 with interference component coming only from PU, only from SU-2 and from both PU and SU-2, respectively. Similarly, let $S_{f1,i}$ stand for the signal-to-noise ratio (SNR) for the ith collision-free subcarrier of the SU-1.

Mathematically, the SINRs and SNR are defined as

$$S_{p1,i}^o = \frac{h_{1,i} P_{1,i}}{g_{s1,i} P_i + \sigma^2},$$

$$S_{12,i}^o = \frac{h_{1,i} P_{1,i}}{h_{2s,i} P_{2,i} + \sigma^2},$$

$$S_{p12,i} = \frac{h_{1,i} P_{1,i}}{g_{s1,i} P_i + h_{2s,i} P_{2,i} + \sigma^2},$$

$$S_{f1,i} = \frac{h_{1,i} P_{1,i}}{\sigma^2}.$$

Then, the instantaneous capacity of SU-1 with the random access method is expressed as

$$\begin{aligned} C_{S_1} &= \sum_{i \in \mathcal{K}_{p1}^o} C_{p1,i}^o + \sum_{i \in \mathcal{K}_{12}^o} C_{12,i}^o + \sum_{i \in \mathcal{K}_{p12}} C_{p12,i} + \sum_{i \in \mathcal{K}_{f1}} C_{f1,i} \\ &= \sum_{i \in \mathcal{K}_{p1}^o} \log\left(1 + S_{p1,i}^o\right) + \sum_{i \in \mathcal{K}_{12}^o} \log\left(1 + S_{12,i}^o\right) \\ &\quad + \sum_{i \in \mathcal{K}_{p12}} \log\left(1 + S_{p12,i}\right) + \sum_{i \in \mathcal{K}_{f1}} \log\left(1 + S_{f1,i}\right), \end{aligned}$$

and the mean value of C_{S1} is given by

$$\mathbb{E}[C_{S_1}] = \frac{T_{s1}q_{s1}}{F^2}\{(F-T_{s2}q_{s2})[T_pq_p\mathbb{E}[C^o_{p1,i}] + (F-T_pq_p)\mathbb{E}[C_{f1,i}]] + T_{s2}q_{s2}[(F-T_pq_p)\mathbb{E}[C^o_{12,i}] + T_pq_p\mathbb{E}[C_{p12,i}]]\},$$

where the expected values of capacities at the ith subcarrier for the four different cases, $C^o_{p1,i}, C^o_{12,i}, C_{p12,i}$ and $C_{f1,i}$ over the Rayleigh channel fading model are investigated.

6.2 Capacity Loss Due to Collisions

The upper bounds for the SU-1 instantaneous and average capacity loss due to subcarrier collisions are given by the following result.

Corollary 1: The maximum capacity (rate) loss of SU-1 due to subcarrier collisions is upper-bounded by

$$\frac{1}{\sigma^2}\left[\sum_{i\in\mathcal{K}_{p1}} g_{s1,i}P_i + \sum_{i\in\mathcal{K}_{12}} h_{2s,i}P_{2,i}\right]$$

Mathematically,

$$\Delta C_{S_1} = C^f_{S_1} - C_{S_1} \le \frac{1}{\sigma^2}\left[\sum_{i\in\mathcal{K}_{p1}} g_{s1,i}P_i + \sum_{i\in\mathcal{K}_{12}} h_{2s,i}P_{2,i}\right]$$

Where C^f_{S1} is the capacity of SU-1 when all of its subcarriers are collision-free, and is defined as $C^f_{S1} =$ " $\sum_{i=1}^{f^S_1}\log(1 + h_{1,i}P_{1,i}/\sigma^2)$. The upper bound for the maximum average capacity loss is given by

$$\mathbb{E}[\Delta C_{S_1}] \le \frac{T_{s1}q_{s1}(T_pq_pP_i + T_{s2}q_{s2}P_{2,i})}{\sigma^2 F}.$$

Note that during the derivation of the maximum capacity loss expression, the aim was to provide a simple and concise expression for the maximum capacity loss that occurs due to collision in terms of a few system parameters. In addition, further insights definitely can be provided. The tightness of the bound depends of the number of subcarriers required by the users and their transmit power levels. The behavior of the bound can be viewed as tight when the capacity (rate) loss is high due to collisions, while it is loose when the loss is low. This fact stems from utilizing the inequality $log(1 + x) < x, \forall x \ge 0$. The objective for deriving the maximum capacity loss bound expression was to provide a bound that consists of a few system parameters. Therefore, we termed the expression as the bound for maximum capacity loss.

Note that the expressions of the exact capacity loss can be obtained as

$$\mathbb{E}[\Delta C_{S_1}] = \mathbb{E}[C_{S_1}] - \mathbb{E}[C^f_{S_1}],$$

where $E[C_{S1}]$ is previously defined and

$$\mathbb{E}\left[C^f_{S_1}\right] = \mathbb{E}\left[f^S_1\right]\mathbb{E}\left[C_{f1,i}\right] = T_{s1}q_{s1}\mathbb{E}\left[C_{f1,i}\right].$$

On the other hand, even though the derived bound is not tight, comparing it with the exact capacity loss expression above, one can observe that the derived bound is much simpler and does not involve any logarithm terms and does not require additional expectation operation as further expectation operations are needed in the exact loss expression. In practice, it is well known that the bounds, possibly simple, are utilized as very important predictors of the system performance and comparison benchmarks.

6.3 Capacity Over Rayleigh Channel Fading

The average capacity expressions at the ith subcarrier for the four different collision cases, given in previous chapters, will be studied. Various methods have been proposed to protect the operation of PU by maintaining the QoS requirements above some predefined threshold, and in this regard peak or average interference power constraints are two well known methods which can be found in literature. Here, to investigate the performance of the proposed random access scheme, the well known peak interference power constraint at each subcarrier is adapted. It is assumed that the peak transmit powers of SUs are the same for a tractable analysis $P_1 = P_2 = P_s$. Therefore, the transmit power of the SU-1 is adapted to protect PU, and is given by

$$P^T_s = \begin{cases} P_s & \beta P_s \le I \\ \frac{I}{\beta} & \beta P_s \ge I \end{cases} \min\{P_s, \frac{I}{\beta}\}$$

where $\beta = h_{1p} + h_{2p}$, and I is the interference power constraint. It is worth to note that due to the random access scheme, the transmit power is adopted (regulated) considering the worst case scenario, as if there are collisions between both SUs and PU (interference from both SUs at PU-Rx, i.e., $h_{1p}P_s + h_{2p}P_s$). This condition assures the QoS requirement of PU. Notice also that considering the Rayleigh channel fading model, all the channel power gains are exponentially distributed with unit mean, e.g., $h_{1p}, h_{2p}, h_1, g_s \sim Exp(1)$. Therefore, the RV β follows Erlang distribution with shape and rate parameters of 2 and 1, respectively, i.e., $\beta \sim Erlang(2,1), f_\beta(x) = xe^{-x}, x \ge 0$. Once the transmit power of the SU-1 is regulated, the received power of SU-1 at the SU-Rx can be defined as $= h_1P^T_s$. Exploiting properties of order statistics, and by following a similar approach, the cumulative distribution function (CDF) of α can be obtained.

$$F_\alpha(x) = 1 - \Pr\left(h_1 > \frac{x}{P_s}\right)\Pr\left(\beta < \frac{\mathcal{I}}{P_s}\right) - \int_{\mathcal{I}/P_s}^{\infty}\Pr\left(h_1 > \frac{xy}{\mathcal{I}}\right)f_\beta(y)\mathrm{d}y$$
$$= 1 - e^{-\frac{x}{P_s}}\left[1 - \left(\frac{\mathcal{I}+P_s}{P_s}\right)e^{-\frac{\mathcal{I}}{P_s}}\right] - \frac{\mathcal{I}^2(x+P_s+\mathcal{I})}{P_s(x+\mathcal{I})^2}e^{-\frac{x+\mathcal{I}}{P_s}}.$$

Also, the probability density function (PDF) of α can be readily expressed as

$$f_{\alpha}(x) = \frac{e^{-\frac{x}{P_s}}}{P_s} - \frac{e^{-\frac{x+\mathcal{I}}{P_s}}}{P_s^2}\left[\mathcal{I} + P_s - \frac{\mathcal{I}^2\left((x+\mathcal{I})^2 + 2(x+\mathcal{I})P_s + 2P_s^2\right)}{(x+\mathcal{I})^3}\right].$$

Once the PDF and CDF of α are obtained, the average capacity expressions can be derived.

V. SIMULATION RESULTS

In this section, the numerical and simulation results are presented to confirm the analytical results by taking all the cases in which included in (section III.A)

- Case 1: collisions between SU-1, PU and SU-2 subcarriers: k_{p12}.

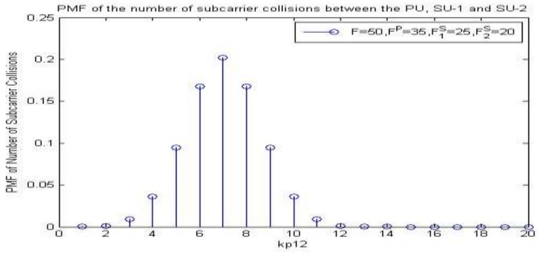

fig 3 collisions between SU-1, PU and SU-2 subcarriers

- Case 2: collisions only between SU-1 and PU subcarriers: $k_{p1}^{o} = k_{p1} - k_{p12}$.

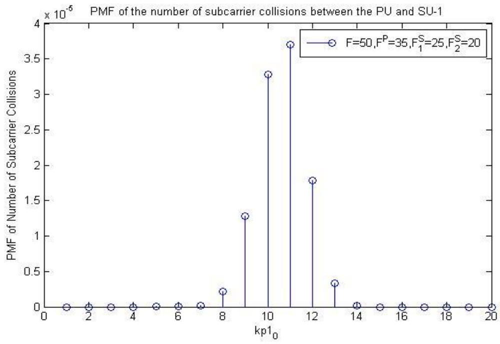

fig 4. collisions only between SU-1 and PU subcarriers

- Case 3: collisions only between SU-1 and SU-2 subcarriers: $k_{12}^{o} = k_{12} - k_{p12}$.

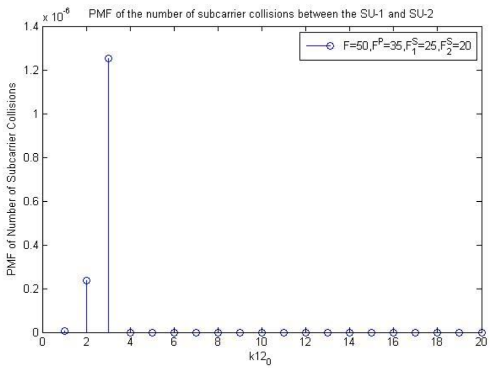

fig 6. collisions only between SU-1 and SU-2 subcarriers

- Case 4: collisions-free subcarriers of SU-1: $k_{f1} = F_1^S - k_{p1}^o - k_{12}^o - k_{p12}$.

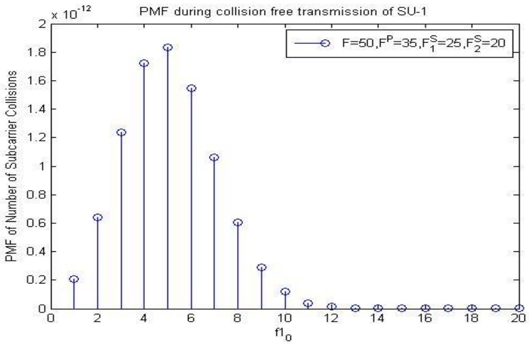

fig 7. collisions-free subcarriers of SU-1 from section IV we obtained

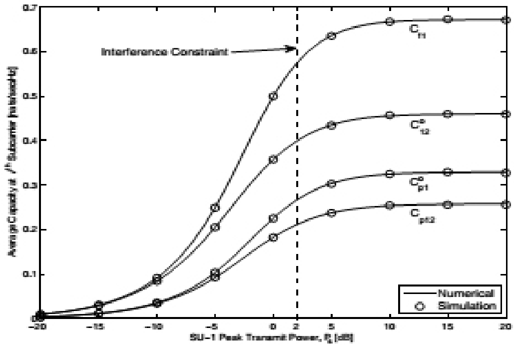

Fig. 8 Average capacity at the ith subcarrier versus peak transmit power, P_s, in case of collision-free (no-interference) and interference from only PU, only SU-2 and both PU and SU-2 with $P = 5$ dB and $\mathcal{I} = 2$ dB.

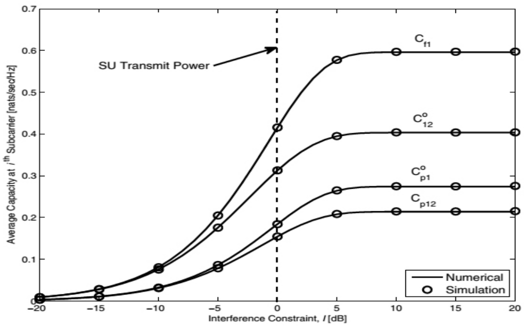

Fig. 9. Average capacity at the ith subcarrier versus interference power constraint, $\mathcal{I}$, in case of collision-free (no-interference) and interference from only PU, only SU-2 and both PU and SU-2 with $P = 5$ dB and $P_s = 0$ dB.

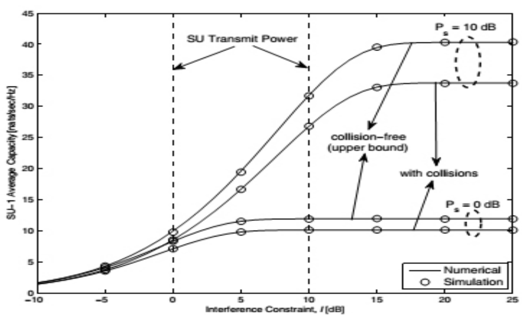

Fig.10. SU-1 average capacity versus interference power constraint, $\mathcal{I}$, for different SU's transmit power with $T_P = T_{s1} = 40$, $T_{s2} = 30$, $F = 100$, and $P = 5$ dB.

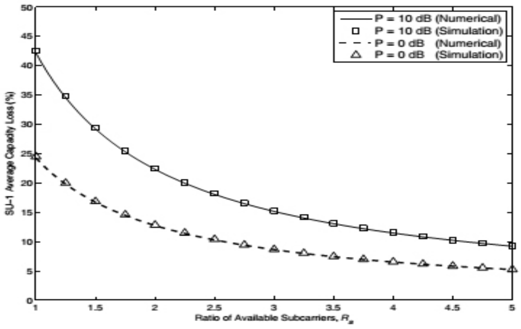

Fig.11. SU-1 average capacity loss versus the ratio of available subcarriers to utilized subcarriers, R_a, for different PU's transmit power with $T_p = T_{s1} = 40$, $T_{s2} = 30$, $\mathcal{I} = 2$ dB, and $P_s = 20$ dB.

VI. CONCLUSION

A random subcarrier access scheme is considered for an OFDM-based CR system with spectrum sharing features and two different secondary networks (cells). It is assumed that no spectrum sensing is performed, i.e., the information about the subcarrier occupation (utilization) by PU is not available at the SUs. It will be shown that the PMFs of the number of subcarrier collisions between any two arbitrary users follows a hypergeometric distribution. It will be further shown that due to the randomness of the access scheme and the absence of cooperation between the SUs, there can be inter-cell collisions between the SUs' subcarriers with a certain probability. The expressions for the PMFs and the average of number of subcarrier collisions, considering both fixed and random (to obtain the long term average) number of subcarriers utilized by PU and SUs, will be derived. The performance of the random access scheme will be analyzed by using the average capacity as performance measure.

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