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Performance Analysis Of Orthogonal Frequency Division Multiplexing (OFDM) Over Fading Channel And Space Time Block Code

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Abstract

Analysis will be carried out for an OFDM wireless communication system using space time block code (STBC) at the transmitter and considering the effect and the wireless channel like delay spread and fading. The analysis will include the effect of imperfect timing recovery at the output of the receiver. The expression for Bit Error Rate with STBC and timing error will be developed. Performance results will be evaluated numerically. Performance degradation due to imperfect timing synchronization will be determined.

1. Introduction

The target of next generation wireless communication is to achieve high data rates with low bandwidth. It should be power efficient. At present time Orthogonal Frequency Division Multiplexing is widely used for its bandwidth efficiency property. Because of its orthogonal characteristic more data can be transmitted at a certain amount of bandwidth compare to the other systems. Increasing the diversity gain is another way to achieve good performance. By using Space Time Block Code the antenna diversity gain can be increased. This paper shows the analysis of STBC-OFDM. At the end of this paper equations for Signal to Noise Ratio (SNR) and Bit Error Rate (BER) have been derived analytically using four transmitting antennas and one receiving antenna and 6 transmitting antennas and one receiving antenna.

2. Space Time Block Code – Orthogonal Frequency Division Multiplexing

Severe attenuation in a multipath wireless environment makes it extremely difficult for the receiver to determine the transmitted signal unless the receiver is provided with some form of diversity i.e. some less-attenuated replica of the transmitted signal is provided to the receiver. In some applications, the only practical means of achieving diversity is deployment of antenna array at the transmitter and/or receiver end. As the current trend of communication systems demands highly power-efficient and bandwidth-efficient schemes, techniques that provide such desirable properties are considered very valuable in next generation wireless systems. Making use of multiple antennas increases the capacity of the system with the associated higher data rates than single antenna systems. Space-Time coding is a power-efficient and bandwidth-efficient method of communication over fading channels by using multiple transmits antennas systems.

2.1. System Model for STBC-OFDM

We consider an OFDM system with transmit diversity, in which the total system bandwidth is divided into *N* equally spaced and orthogonal sub-carriers. We investigate the system with four transmission antennas and one receiving antenna. During the first time instant, the four symbols [X_0 X_1 X_2 X_4] are transmitted from four antennas simultaneously, with X_0, X_1, X_2 and X_3 transmitted from all four antennas. In the second time slot [$-X_1^*$ X_0^* $-X_3^*$ X_2], third time slot [$-X_2^*$ $-X_3^*$ X_0^* X_1^*] and fourth time slot [X_3 $-X_2$ $-X_1$ X_0] are transmitted.

This encoding of the transmitted symbol sequence from the transmit antennas is given by then encoding matrix.

$$\begin{bmatrix} H_0 & H_1 & H_2 & H_3 \\ -H_1^* & H_0^* & -H_3^* & H_2^* \\ -H_1^* & -H_3 & H_0 & H_0 \\ H_0 & -H_0 & -H_0 & H_0 \end{bmatrix}$$

For each transmit antenna, a block of *N* complex-valued data symbols **{X(*k*)} *for k=0 to N-1*** are grouped and converted into a parallel set to form the input to the OFDM modulator, where *k* is the sub carrier index and *N* is the number of sub carriers. The modulator consists of an Inverse Fast Fourier transform (IFFT) block. The output of the IFFT at each transmitter is the complex baseband modulated OFDM symbol in discrete time domain and is given by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}; \quad 0 \le n \le N-1$$

2.2 Channel

The channel is modelled by a tapped delay line with channel coefficients that are assumed to be slowly varying such that they are almost constant over the two transmission instants. The channel frequency response for the *kth* subcarrier is

$$H(k) = \sum_{p=0}^{L-1} h(p) e^{\frac{j2\pi pk}{N}}$$

Where *h (p)* is the complex channel gain of the *pth* multipath component

2.3 Phase Noise

The phase noise θ(n) is modelled as a zero-mean continuous Brownian motion process with variance σ_θ^{2} The phase noise increments take the form of a Wiener process, with independent Gaussian increments.

2.4 Received Signal

The time-domain received signals at the first and second transmission instances at the input to the FFT block are respectively given by

$Y^0(n)=(h_0(n)\Diamond x_0(n)+h_1(n)\Diamond x_1(n)+h_2(n)\Diamond x_2(n)+h_3(n)\Diamond x_3(n)+w(n)^0)e^{j\theta(n)}$

$Y^1(n)=(-h_0(n)\Diamond x_1{*}(n)+h_1(n)\Diamond x_0{*}(n)-h2(n)\Diamond x3{*}(n)+h3(n)\Diamond x2{*}(n)+w(n)^1)\, e^{j\theta(n)}$

$Y^2(n)=(-h_0(n)\Diamond x_2{*}(n)-h_1(n)\Diamond x_3{*}(n)+h_3(n)\Diamond x_0{*}(n)+h_3(n)\Diamond x_2{*}(n)+w(n)^2)\, e^{j\theta(n)}$

$Y^3(n)=(h_0(n)\Diamond x_3(n)-h_1(n)\Diamond x_2(n)-h_2(n)\Diamond x_1(n)+h_3(n)\Diamond x_0(n)+w(n)^3)\, e^{j\theta(n)}$

Where ◊ represents linear convolution, subscripts indicate antenna index, and superscripts indicate transmission instant. The complex Gaussian random variable *w (n)* represents the Additive White Gaussian Noise (AWGN) term with $\sigma_w^2 = E[|w(n)|^2]$, and θ(n) is the phase noise.

2.5 Detection with Imperfect Channel Estimation

In the presence of imperfect channel estimation, we assume a channel estimation model such that the channels estimate *H'* of the true channel *H* is given by

$$\begin{bmatrix} H_0 & H_1 & H_2 & H_3 \\ -H_1^* & H_0^* & -H_3^* & H_2^* \\ -H_1^* & -H_3 & H_0 & H_0 \\ H_0 & -H_0 & -H_0 & H_0 \end{bmatrix} =$$

$$\begin{bmatrix} H_0+\varepsilon 0 & H_1+\varepsilon 1 & H_2+\varepsilon 2 & H_3+\varepsilon 3 \\ -H_1^*+\varepsilon 1^* & H_0^*+\varepsilon 0^* & -H_3^*-\varepsilon 3^* & H_2^*+\varepsilon 3^* \\ -H_1^*+\varepsilon 2 & -H_3-\varepsilon 1 & H_0+\varepsilon 0^* & H_0+\varepsilon 3^* \\ H_0+\varepsilon 3 & -H_0-\varepsilon 2 & -H_1-\varepsilon 1 & H_0+\varepsilon 0 \end{bmatrix}$$

-

Where ε0 , ε1 , ε2 and ε3 are the errors in the channel estimate from the first ,second, third and fourth transmit antennas respectively, and are modeled as independent zero-mean complex Gaussian random variables with variances $2\sigma\ \varepsilon 0^2$, $2\sigma\ \varepsilon 1^2$, $2\sigma\ \varepsilon 2^2$ and $2\sigma\varepsilon 3^2$ respectively.

2.5 Variance

As the noise signal has both positive and negative amplitude, it is squared, and then the mean has been taken, which is variance. We consider the variance of noise for calculation

2.5.1 Variance of Noise

The variance of the noise *W*, after some mathematical manipulations, is given by

$$\sigma_w^{2'} = E[|W|^2] = \sigma_w^2 * \begin{pmatrix} \sum_{i=0}^{3}(\sigma_{Hi}^2+\sigma\varepsilon_i^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma_{Hi}^2+\sigma\varepsilon_i^2) \end{pmatrix}$$

2.5.2 Variance of Channel Estimator Error

The variance of Ψ is given by

$$\sigma_\psi^{2'} = E[|\Psi|^2] = \sigma_{H0}^2 * E_g * \begin{pmatrix} \sum_{i=0}^{3}(\sigma\varepsilon_i^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma_{\varepsilon i}^2) \end{pmatrix} + \sigma_{H1}^2 * E_g * \begin{pmatrix} \sum_{i=0}^{3}(\sigma\varepsilon_i^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma\varepsilon_i^2) \end{pmatrix}$$

$$+ \quad \sigma_{H2}^{2} * E_g * \begin{bmatrix} \sum_{i=0}^{3}(\sigma\varepsilon_i^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma\varepsilon_i^2) \end{bmatrix}$$

$$+ \quad \sigma_{H3}^{2} * E_g * \begin{bmatrix} \sum_{i=0}^{3}(\sigma\varepsilon_i^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma\varepsilon_i^2) \end{bmatrix}$$

2.5.3 Variance of Inter Carrier Interference

Similarly, the variance of the ICI term β' is given by

$$\sigma_{\beta'}^{2} = E[|\beta|^2]$$
$$= E_g * \begin{bmatrix} \sum_{i=0}^{3}(\sigma_{Hi}^2) & 0 \\ 0 & \sum_{i=0}^{3}(\sigma_{Hi}^2) \end{bmatrix}$$

3. Calculation of SNR and BER

We present the bit error rate analysis for the case of 16QAM modulation using Gray code mapping for *(b_1 b_2 b_3 b_4)*. It is important to note that although the presentation is only for 16QAM, the following analysis is valid for all square QAM constellations. The conditional BER for bit *b1*, condition on H_0, H_1, H_2, H_3 is given by

$$P1(b1|H0,H1,H2,H3) = \frac{1}{2} * [Q\left(\sqrt{\frac{\left(2*\frac{Eg}{5}\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)$$

$$+Q\left(\sqrt{\frac{\left(2*\frac{Eg}{5}\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)]$$

and for bit b_3 is given by

$$P1(b1|H0,H1,H2,H3) = \frac{1}{2} * [Q\left(\sqrt{\frac{\left(9*\left(\frac{2*Eg}{5}\right)\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)$$

$$+Q\left(\sqrt{\frac{\left(2*\frac{Eg}{5}\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)$$

$$+Q\left(\sqrt{\frac{\left(2*\frac{Eg}{5}\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)$$

$$+ Q\left(\sqrt{\frac{\left(25*\left(\frac{2*Eg}{5}\right)\right)(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{6\psi^2+6\beta^2+6w^2}}\right)]$$

From which the SNR γ is given by

$$\gamma = \frac{(|H0|^2+|H1|^2+|H2|^2+|H3|^2)}{(6\psi^2+6\beta^2+6w^2)}$$

It follows a Chi-square distribution with probability density function (PDF) given by

$$P(\gamma) = \frac{1}{2*6\gamma^2} * \exp\left[-\frac{\gamma}{2*6\gamma^2}\right]$$

Due to the symmetry of square M-QAM constellations, the BER for the in-phase and quadrature bits are equal such that $Pe(b_1) = Pe(b_2)$ and $Pe(b_3)=Pe(b_4)$. Therefore the average BER is obtained by averaging the conditional BER of b1 and *b3* over the PDF of the SNR γ. The average BER is therefore given by

$$Pe = \frac{1}{2} * \int_0^{\infty}[Pe(b1|H0,H1,H2,H3) + Pe(b3|H0,H1,H2,H3)]p(\gamma)d(\gamma)$$

Similarly for 6:1 transmission system SNR γ is given by

$$\gamma = \frac{(|H0|^2+|H1|^2+|H2|^2+|H3|^2+|H4|^2+|H5|^2)}{(6\psi^2+6\beta^2+6w^2)}$$

And BER is giver by

$$Pe = \frac{1}{2} * \int_0^{\infty}[Pe(b1|H0,H1,H2,H3,H4,H5) + Pe(b3|H0,H1,H2,H3,H4,H5)]p(\gamma)d(\gamma)$$

4. Results (STBC-OFDM):

The parameters that uses in the calculation are shown in table-1

Table 1: System and Channel Parameters (STBC-OFDM)

Parameters	Values
σh^2	0.1 0.2 0.02 0.4
$\sigma \varepsilon^2$	0.1 0.04 0.06
Subcarriers (N)	64
Channel Path Gains	-9.7 -0.9 -8.5 -0.5

Figure: 7 shows the SNR Vs BER graph for 2:1 transmission system. Figure: 8 shows the same graph for different value of noises. Then figure: 9 shows the graph of SNR Vs BER for 4:1 transmission system along with 2:1 and 1:1 system. It is seen that 4:1 graph is closer to the two axes than the others. It means that the BER is decreasing fast when the number of antennas increases. Figure: 10 shows the SNR Vs BER graph of 4:1 for different value of noises. Then figure: 11 shows the performance of 6:1 along with 4:1, 2:1, 1:1 transmission system. Figure: 12 shows the SNR Vs BER graph of 6:1 for different value of noise. From the figure: 11 receiver sensitivity graphs are plotted for different value of noises. It shows that transmission power decreases when number of antennas increase. The graph is shown in figure: 13. Another analysis can be drawn from the graph shown in figure: 13 is that for a fixed value of transmission power the noise term can be reduced by increasing the number of antennas.

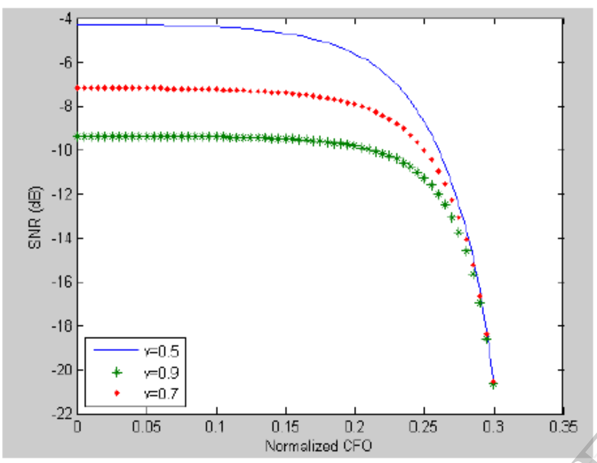

Figure 2: Graph shows SNR Vs Normalized form of CFO

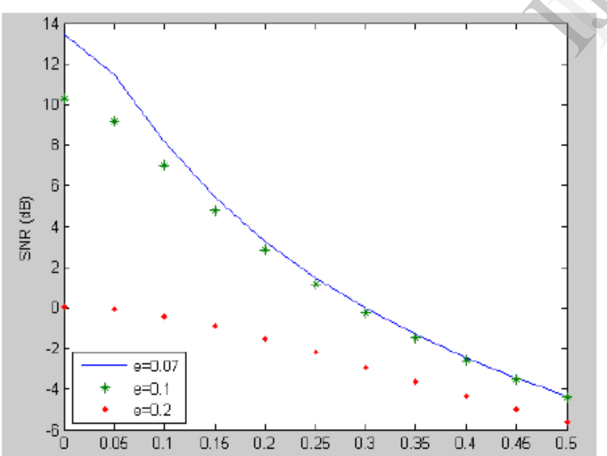

Figure 3: Graph shows SNR Vs variance of Phase Noise

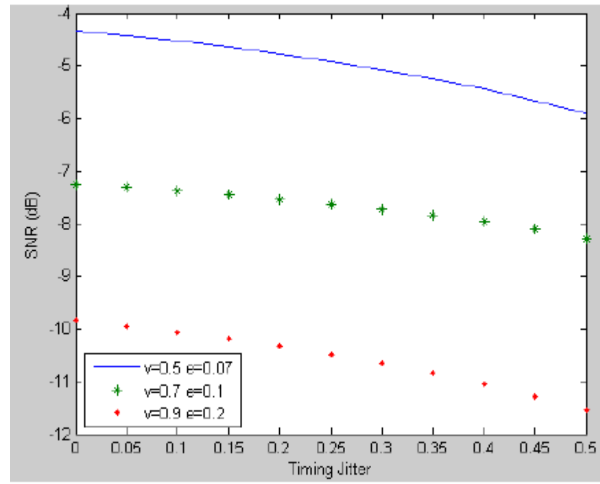

Figure 4: Graph shows SNR Vs Timing Jitter

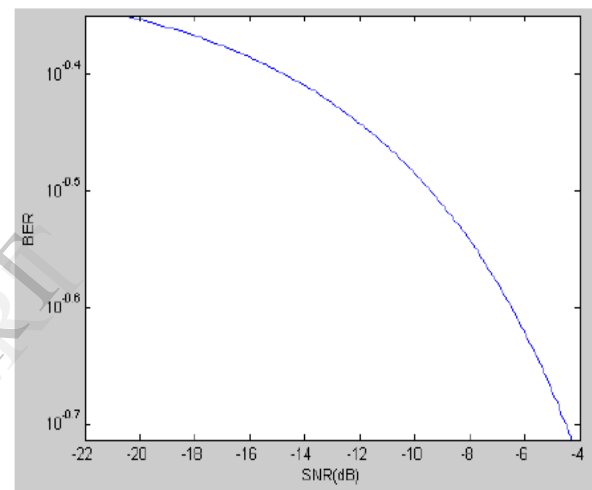

Figure 5: Graph shows the BER Vs SNR (dB)

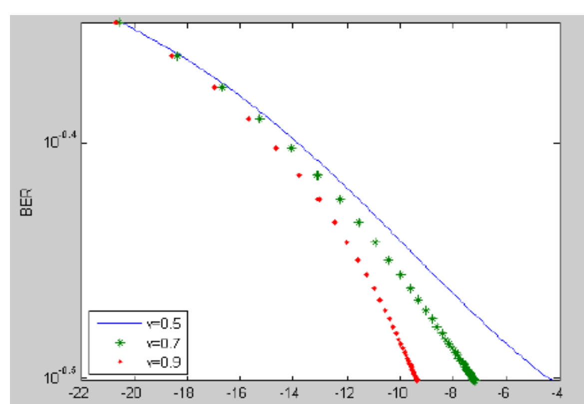

Figure 6: Graph shows different plotting of BER Vs SNR of different value of variance of Phase Noise

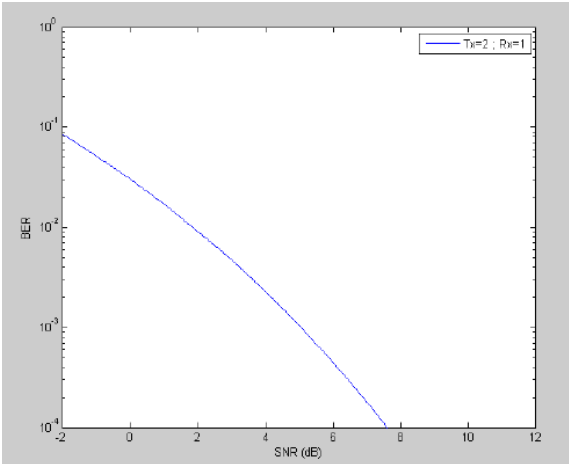

Figure 7: SNR Vs BER graph for 2:1 transmission system

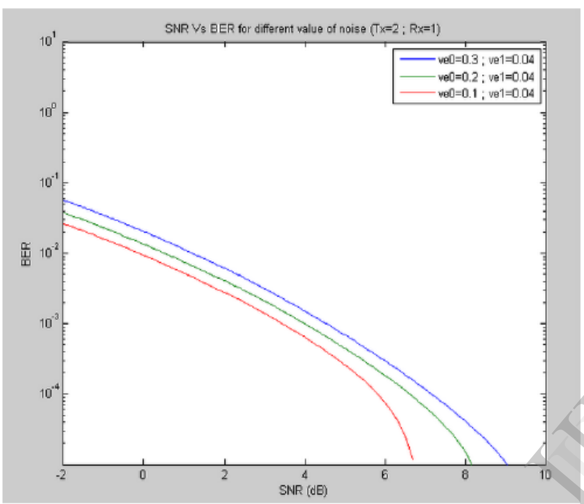

Figure 8: SNR Vs BER graph for different value of noises (Tx=2 ; Rx=1)

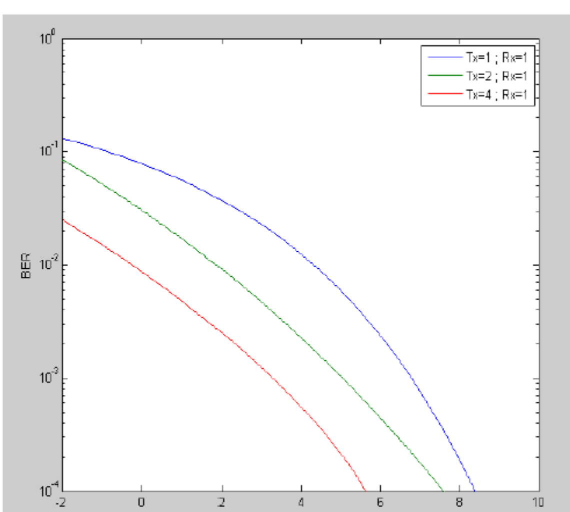

Figure 9: SNR Vs BER for 4:1, 2:1, 1:1 transmission system

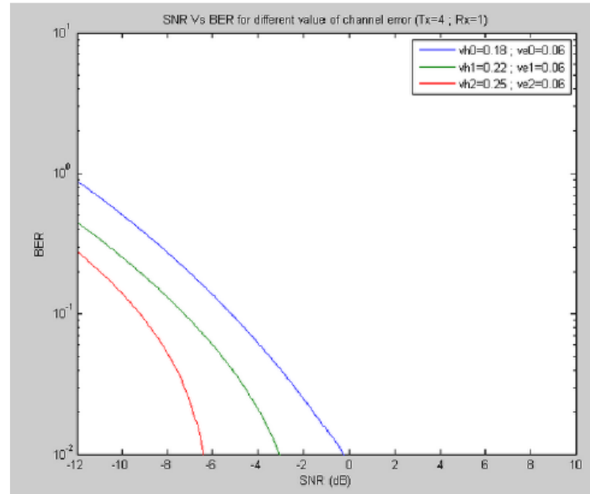

Figure 10: SNR Vs BER graph for different value of noises (Tx=4 ; Rx=1)

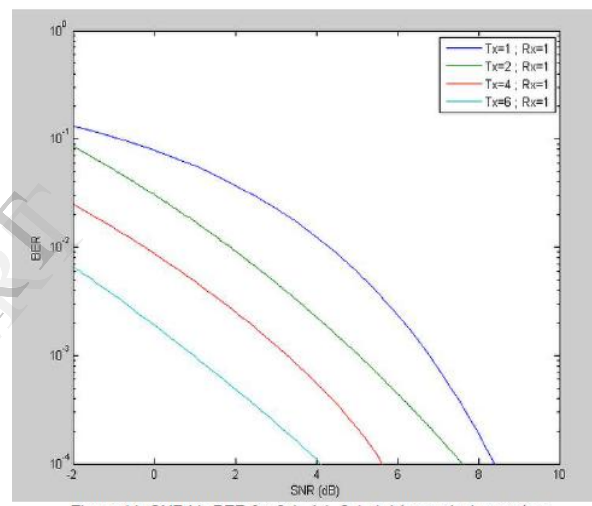

Figure 11: SNR Vs BER for 6:1, 4:1, 2:1, 1:1 transmission system

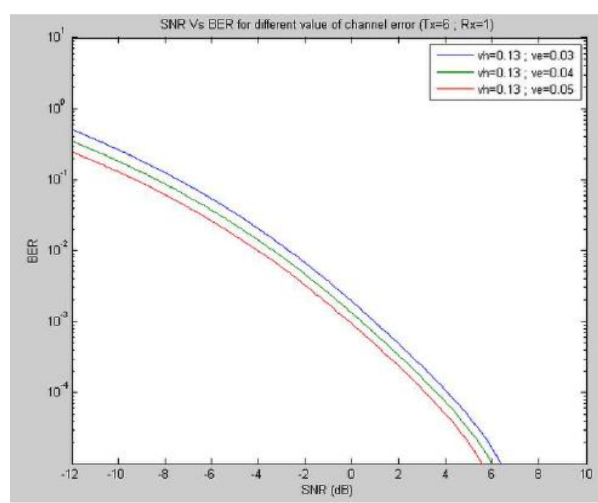

Figure 12: SNR Vs BER graph for different value of noises (Tx=6 ; Rx=1)

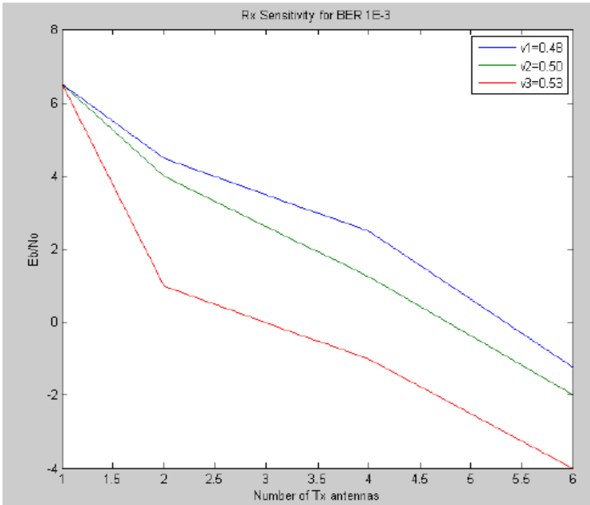

Figure 13: Rx Sensitivity for BER 1e-3

5. Conclusion

The equations for SNR and BER using 4:1 and 6:1 transmission system are derived. The effects of noise, carrier interference and channel estimator error on the system are analyzed. The SNR Vs BER curve shows that increasing the diversity gain improve the performance of the system. Receiver sensitivity graph shows the power efficiency characteristic of the system

6. References

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