Performance Analysis of Peak to Average Power Ratio Reduction Techniques in OFDM System

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Abstract **- Orthogonal Frequency Division Multiplexing (OFDM) is an efficient modulation scheme proposed for high data rate applications. OFDM is a multiple carrier modulation technique. It distributes data over a large number of sub-carriers spaced apart at precise frequencies, such that they are orthogonal to one another. OFDM has been successfully applied to a wide variety of digital communication applications over the past several years including digital TV broadcasting, digital audio broadcasting, Asynchronous Digital Subscriber Line (ADSL) modems and wireless networking worldwide. Its application in mobile communication is more complex especially because of the mobility of the mobile user; thus more exact symbol timing and frequency-offset control must be used to ensure that sub-carriers remain orthogonal. An OFDM signal consists of a number of independently modulated subcarriers, which can give a large crest factor or Peak-to-Average Power Ratio (PAPR) when added up coherently. This poses problem regarding the linear bandwidth of the RF amplifiers. In this paper, different PAPR reduction techniques are analysed and compared with standard OFDM.**

Keywords: Peak to Average Power Ratio (PAPR), Orthogonal Frequency Division Multiplexing (OFDM), Selective Level Mapping (SLM), Partial Transmit Sequence (PTS), Decimation in Frequency Fast Fourier Transform (DIFFFT)

I. INTRODUCTION

Multicarrier Modulation (MCM) actually divides the input signal in low rate parallel bit streams and uses these bit streams to perform the modulation for several carriers. It divides the complete input frequency band in N channels in such a way, no over lapping of the signal will be done. These N sub channels are frequency multiplexed and to modulate each channel a separate symbol is used. If some kind of overlapping occur over the signal it will cause the channel interference, so that the MCM is also responsible for the non-overlapping of the channels. But the utilization of the available spectrum is also a challenge for the OFDM systems. To manage along with inefficiency, one approach was proposed in 1960 that collect MCM along with Frequency Division Multiplexing (FDM). It gives the frequency division with overlapping channels. OFDM provide such kind of multicarrier transmission over the signal. Here the word orthogonal defines the relation between the frequencies of carrier in the system. To receive the signals in MCM systems, some traditional filters and demodulators are spaced apart and in theses receivers, some guard bands are define between the carriers and the frequency domain. But inclusion of such bands in the system reduces the efficiency of the spectrum

Most of the problems of traditional MCM is been rectified by OFDM. It actually arranges the carriers in such a way that even in case of overlapping of the carriers, it will return the effective signal throughput without any carrier interference. And this is done only by selecting the carriers mathematically orthogonal. In OFDM systems, on the receiver side the translation of the carrier is performed to DC and then the integration of the signal is performed over a symbol period to get the raw data. OFDM waveforms can be generated using a Discrete Fourier Transform (DFT) at the transmitter and receiver for the modulation and demodulation [1]. For a long time, usage of OFDM in practical systems was limited. Main reasons for this limitation were the complexity of real time Fourier Transform and the linearity required in RF power amplifiers. The most important disadvantage of OFDM systems is that highly linear RF amplifiers are needed [2]. To average the signal power, the N Times power is added to N signals.

II. OFDM GENERATION

The idea behind the analog implementation of OFDM can be extended to the digital domain by using the discrete Fourier Transform (DFT) and its counterpart, the inverse discrete Fourier Transform (IDFT). These mathematical operations are widely used for transforming data between the time-domain and frequency-domain. These transforms are interesting from the OFDM perspective because they can be viewed as mapping data onto orthogonal subcarriers. For example, the IDFT is used to take in frequency-domain data and convert it to time-domain data. In order to perform that operation, the IDFT correlates the frequency-domain input data with its orthogonal basis functions, which are sinusoids at certain frequencies. This correlation is equivalent to mapping the input data onto the sinusoidal basis functions.

OFDM systems are implemented using a combination of fast Fourier Transform (FFT) and Inverse fast Fourier Transform (IFFT) blocks that are mathematically equivalent versions of the DFT and IDFT, respectively, but more efficient to implement. An OFDM system treats the source symbols (e.g., the QPSK or QAM symbols that would be present in a single carrier system) at the transmitter as though they are in the frequency-domain. These symbols are used as the inputs to an IFFT block that brings the signal into the time domain. The IFFT takes in N symbols at a time where N is the number of Subcarriers in the system. Each of these N input symbols has a symbol period of T seconds. Recall that the basic functions for an IFFT are N orthogonal sinusoids. These sinusoids each have a different frequency and the lowest frequency is DC. Each input symbol acts like a complex weight for the corresponding sinusoidal basis function. Since the input symbols are complex, the value of the symbol determines both the amplitude and phase of the sinusoid for that subcarrier. The IFFT output is the summation of all N sinusoids. Thus, the IFFT block provides a simple way to modulate data onto N orthogonal subcarriers. The block of N output samples from the IFFT make up a single OFDM symbol. The length of the OFDM symbol is NT where T is the IFFT input symbol period mentioned above. After some additional processing, the time-domain signal that results from the IFFT is transmitted across the channel. Fig. 1 shows the block diagram of OFDM transmitter.

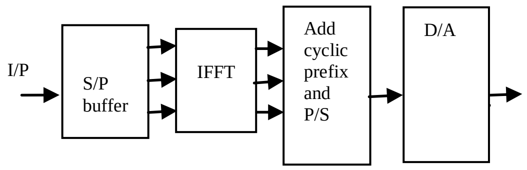

Fig. 1 OFDM transmitter block diagram

At the receiver, an FFT block is used to process the received signal and bring it into the frequency domain [3]. Ideally, the FFT output will be the original symbols that were sent to the IFFT at the transmitter. When plotted in the complex plane, the FFT output samples will form a constellation, such as 16-QAM. However, there is no notion of a constellation for the time-domain signal. When plotted on the complex plane, the time-domain signal forms a scatter plot with no regular shape. Thus, any receiver processing that uses the concept of a constellation (such as symbol slicing) must occur in the frequency- domain. OFDM receiver is shown fig. 2.

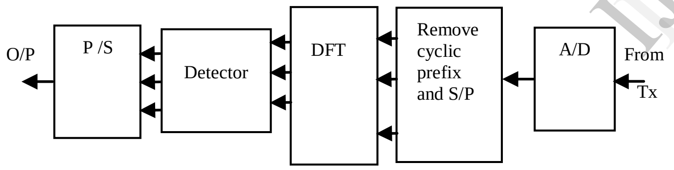

Fig. 2 OFDM transmitter block diagram

A major disadvantage of OFDM is that the envelope is not constant. Due to the summation of sine waves, we have a large peak to average ratio (PAPR).

$$\text{PAPR} = \frac{\max(|S[n]|^2)}{E\{|S[n]|^2\}} \qquad (1)$$

S|n| represents the signal samples, max $(|S|n||^2)$ denotes the maximum instantaneous power and $E\{|S|n||^2\}$ is the average power of the signal.

III. PAPR REDUCTION TECHNIQUES

PAPR reduction is a well-known signal processing topic in multi-carrier transmission and large number of techniques appeared in the literature during the past decades. These techniques include amplitude clipping and filtering, coding, tone reservation (TR) and tone injection (TI), active constellation extension (ACE) and multiple signal representation methods such as partial transmit sequence (PTS), selected mapping (SLM) and interleaving. The existing approaches are different from each other in terms of requirements, and most of them enforce various restrictions to the system. Therefore, careful attention must be paid to choose a proper technique for each specific communication system. In this section we focus more closely on the PAPR reduction techniques for multi-carrier transmission. In order to evaluate the performance of these techniques, we need to look at the application and existing restrictions of a communication system.

A. SELECTIVE LEVEL MAPPING (SLM)

Selective Level Mapping (SLM) approaches have been proposed by Bauml in 1965 [4]. Selective level mapping method is used for minimization of peak to average transmit power of multicarrier transmission system. A set of signal is generated signifying the same information in selected mapping, and then most favorable signal is selected as consider to PAPR and transmitted. The input data structure is multiplied by random series and resultant series with the lowest PAPR is chosen for transmission. To recover the original data to the multiplying sequence can be sent as 'side information'.

While performing the transmission, a slightly different set of data blocks is generated to represent the information alike the data blocks and it performs the favourable transmission over the system. As shown in the fig. 3, the data blocks are multiplied by U different sequences of length N. Each sequence is different in terms of phase and after the variation in the phase, N new modified data blocks are obtained. These modified data blocks are then included along with the unmodified data blocks.

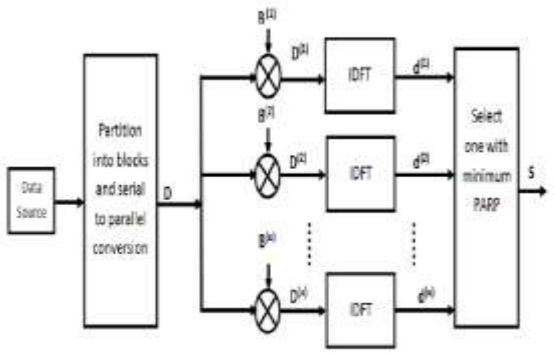

Fig. 3 Block diagram of the SLM technique

Let us denote the modified data block for the u_{th} phase sequence $D^{(u)} = [D_0 b_{u,0}, D_1 b_{u,1} \cdots, D_{N-1} b_{u,N-1}]^T$ $u = 1, 2, \cdots, U$, After applying SLM to X, the multi-carrier signal becomes

$$d_k^u = \sum_{n=0}^{N-1} D_n \, b_{u,n} \, e^{j2\pi n\frac{k}{N}}, \; 0 \le k \le N-1 \qquad (2)$$

A selection of the lowest PAPR value is identified over all the blocks after obtaining modified data blocks. Along with this, the phase sequence is required to transmit to the receiver side and on this side the operation is been performed to get back the original data blocks. SLM is the parameter approach that basically requires the IDFT operation along with the number of required side information bits. The IDFT is defined for U bits and information bits are defined for each data blocks. Same process can be performed under different modulation schemes and for different number of subcarriers. Respective to this the PAPR reduction amount is identified and it depends on the number of phase sequences called U and the design of the phase sequences.

The good side of selected mapping method is that it doesn't eliminate the peaks, and can handle any number of subcarriers. The drawback of this method is the overhead of side information that requires to be transmitted to the receiver of the system in order to recover information.

B. PARTIAL TRANSMIT SEQUENCE (PTS)

Partial Transmit Sequence is a technique for improving the PAPR of multi-carrier signal. The basic idea of partial transmit sequence algorithm is to divide the original OFDM sequence into several sub-sequences and for each sub-sequences multiplied by different weights until an optimum value is chosen. The input frame is divided in N Non-overlapping sub blocks and for each sub block a phase shift is performed by a constant factor so that the reduction of the PAPR is performed. It is basically a probabilistic method which performs the reduction in the PAPR. It is actually improved form of SLM approach as it gives better results than the SLM approach. The good side of PTS is that no information is required to transfer at any side during the modulation phase.

In this approach, the input signal is first divided in to sub blocks so that the serial to parallel conversion is performed. Now these parallel constructed data block is sub divided to N sub blocks each of length M. With each sub block some a phase vector is identified a relative weight is assigned to each block such that the optimization of sub blocks will be performed. Finally all these sub blocks are collected so that the combined signal can be transferred. The block diagram of the approach is shown in fig. 4. In the conventional PTS, the input data block D is partitioned into M disjoint sub-blocks Dm = $[D_{m,0}, D_{m,1}, \cdots, D_{m,N-1}]^T$, m = 1, 2, · · · ,M, such that $\sum_{m=1}^{M} D_m = D_i$ and the sub-blocks are finally collected so that the overall reduction of the PAPR is done. The overall signal is then over-sampled for L-Times so that the IDFT can be driven for all N blocks. It will gives a IDFT of length NL. All the obtained phase variated blocks for m data blocks is represented by $b_m = [b_{m,0}, b_{m,1}, \cdots, b_{m,LN-1}]^T$, m = 1, 2, · · · ,M; these are called the partial transmit sequences. On these transmitted sequences, some phase oriented weighted value is assigned represented by $W = [W_1, W_2, \cdots, W_M]^T$ in the block diagram.

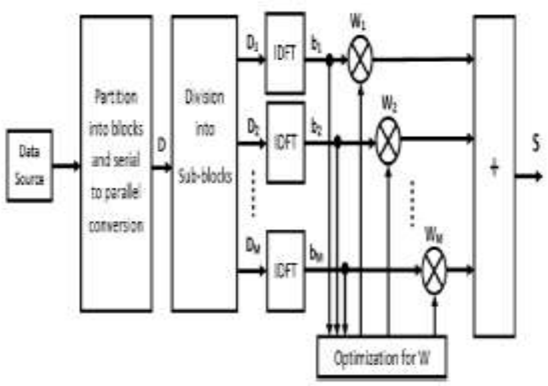

Fig. 4 Block diagram of the PTS technique with *M* disjoint sub-blocks

Finally the composed signal under the time frame is represented by the equation 3.

$$s(n) = \sum_{m=1}^{M} W_m b_m(n) \quad (3)$$

Along with this, the selection of phase reduction factors are required to identified so that the PAPR can be reduced upto maximum possible level. The vectors that affect the phase variation are given as $P = e^{\frac{j2\pi l}{K}}$, l = 0, 1, · · · ,K − 1, where K is the number of allowed phases. The first phase weight is set to 1 without any loss of performance, so a search for choosing the best one is performed over the (M − 1) remaining places. As the number of blocks increases in the system, the overall complexity of the system is also increased. Based on it the possible phase vector is identified an the optimality of the system depends on how effectively the phase factors are evaluated. These vectors are applied for each data blocks and relatively information bits are modified for each block along with weightage assigned for the reduction. These factors are also responsible to identify the reduction of PAPR that can be performed to each sub block. Sub block partitioning is the another vector used to perform the reduction of PAPR over the signal. Based on this vector the multiple disjoing sub blocks are been identified and relatively the transmission blocks are optimized. To perform these partitions we have three basic methods called adjacent approach, interleaved and the pseudo random partitioning approach. These values are effectively and exponentially increase the complexity of the system.

The main advantage of this PTS is that it can work on different number of carriers as well as for different modulation schemes effectively. It also reduces the complexity of the system and provide and optimized throughput over the signal. It defined different techniques to perform the search on complexity. The most common approach is the iteratively, it means it will substitute the different values one by one till the required optimization over the signal is not obtained. It will give the appropriate reduction to the PAPR as well the search complexity and will not affect the efficiency of the system.

C. RADIX-2 DECIMATION-IN-FREQUENCY (DIF) FFT

Let X_r is DFT of sequence x_l given as

$$X_r = \sum_{l=0}^{N-1} x_l w^{rl}, \ r = 0,1,\ldots,N-1 \quad (4)$$

The radix-2 DIF FFT algorithm is decimated into an even-indexed set $\{X_{2k} \,|k = 0, \ldots, N/2 - 1\}$ and an odd-indexed set $\{X_{2k+1}|k = 0, \ldots, N/2 - 1\}$. Equation (4) is rewritten as

$$X_r = \sum_{l=0}^{\frac{N}{2}-1} x_l w_N^{rl} + \sum_{l=\frac{N}{2}}^{N-1} x_l w_N^{rl}$$

$$= \sum_{l=0}^{\frac{N}{2}-1} x_l w_N^{rl} + \sum_{l=0}^{\frac{N}{2}-1} x_{l+\frac{N}{2}} w_N^{r(l+\frac{N}{2})}$$

$$= \sum_{l=0}^{\frac{N}{2}-1} (x_l + x_{l+\frac{N}{2}} w_N^{r\frac{N}{2}}) w_N^{rl}, \ r = 0,1,\ldots\ldots,N-1 \quad (5)$$

Hence $Y_k = X_{2k}$ and $y_l = x_l + x_{l+N2}$ yields the first half-size sub problem

$$Y_k=\sum_{l=0}^{\frac{N}{2}-1} y_l W_{\frac{N}{2}}^{kl},\ k = 0,1,\ldots,N/2 - 1 \qquad (6)$$

$Z_k = X_{2k+1}$ and $X_l = (x_l - x_{l+\frac{N}{2}})W_N^l$ yields the second half-size problem

$$Z_k=\sum_{l=0}^{\frac{N}{2}-1} x_l W_{\frac{N}{2}}^{kl},\ k = 0,1,\ldots,N/2 - 1 \qquad (7)$$

No more computation is needed to obtain the solution for the original problems after the two sub problems are solved. Therefore, in the implementation of the DIF FFT, the bulk of the work is done during the subdivision step, i.e., the set-up of appropriate sub problems, and there is no combination step. Consequently, the computation of $y_l = x_l + x_{l+N2}$ and $X_l = (x_l - x_{l+\frac{N}{2}})W_N^l$ completes the first (subdivision) step. The computation of y_l and z_l in the subdivision step as defined above is referred to as the Gentleman-Sande butterfly and is depicted by the annotated butterfly symbol in fig. 5.

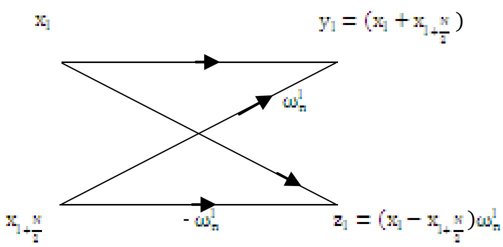

Fig. 5 The Gentleman-Sande butterfly

The block diagram of radix-2 Decimation in Frequency Fast Fourier Transform is shown in fig. 6.

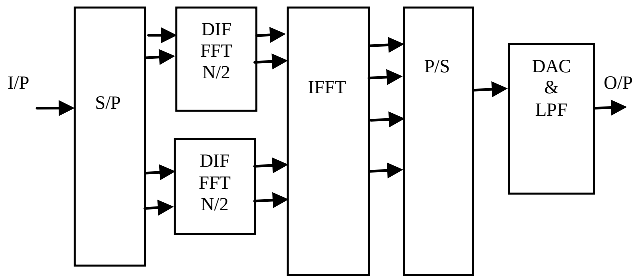

Fig. 6 Radix-6 Decimation in Frequency Fast Fourier Transform block diagram

In this scheme N/2 FFT is taken first at output of serial to parallel data convertor block. Then IFFT is taken and Parallel to serial conversion is performed.

IV. SIMULATION RESULTS

In fig. 7 PAPR comparison of standard OFDM and OFDM using SLM technique is simulated with 10000 OFDM symbols, QPSK modulated 64-subband OFDM symbols.

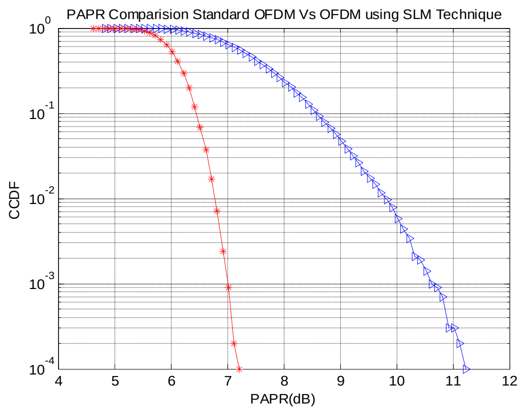

Fig. 7 PAPR comparison OFDM Vs SLM_OFDM

Table I PAPR OFDM Vs SLM_OFDM

CCDF	OFDM	SLM_OFDM
10^{-3}	10.5db	7
10^{-2}	9.8db	6.8db
10^{-1}	8.5db	6.5db

It is clear from fig. 7, PAPR power is reduced 3.5 dB using SLM_OFDM technique at 10^{-3} and 2.2 db at 10^{-2} than standard OFDM.

In fig. 8 PAPR comparison of standard OFDM and OFDM using PTS technique is simulated with 10000 OFDM symbols, QPSK modulated, 64-subband OFDM symbols.

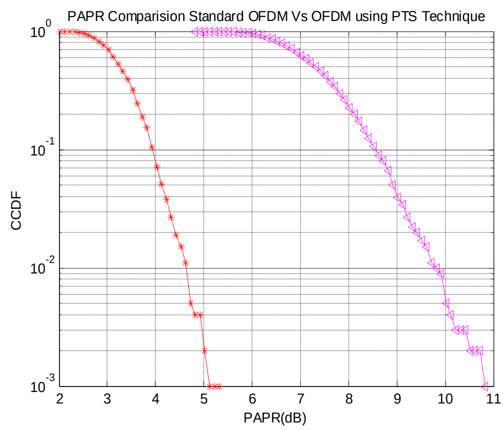

Fig. 8 PAPR comparison OFDM Vs PTS_OFDM

Table II PAPR OFDM Vs PTS_OFDM

CCDF	OFDM	PTS_OFDM
10^{-2}	9.8db	4.5db
10^{-1}	8.5db	4db

It can be seen clearly from fig. 8, PAPR power is reduced 5.3 dB using PTS_OFDM technique at 10^{-2} and 4.5 db at 10^{-1} than standard OFDM.

In fig. 9 PAPR comparison of standard OFDM and OFDM using Radix-2 Decimation in Frequency FFT technique is simulated with QPSK modulated, 128 data block size.

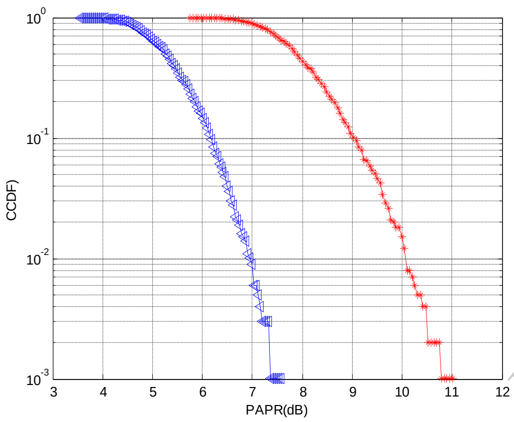

Fig. 9 Standard OFDM Vs radix-2 DIFFFT_OFDM

Table III PAPR OFDM Vs radix-2 DIFFFT_OFDM

CCDF	OFDM	Radix-2 DIFFFT_OFDM
10^{-2}	9.8db	7db
10^{-1}	8.5db	6.1db

It can be seen clearly from fig. 8, PAPR power is reduced 2.8 dB using Radix-2 DIFFFT_OFDM technique at 10^{-2} and 2.4 db at 10^{-1} than standard OFDM.

V. CONCLUSION

All PAPR reduction techniques have some advantages and disadvantages. These PAPR reduction techniques should be chosen carefully for getting the desirable minimum PAPR. All PAPR reduction techniques are based on particular situation of system.

Among signal Scrambling Techniques Block Coding Technique is simple and accurate for short codes because it needs extreme computation when the frame size is bigger. The good side of selected mapping (SLM) method is that it doesn't eliminate the peaks, and can handle any number of subcarriers. The drawback of this method is the overhead of side information that requires to be transmitted to the receiver of the system in order to recover information. PTS method is a modified method of SLM. PTS method works better than SLM method. The main advantage of this scheme is that there is no need to send any side information to the receiver of the system, when differential modulation is applied in all sub blocks. In these schemes data rate loss exists. The radix 2 Decimation in Frequency FFT algorithm provides 2-3dB performance improvement than standard OFDM system as shown by simulated results carried out in MATLAB.

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