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Performance Analysis of OFDM Systems Subjected to Carrier Frequency Offset in Fading Communication Channels

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) systems are very sensitive to the frequency synchronization errors in form of carrier frequency offset (CFO). CFO can lead to the Inter-Carrier interference (ICI) and also destroys the orthogonality between sub-carriers. Therefore, CFO plays a key role in Frequency synchronization. Basically for getting a good performance of OFDM, the CFO should be estimated and compensated. In this paper we investigate the algorithms for the CFO estimation in OFDM systems. Four types of estimators are investigated: cyclic prefix (CP) and Training Sequence based estimators in time domain in addition to Training Symbol based and pilot tone based estimators in frequency domain. Mean Square Error (MSE) is the comparison criteria used in studying the performance. Simulation results indicate the performance of the different estimators over both additive white Gaussian noise (AWGN) and four taps multipath fading channels.**

Keywords—Orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), Inter-Carrier interference (ICI), cyclic prefix (CP), Training Sequence, Training Symbol, mean square error (MSE).

I. INTRODUCTION

As high data rate transmission is one of the major challenges in modern wireless communications, there is a substantial need for a higher frequency bandwidth. Meanwhile, with the increase of data rate the distortion of the received signals caused by multipath fading channel becomes a major problem. Orthogonal Frequency Division Multiplexing (OFDM) gives higher bandwidth efficiency by using the orthogonality principle and overcomes the effect of multipath fading channel by dividing the single high data rate stream into a several low data rate streams. So OFDM is a multicarrier transport technology for high data rate communication system. The OFDM concept is based on spreading the high speed data to be transmitted over a large number of low rate carriers. The carriers are orthogonal to each other and frequency spacing between them are created by using the Fast Fourier transform (FFT). OFDM is being used in a number of wired and wireless voice and data applications due to its flexible system architecture. OFDM adopted as modulation mechanism at physical layer (or air access) in Modern wireless digital transmission systems, such as Wireless Local Area Network (WLAN) systems based on the IEEE 802.11a or Hiperlan2 [1]-[2], Wireless Metropolitan Area Network (WMAN) systems based on the standard IEEE 802.16e, Worldwide Interoperability for Wireless Microwave Access (WiMAX) and Long-Term Evolution (LTE) systems. There are two ways to manage the air access in wireless systems: unframed and framed. The unframed solution is used in WLAN, while the framed solution is used by WiMAX and LTE [3]-[4].

OFDM systems are very sensitive to frequency synchronization errors in form of carrier frequency offset (CFO) [5]. The CFO violates the OFDM sub-carriers (SCs) orthogonality and hence the received signal suffers from attenuation, phase rotation, and inter-carrier interference (ICI) from other SCs in the OFDM signal [5], leading to detection errors. In literature, CFO mitigation techniques can be broadly categorized into two groups. The first group includes the CFO estimation and correction techniques [6]-[18] and the second group includes the CFO sensitivity reduction techniques [19] - [21]. In CFO estimation and correction techniques, the CFO is estimated and corrected at the receiver side. In general, CFO estimation can be divided into two main categories, namely data-aided [6]-[11] and non-data aided (blind) [12]-[17] techniques.

The paper is organized in the following way. The problem of carrier frequency offset in OFDM is described in section II. The CFO problem modeling and the effect of CFO on the received signal and the effect of integer and fractional CFO are analyzed in section III. Two time domain estimation techniques and two frequency domain estimation techniques are proposed in section IV. In section V, we proposed the simulation results in AWGN and four taps multipath fading channels for the performance of CP-based estimator with different CP length and the performance of training sequence with two and four repetitive patterns, finally we gave a comparative simulation between the above two estimators with training symbol estimator and pilot tone estimator. At the end the conclusions are presented in section VI.

II. CARRIER FREQUENCY OFFSET IN OFDM

In OFDM systems, subcarriers (SCs) will sample at their peak, and this can only occur when there is no frequency offset, however if there is any frequency offset, the sampling will be done at the offset point, which is not the peak point. This causes to reduce the amplitude of the anticipated subcarriers, which can result to raise the Inter Carrier Interference (ICI) from the adjacent subcarriers (SCs) as shown in Fig.1. There are two main causes of CFO. The first is a frequency mismatch between the local oscillators at the transmitter and receiver. The second cause is the Doppler shift due to motion between the transmitter and receiver in mobile environments. The carrier frequency difference between the transmitter and receiver can be written as $f_{offset} = f_c - f_c'$, where f_c and f_c' are the carrier frequency in the transmitter and receiver respectively. The normalized CFO, ε, can be expressed as $\varepsilon = f_{offset}/\Delta f$, where Δf is the subcarrier spacing. The normalized CFO can be divided into two parts; integral CFO [9] (IFO) ε_i and fractional. CFO (FFO), ε_f, therefore $\varepsilon = \varepsilon_i + \varepsilon_f$.Accordingly, we can divide the problem into two parts; fine frequency offset estimation by estimating the center frequencies of each sub-channel (sub-carrier) and coarse frequency offset estimation by estimating the tone numbering index.

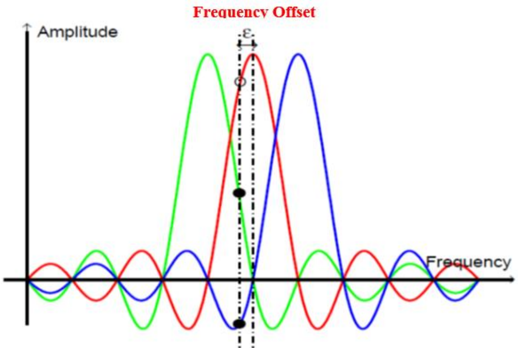

Fig. 1. Impact of carrier frequency offset (CFO).

III. CFO PROBLEM MODELING

CFO problem in OFDM systems can be modeled as shown in Fig.2. A baseband discrete-time OFDM system with N subcarriers is considered. The OFDM symbol is generated by taking the N-point inverse fast Fourier transform (IFFT) on the information symbol $X[k]$, $0 \le k < N$. Hence, the OFDM symbol at the transmitter side $x[n]$ can be written as

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\cdot e^{j2\pi nk/N} \quad ,n = 0,1,\dots,N-1 \qquad (1)$$

After passing through the channel, the received OFDM signal $y[n]$ in time domain with the effect of CFO can be written as

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k]X[k]e^{\frac{j2\pi(k+\varepsilon)n}{N}} + w[n] \qquad (2)$$

Where ε is the normalized carrier frequency offset, $H[k]$ is the transfer function of the channel, $w[n]$ is AWGN and n is time index.

The demodulated signal $Y[k]$for the k^{th} subcarrier (i.e., at the FFT output) consisting of three components as shown in eq.4.

$$Y[k] = \sum_{n=0}^{N-1} y[n]\cdot e^{-j2\pi nk/N} \;, k = 0,1,\dots,N-1 \qquad (3)$$

Where, k represents the frequency index.

$$Y[k] = \sum_{n=0}^{N-1}\frac{1}{N}X[k]\cdot H[k]e^{j2\pi n\varepsilon/N} + \sum_{l=0,l\neq k}^{N-1}\frac{1}{N}X[l]\cdot H[l]e^{j2\pi n(l-k+\varepsilon)/N} + \sum_{n=0}^{N-1} W[n]\cdot e^{-j2\pi kn/N} \qquad (4)$$

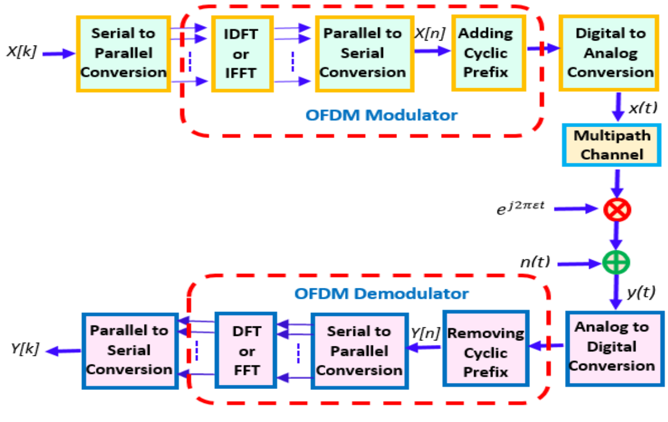

Fig. 2. CFO modeling of OFDM system

The first term is the estimate of $\hat{X}[k]$ which experiences an amplitude reduction and phase shift due to the carrier frequency offset ε.

$$\hat{X}[k] = e^{j\pi\varepsilon(N-1)/N}\left\{\frac{\sin(\pi\varepsilon)}{N\sin(\pi\varepsilon/N)}\right\}X[k]\cdot H[k] \qquad (5)$$

The second term *I[k]* represents the inter-carrier interference caused by the frequency offset:

$$I[k] = e^{j\pi\varepsilon(N-1)/N}\sum_{l=0,l\neq k}^{N-1}\frac{\sin(\pi(l-k+\varepsilon))}{N\sin(\pi(l-k+\varepsilon)/N)}H[l]X[l]e^{\frac{j\pi(l-k)(N-1)}{N}} \qquad (6)$$

The third term *W[k]* is the added white Gaussian noise (AWGN).

A. *Effect of Integer Carrier Frequency Offset (IFO)*

As seen in the Fig. 3, the time domain transmitted signal $\{x_l[n]\}_{n=0}^{N-1}$affects by integer CFO. This integer CFO causes phase shifting in the received signal $e^{j2\pi\varepsilon_i n/N}x[n]$ where channel effect is not considered. It causes frequency shifting (cyclic shift) by ε_i of the frequency domain signal which is $X[k-\varepsilon_i]$. For example, if $\varepsilon_i = 1$then $X[1]$ will be moved to the second subcarrier index and $X[2]$ will be moved to the third subcarrier index. Thus, the integer CFO can significantly reduce the BER performance. However, integer CFO does not cause the loss of orthogonality among the subcarrier frequency components and thus, ICI does not occur [1], [3].

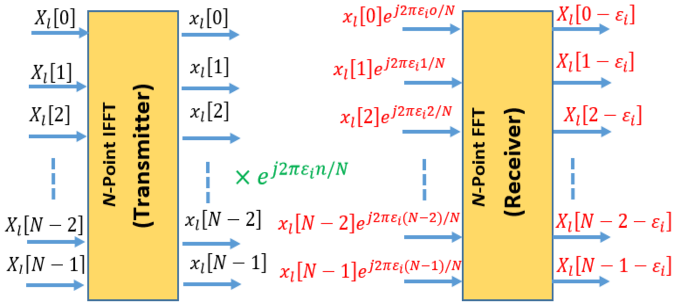

Fig. 3. Effect of integer CFO on the received signal.

B. *Effect of Fractional Carrier Frequency Offset (FFO)*

The effect of fractional CFO can be described by replacing ε with ε_f in eq. (5) and eq. (6) then we get:
The desired signal with phase distortion and amplitude attenuation due to the fractional CFO as:

$$\hat{X}[k] = e^{j\pi\varepsilon_f(N-1)/N}\left\{\frac{\sin(\pi\varepsilon_f)}{N\sin(\pi\varepsilon_f/N)}\right\}X[k]\cdot H[k] \qquad (7)$$

The inter-carrier interference caused by the fractional carrier frequency offset as

$$I[k] = e^{j\pi\varepsilon_f(N-1)/N}\sum_{l=0,l\neq k}^{N-1}\frac{\sin\left(\pi(l-k+\varepsilon_f)\right)}{N\sin\left(\pi(l-k+\varepsilon_f)/N\right)}H[l]X[l]e^{\frac{j\pi(l-k)(N-1)}{N}} \qquad (8)$$

So, we can see that the original symbol $X[k]$ suffers at reception from amplitude attenuation due to the fractional CFO by $\left(\sin(\pi\varepsilon_f)/N\sin(\pi\varepsilon_f/N)\right)$ and phase rotation or phase distortion by $\left(e^{j\pi\varepsilon_f(N-1)/N}\right)$, and ICI from other subcarriers into k^{th} subcarrier frequency component, which implies that the orthogonality among subcarrier frequency components is not maintained any longer due to the FFO. So, if the fractional CFO exists, it destroys the orthogonal property of an OFDM system In order to compensate for these effects, the CFO value,ε_f, needs to be estimated and removed at the receiver [1], [3].

Figure (4) show the QPSK received signal constellation in the absence of frequency offset. Where Fig. 5 and Fig.6 show the QPSK received signal constellation with frequency offset 0.3 and 0.5 respectively. As shown from fig. 5 and fig. 6, when the frequency offset is increased, the distortion in the received signal is increased and for bigger values of frequency offset the received data are unreadable.

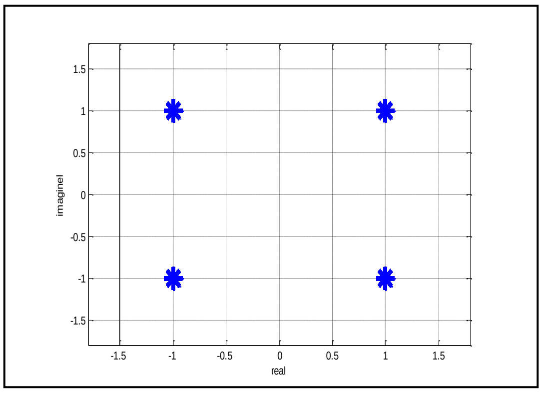

Fig. 4. Received signal constellation with 0% frequency offset

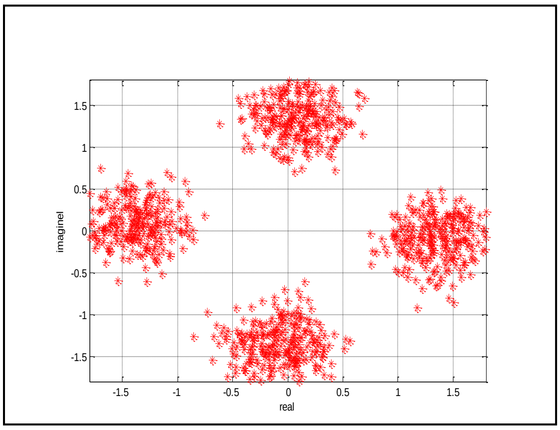

Fig. 5. Received signal constellation with 0.3% frequency offset

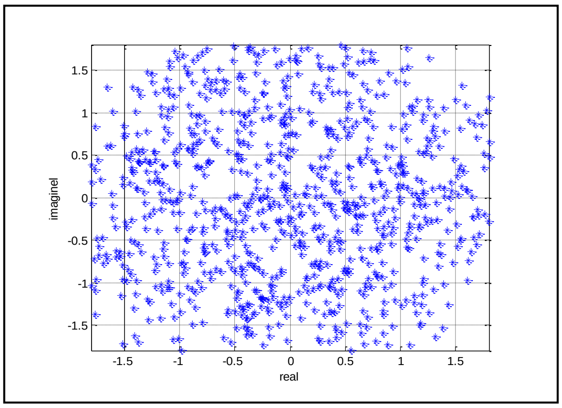

Fig. 6. Received signal constellation with 0.5% frequency offset

Figure (7) shows BER vs. SNR (E_b/N_0) of OFDM in the AWGN channel for BPSK modulation under different CFOs compared to theoretical BER curve. As shown in Fig.6 with CFO increases the BER increases and degrade the performance of OFDM system.

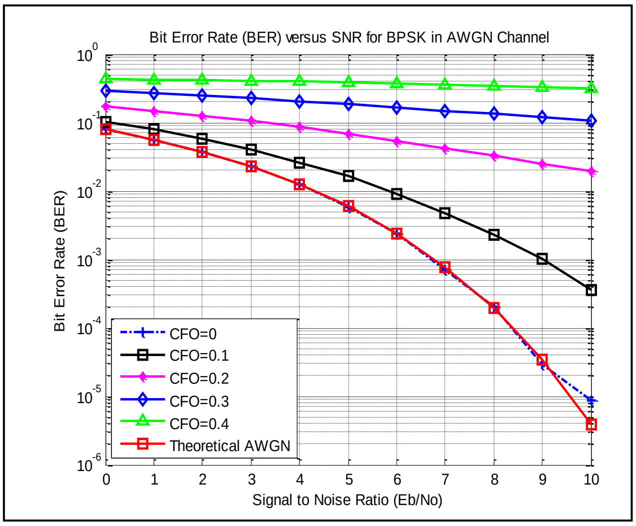

Fig. 7. BER vs. SNR (E_b/N_0) of OFDM in the AWGN channel for BPSK

IV. CFO ESTIMATION TECHNIQUES

The goal in carrier frequency offset (CFO) estimation is to maintain or preserve the orthogonality properties of the sub-carriers. There are many methods for CFO estimation in OFDM systems have been developed, which can be separated into two categories: data-aided methods (DA) and non-data aided methods (NDA). DA category uses a training symbol or pilot tone for estimation. It has high accuracy but reduces the usable bandwidth or reduces the data transmission speed. The synchronization time needs to be as short as possible, and the accuracy must be as high as possible for high packet rate transmission. However, the use of pilot symbols inevitably decreases the capacity and/or throughput of the overall system, thus making them suitable only in a startup/training mode. Also, the SNR at the front of the receiver is often too low to effectively detect pilot symbols, thus a blind approach is usually much more desirable. NDA category often uses the cyclic prefix correlation. It doesn't waste bandwidth and reduce the transmission speed, but its estimation range is too small, to be suitable for acquisition.

CFO can be estimated and compensated in time domain or frequency domain, sometimes called *pre-FFT* and *post-FFT* synchronization, respectively. Pre-FFT synchronization performs the estimation of CFO before OFDM demodulation (FFT processing) but post-FFT synchronization performs the estimation after FFT processing. The pre-FFT algorithms provides fast synchronization and requires less computing power due to the fact that no FFT processing is needed but post-FFT algorithms has a higher throughput spectral.

So, both time-domain and frequency-domain frequency synchronization play important roles in correcting carrier frequency offset in OFDM systems. It's up to the system designer to decide on the appropriate approach to use, depending on the requirements and specifications of the system.

Estimation techniques in the time domain (TD) are based on introducing a cyclic prefix, or training sequence, and the use of correlation [15]-[17]. Frequency domain (FD) CFO estimation techniques are applied under assumption that perfect time synchronization is achieved. Furthermore, the FD techniques are based on transmitting two identical symbols or pilot tone (pilot insertion) [6]-[7]. Each of these techniques is discussed as follows.

A. Cyclic Prefix CFO Estimation Technique

Cyclic prefix (CP) is a portion of an OFDM symbol used to absorb inter-symbol interference (ISI) caused by any transmission channel time dispersion and it can be used in CFO estimation [1], [15], [17]. Figure (8) shows OFDM symbol with CP. CP based estimation method exploits CP to estimate the CFO in time domain.

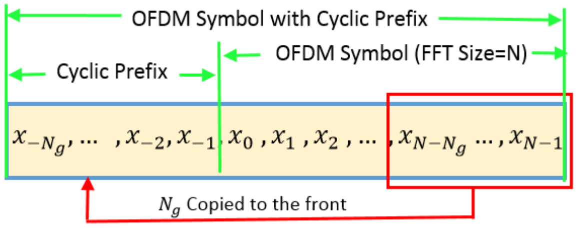

Fig. 8. OFDM Symbol with Cyclic Prefix

Considering the channel effect is minimal and can be neglected, then, the l^{th} OFDM symbol affected by CFO can be written as

$$y_l[n] = x[n]e^{\frac{j2\pi\varepsilon n}{N}} \quad (9)$$

Replacing n by *(n+N)* in eq.9 the corresponding CP in the OFDM symbol can be written as

$$y_l[n+N] = x[n+N]e^{\frac{j2\pi\varepsilon}{N}(n+N)} \quad (10)$$

$$y_l[n+N] = x(n)e^{\left[\frac{j2\pi\varepsilon n}{N}+j2\pi\varepsilon\right]} \quad (11)$$

By comparing eq.9 and eq.11, we can find that the phase difference between CP and the OFDM symbol is $2\pi\varepsilon$. Therefore, the amount of CFO can be found from the argument of the multiplication of OFDM symbol by the conjugate of its CP:

$$\hat{\varepsilon} = \frac{1}{2\pi} arg\{y_l^*[n]y_l[n+N]\}, \text{n} = -1, -2, \ldots, -N_g \quad (12)$$

In order to reduce the noise effect, its average can be taken over the samples in a CP interval as:

$$\hat{\varepsilon} = \frac{1}{2\pi} arg\left\{\sum_{n=N_g}^{-1} y_l^*[n]y_l[n+N]\right\}, \text{n} = -1, -2, \ldots, -N_g \quad (13)$$

Since the argument operator arg( ) is performed by using $\tan^{-1}(\,)$, the range of CFO Estimation in eq.13 is $[-\pi,+\pi]/2\pi$ = [-0.5, +0.5] so that $|\hat{\varepsilon}| \leq 0.5$. Therefore, CP results CFO estimation in the range, $|\hat{\varepsilon}| \leq 0.5$. Hence, this technique is useful for the estimation of Fractional CFO (FFO). CFO estimation technique using CP does not estimate the integer offset. To overcome this drawback, the training sequence technique is used to estimate CFO. This is helpful in increasing the range of the CFO estimation.

B. *Training Sequence CFO Estimation Technique*

It has been shown that the CFO estimation technique using CP can estimate the CFO only within the range ($|\hat{\varepsilon}| \leq 0.5$). Since CFO can be large at the initial synchronization stage, we may need estimation techniques that can cover a wider CFO range. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. Let D be an integer that represents the ratio of the OFDM symbol length to the length of a repetitive pattern as shown in Fig.9.

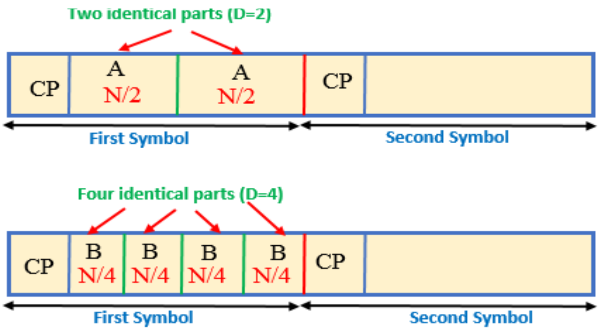

Fig. 9. Training Sequence in OFDM Symbol

Let a transmitter sends the training symbols with D repetitive patterns in the time domain, which can be generated by taking the IFFT of a comb-type signal in the frequency domain given as

$$X_l[k] = \begin{cases} A_m, & if\ k = D.i\ , i = 0,1,2,\dots,\left(\frac{N}{D-1}\right) \\ 0 & otherwise \end{cases} \quad (14)$$

Where A_m represents an M-ary symbol and *N/D* is an integer. As $x_l[n]$ and $x_l[n+N/D]$ are identical (i.e.,$[y_l^*[n]y_l\left[n+\frac{N}{D}\right] = |y_l[n]|^2 e^{j\pi\varepsilon}$, a receiver can make CFO, estimation as follows.

$$\hat{\varepsilon} = \frac{D}{2\pi} arg\left\{\sum_{n=0}^{N/D-1} y_l^*[n]y_l[n+N/D\,]\right\} \quad (15)$$

The CFO estimation range covered by this technique is $|\hat{\varepsilon}| \leq D/2$ .Which becomes wider as D increases. Increasing in estimation range is obtained at the sacrifice of mean square error (MSE) performance. Hence, there is a trade-off relationship between the MSE performance and estimation range of CFO is clearly shown. This is due to reduction in the correlation samples by a factor of *1/D*. The solution to this problem is to calculate average of all the estimates over repetitive short periods as in eq. 16.

$$\hat{\varepsilon} = \frac{D}{2\pi} arg\left\{\sum_{m=0}^{D-2}\sum_{n=0}^{N/D-1} y_l^*[n+mN/D]y_l[n+(m+1)N/D\,]\right\} \quad (16)$$

C. *Training Symbol CFO Estimation Technique.*

The maximum likelihood (ML) CFO estimation method is based on two consecutive and identical training symbols [1], [6]. The same data frame is repeated and the phase value of each carrier between consecutive symbols is compared as shown in Fig.10. The offset is determined by maximum likelihood estimation algorithm (MLE). Let OFDM signal at the receiver, in the absence of noise, after repeating the same data frame is given by:

$$r_n = (1/N)\left[\sum_{k=-K}^{K} X_k H_k e^{\frac{2\pi jn(k+\varepsilon)}{N}}\right], n = ,1,\dots,2N-1 \quad (17)$$

In eq.17, X_k is the transmitted signal, H_k is the transfer function of the channel at the k^{th} carrier, and ε is the frequency offset. In order to determine the value of frequency offset, ε, compare the two consecutive received data symbols at a given frequency.

The k^{th} element of the N point DFT of the first N points of equation (17) is

$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-2\pi jnk/N} \quad ; k = 0,1,2,\dots,N-1 \quad (18)$$

And the k^{th} element of the DFT of the second half of the sequence is

$$R_{2k} = \sum_{n=N}^{2N-1} r_n e^{-\frac{2\pi jnk}{N}}$$
$$= \sum_{n=0}^{N-1} r_{n+N} e^{-2\pi jnk/N} \; ; k = 0,1,2,\dots,N-1 \quad (19)$$

From eq.17 we can see

$$r_{n+N} = r_n e^{2\pi j\varepsilon} \rightarrow R_{2k} = R_{1k} e^{2\pi j\varepsilon} \quad (20)$$

If AWGN noise W_{1k} and W_{2k} is added the signal at the receiver becomes;

$$Y_{1k} = R_{1k} + W_{1k}$$
$$Y_{2k} = R_{1k} e^{2\pi j\varepsilon} + W_{2k}\,, \quad k = 0,1,2,\dots,N-1 \quad (21)$$

Therefore, we can use a maximum likelihood approach to determine the relative frequency offset ε as

$$\hat{\varepsilon} = \frac{1}{2\pi} tan^{-1}\left[\frac{(\sum_{k=-K}^{K} Im[Y_{2k}Y_{1k}^*])}{(\sum_{k=-K}^{K} Re[Y_{2k}Y_{1k}^*])}\right] \quad (22)$$

Where $\hat{\varepsilon}$ is the maximum likelihood estimate of the relative frequency offset defined as $\varepsilon = N\Delta f/B$, where B is bandwidth, N is the number of subcarriers and Δf is the frequency offset in Hz. The estimation range of this technique equals one half (±1/2) of the sub-carrier spacing.

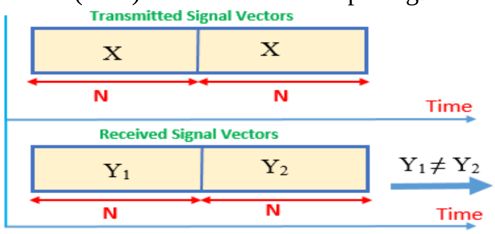

Fig. 10. Schematic of training symbol technique

D. Pilot tone CFO Estimation Technique

CFO estimation in this technique is based on inserting some pilot tones in the frequency domain then transmitted in every OFDM symbol. CFO estimation is performed in two stages: an acquisition stage and a tracking stage [7]. During the acquisition stage, large frequency offsets (multiples of subcarrier spacing) are estimated whereas, the tracking stage deals with small fractional frequency offset [1] as shown in Fig.11.

After time synchronization, N samples of two OFDM symbols, $y_l[n]$ and $y_{l+D}[n]$ are saved in the memory. The FFT unit transforms these symbols into frequency domain signals $Y_l[k]$ and $Y_{l+D}[k]$ with k= 0, 1, 2,…, N-1.These pilot subcarriers are then utilized for CFO estimation in frequency domain and subsequently, the frequency correction is done in time domain. Stage 1 is uses to obtain a coarse CFO estimate as quickly as possible. Stage 2 uses coarse estimate from stage 1 and performs the tracking. Therefore, stage 1 can be optimized for higher estimation range and speed while stage 2 can be optimized for higher accuracy.

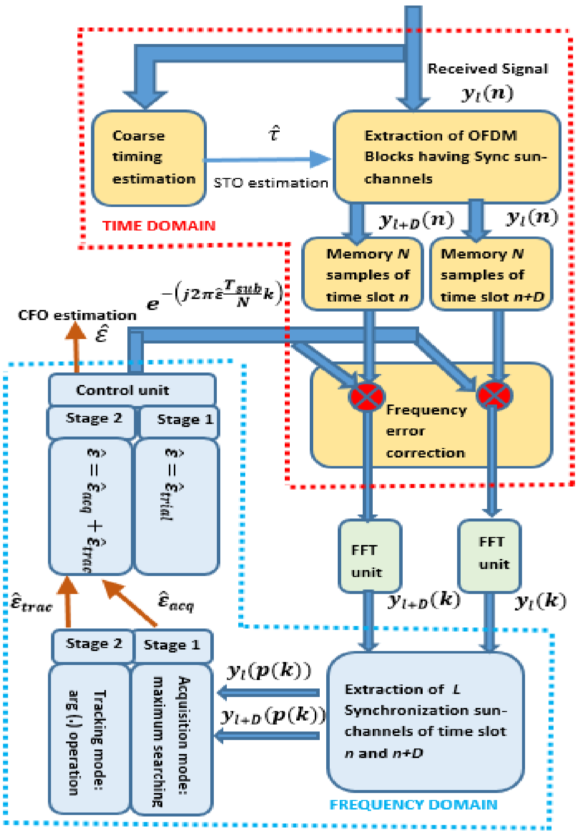

Fig. 11. Data-Aided CFO synchronization scheme using pilot tones.

The integer CFO is estimated by

$$\hat{\varepsilon}_{acq} = \frac{1}{2\pi T_{sub}} \max_{\varepsilon_{trial}} \left\{ \left| \sum_{j=0}^{L-1} Y_{l+D}[p(j), \varepsilon_{trial}] Y_l^*[p(j), \varepsilon_{trial}] X_{l+D}^*[p(j)] X_l[p(j)] \right| \right\} \quad (23)$$

Where $p(j)$ denotes the location of the jth pilot tone, $X_l[p(j)]$ denotes the pilot tone located at $p(j)$ in the frequency domain at the lth symbol period and L denotes the number of pilots. ε_{trial} is the trial frequency values and $Y_l^*[p(j), \hat{\varepsilon}_{trial}]$ is the FFT output with ε_{trial}-offset-corrected input. Moreover the complex conjugate of known pilot tones $X_l[k]$and $X_{l+D}[k]$ is used to undo the effect of modulation at pilot tones. Since the frequency correction by counter rotating the received samples in time domain by multiplying with $e^{-\left(j2\pi\hat{\varepsilon}\frac{T_{sub}}{N}k\right)}$is done before FFT unit, hence the variable $\hat{\varepsilon}_{acq}$ is also shown as an argument of the FFT output. Meanwhile, the fine CFO can be estimated from the phase shift between two successive subcarriers as the following.

$$\hat{\varepsilon}_f = \frac{1}{2\pi T_{sub}.D} \arg \left\{ \left| \sum_{j=0}^{L-1} Y_{l+D}[p[j], \hat{\varepsilon}_{acq}] Y_l^*[p[j], \hat{\varepsilon}_{acq}] X_{l+D}^*[p[j]] X_l[p[j]] \right| \right\} \quad (24)$$

where integer D= n+D-n represents the difference between time index of two symbols used for estimation and L represents the known pilot symbol pairs in nth and $(n + D)^{th}$ time slots. In the acquisition mode, $\hat{\varepsilon}_{acq}$ and $\hat{\varepsilon}_f$ are estimated and then, the CFO is compensated by their sum. In the tracking mode, only $\hat{\varepsilon}_f$ is estimated and then compensated.

V. Simulation Results

In this section, we have introduced the simulation results of CFO estimation in time domain and frequency domain in the case of three different channels, AWGN channel, Rayleigh fading channel and Rician Fading channels (four taps). First, we analyzed the effect of cyclic prefix length in OFDM symbol in CFO estimation by using CP-based estimator in time domain. Then, we analyzed the effect of repetitive pattern in OFDM symbol in CFO estimation by using training sequence estimator in time domain. Finally, we gave a simulation comparison between the above two method in time domain and another two methods, symbol based and pilot tone, in frequency domain.

A. Simulation results of CP based estimator

Fig.12, Fig.13 and Fig.14 show the simulation results of CFO estimation using CP-based in time domain for AWGN Channel, Rayleigh Fading Channel and Rician Fading Channel (four taps) at the normalized CFO equal 0.2. These simulation results are based on 1000 iterations with an OFDM system of 128 subcarriers, SNR from 0 to 30 dB. The simulation results are considered for different values of CP-length as 8, 16, 32 and 50 samples of an OFDM symbol.

As evident from Fig.12 to Fig.14 the performance of CP-based estimation technique improves with the CP-length increased as shown in the three different types of channel. In Rayleigh fading channel MSE is large for 8 CP-length and at MSE equal 10^{-4} estimation using 50 CP-length better than using 32, 16 CP-length by 2.399 dB, 11.619 dB respectively as shown in fig.13. Since numbers of samples of an OFDM symbol are used for correlation increase, the estimation performance improved. However, there is a trade-off between CP-length in OFDM symbol and the bandwidth.

Figure (15) shows the range of CP-based estimation technique at 5dB for 8, 16, 32, and 50 CP-length in Rayleigh Fading channel. It is clear that from the figure the estimation range of CP estimator is ±0.5 of subcarrier spacing. Therefore, CP based estimation is suitable for fine CFO estimation.

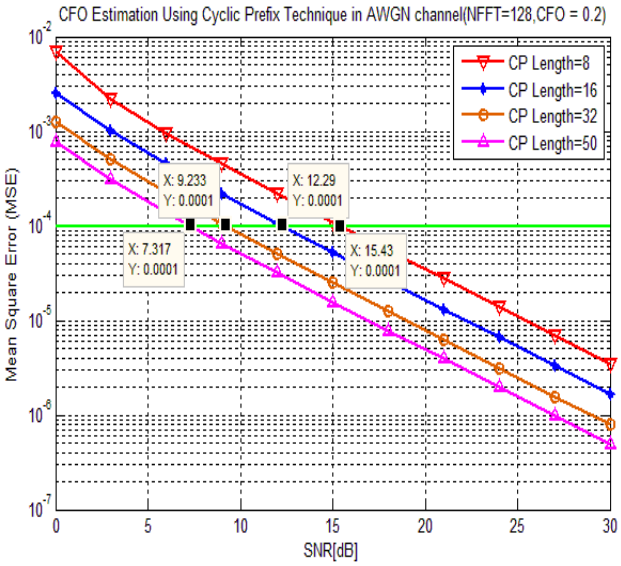

Fig. 12. CFO estimation using CP estimator in AWGN channel at CFO= 0.2

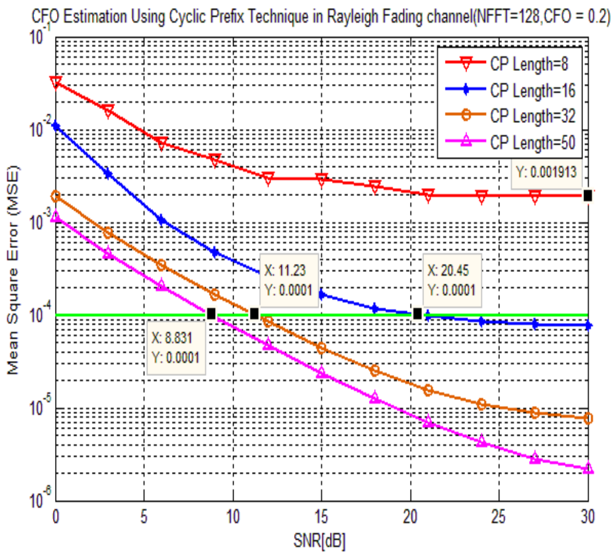

Fig. 13. CFO estimation using CP estimator in Rayleigh Fading channel at CFO= 0.2

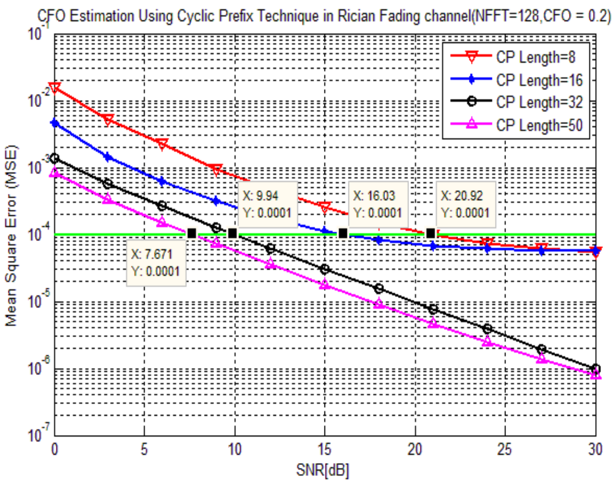

Fig. 14. CFO estimation for CP estimator in Rician Fading channel at CFO= 0.2

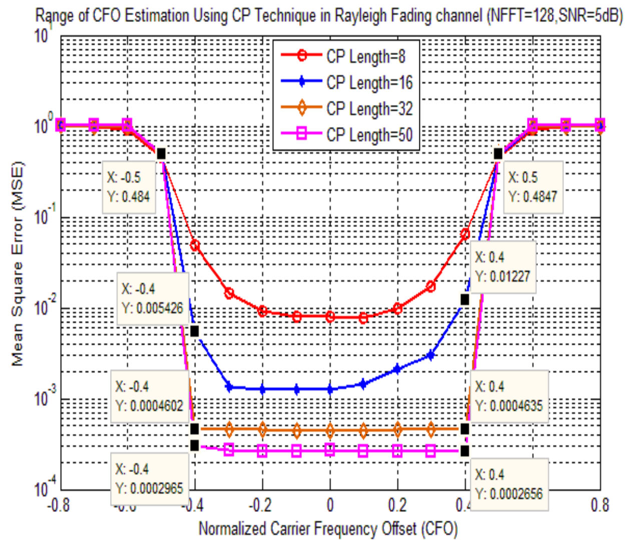

Fig. 15. CFO estimation range for CP estimator in Rayleigh Fading Channel at SNR= 5dB

B. Simulation results of Training Sequence based estimator

The simulation results for CFO estimation in time-domain using training sequence are presented are based on 128 subcarriers and 32 cyclic prefix length in AWGN channel , Rayleigh fading channel and Rician Fading channels (four taps) at SNR = 0:30 dB for 0.3 normalized CFO based on 1000 iterations. The number of repetitive patterns in the simulation are considered D= 2 and D=4.

Fig.16 to Fig.18 show the relation between MSE versus SNR in the case of repetitive pattern D=2 and D=4 in three different types of channels. It is clear from simulation results MSE increased with increasing D in the training sequence estimator but there are a trade-off between CFO estimation range and MSE as shown in Fig.19. Hence the estimation range increases with increasing *D*, the number of repetitive patterns in OFDM symbol. As shown in Fig.19 CFO estimation rage at *D* = 2 reach to ±0.9 subcarrier spacing but the range at *D* =4 increase to reach to ±1.9 at SNR equal 5dB. However, this broader estimation range comes at the cost of MSE performance degradation. So, the training sequence estimator can be used for coarse as well as fine CFO estimation.

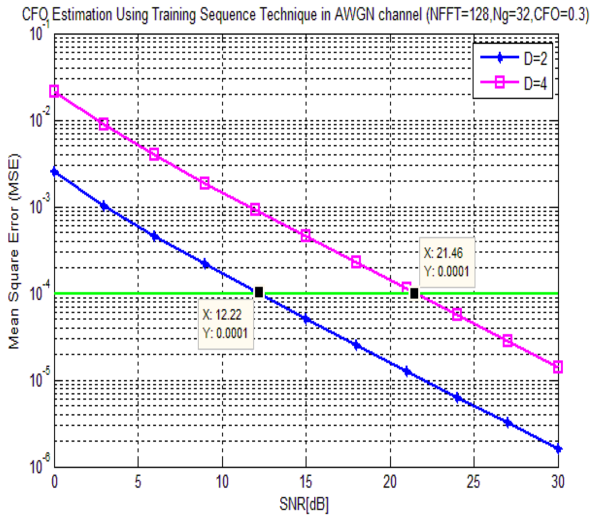

Fig. 16. CFO estimation using training sequence estimatorin AWGN channel at CFO= 0.3

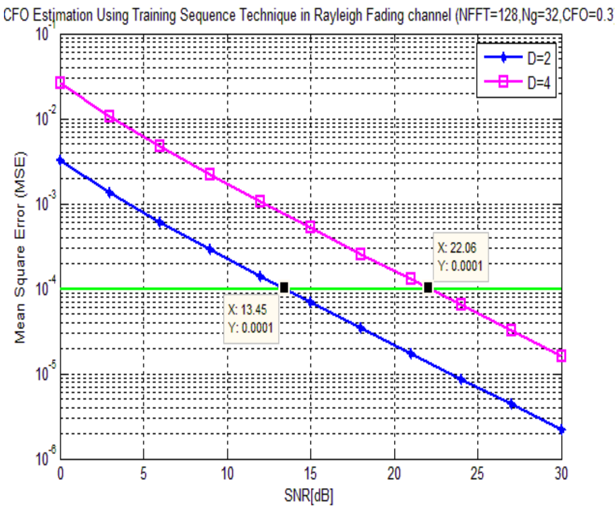

Fig. 17. CFO estimation using training sequence estimator in Rayleigh Fading channel at CFO= 0.3

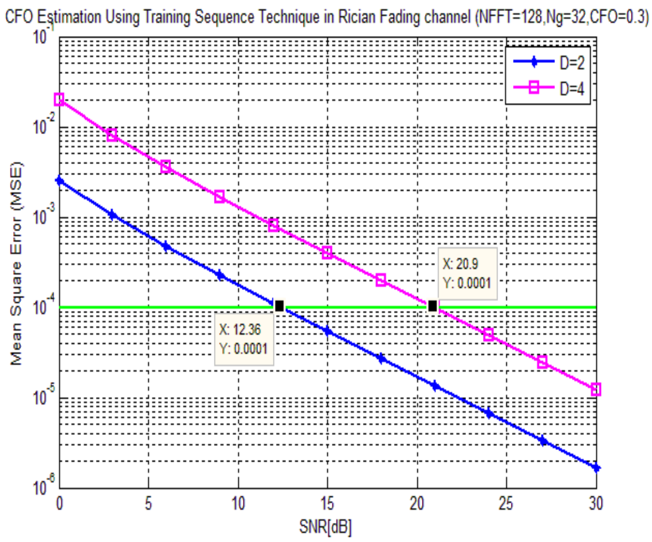

Fig. 18. CFO estimation using training sequence estimatorin Rician Fading channel at CFO= 0.3

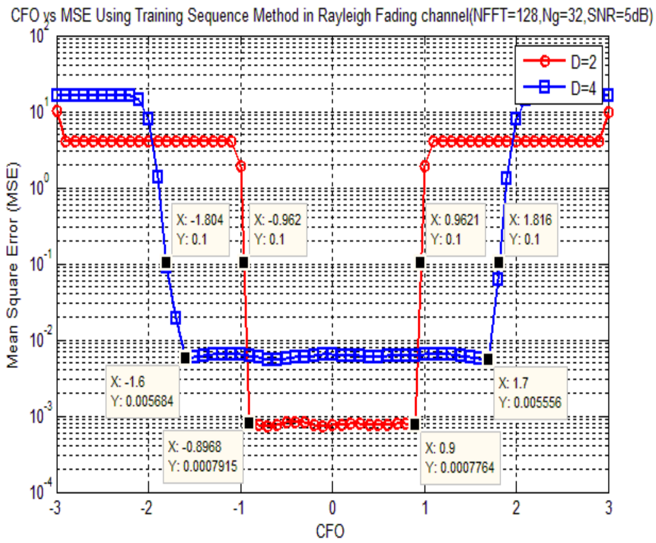

Fig. 19. CFO estimation range for training sequence estimator in Rayleigh Fading channel at SNR= 5dB

C. Simulation Comparison between Estimation Techniques

The assumed simulation parameters used in comparison between CP-based and training sequence estimators in time domain and training symbol and pilot tone estimators in frequency domain are shown in table (1). By comparing between different estimators at 10^{-4} MSE for 0.2 normalized CFO. We find Pilot tone estimator outperforms than symbol based ,CP based and training sequence based estimators by 1.652 dB, 7.168 dB, 10.295 dB in AWGN channel and 2.132 dB, 8.429dB 10.59 dB in four taps Rayleigh fading channel in addition to 2,231dB, 7.994dB and 10.384 dB in four taps Racian channel respectively as shown in Fig 20 to Fig 22 .

Table 1.Assumed Simulation Parameters

No	Parameter	Simulation Value
1	NFFT Size	128
2	Cyclic Prefix Length (Ng)	32
3	Modulation Scheme	QAM (QPSK)
4	Channel	AWGN, Rayleigh, Rician
5	Normalized Frequency offset (CFO)	0.1, 0.2, 0.3
6	Number of Taps in multipath Fading channel	4
7	Number of Iterations	1000
8	Symbol Duration Length	3
9	Number of Bits per Symbol	2
10	Pilot Spacing (Nps)	4
11	Number of identical Parts (D)	2
12	Signal to Noise Ratio (SNR)	0-30 dB

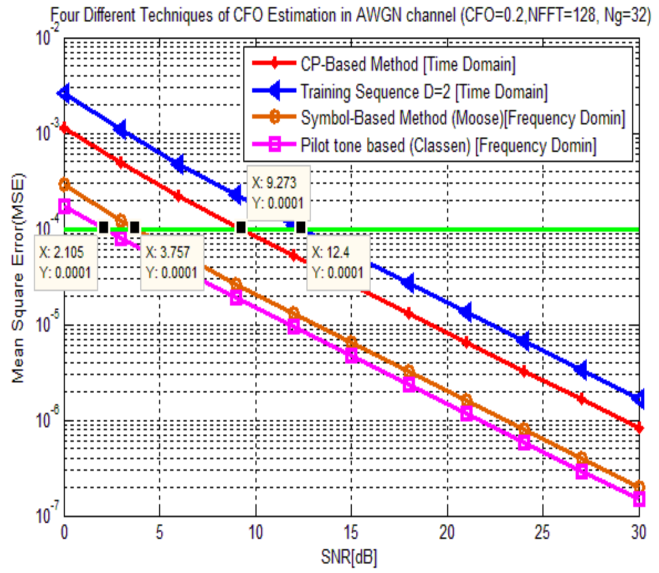

Fig. 20. Comparison between CFO Estimation techniques in AWGN Channel at normalized CFO=0.2

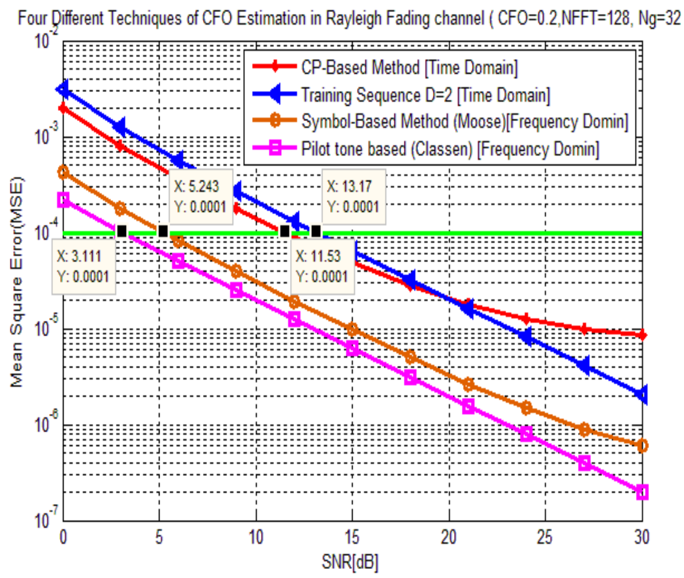

Fig. 21. Comparison between CFO Estimation techniques in Rayleigh Fading Channel at normalized CFO=0.2

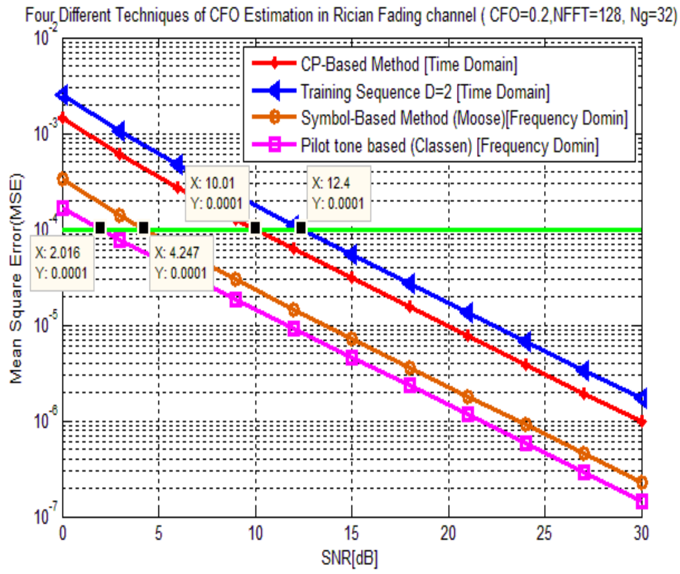

Fig. 22. Comparison between CFO Estimation techniques in Rician Fading Channel at normalized CFO=0.2

Table (2) shows the numerical comparison of the simulation results for the four different estimators at 10^{-4} MSE when the normalized CFO equal 0.1,0.2 and 0.3. Fig.23 to Fig.25 summarize the outperforms of Pilot-based estimator compared to Symbol-based, training sequence-based and CP-based in AWGN, Rician and Rayleigh fading channels in the case of MSE=10^{-4} at CFO equal 0.1, 0.2 and 0.3 respectively.

Table 2. Simulation results comparison between Four different Estimator at MSE=10^{-4}

Estimation Method	Rayleigh Fading Channel	Rician Fading Channel	AWGN Channel	CFO
Pilot Tone Based (FD)	1.226 dB	2.558 dB	1.349 dB	**Normalized CFO=0.1**
Symbol Based (FD)	3.823 dB	5.107 dB	4.043 dB	
Training Sequence (TD)	9.334 dB	11.35 dB	9.769 dB	
CP-Based (TD)	12.36 dB	13.28 dB	12.48 dB	
Pilot Tone Based (FD)	2.105 dB	3.111 dB	2.016 dB	**Normalized CFO=0.2**
Symbol Based (FD)	3.757 dB	5.243 dB	4.247 dB	
Training Sequence (TD)	9.273 dB	11.53 dB	10.01 dB	
CP-Based (TD)	12.4 dB	13.17 dB	12.4 dB	
Pilot Tone Based (FD)	2.642 dB	3.82 dB	2.823 dB	**Normalized CFO=0.3**
Symbol Based (FD)	3.66 dB	5.249 dB	4.382 dB	
Training Sequence (TD)	9.335 dB	11.66 dB	10.1 dB	
CP-Based (TD)	12.19 dB	13.15 dB	12.22 dB	

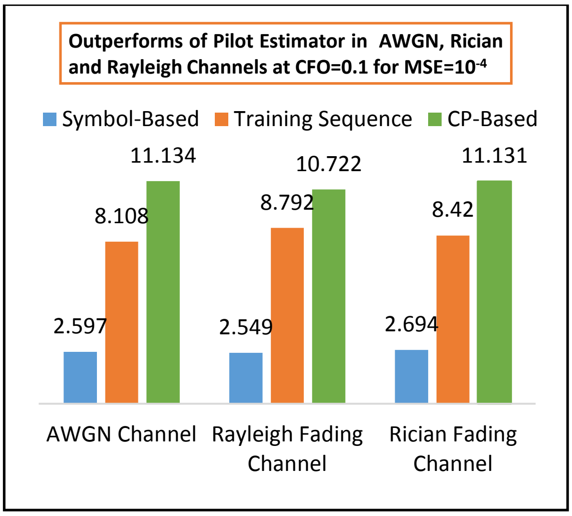

Fig. 23. Outperforms of Pilot Estimator at CFO=0.1 in dB

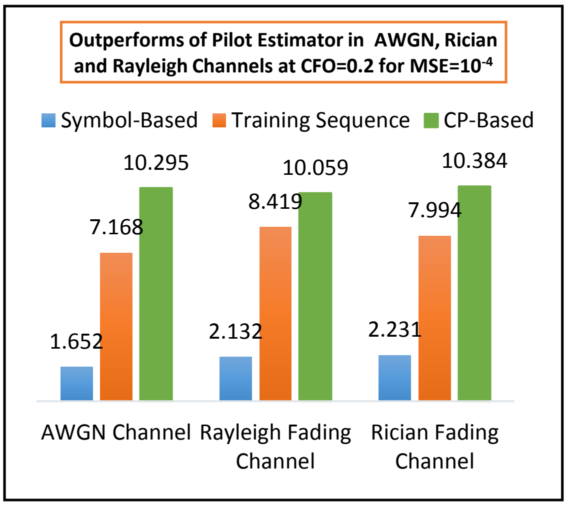

Fig. 24. Outperforms of Pilot Estimator at CFO=0.2 in dB

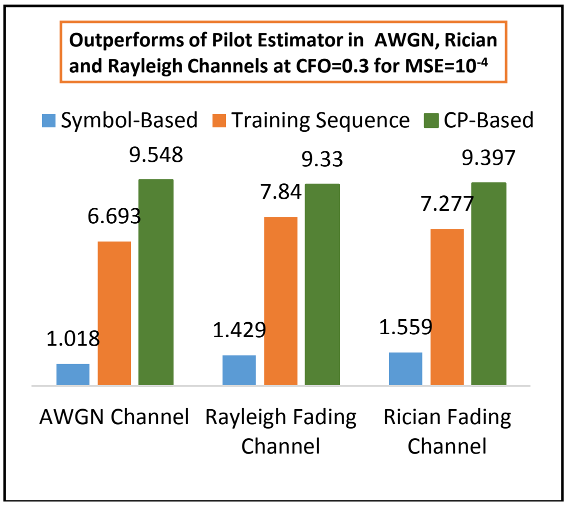

Fig. 25. Outperforms of Pilot Estimator at CFO=0.3 in dB

VI. CONCLUSION

CFO in OFDM systems destroys the orthogonality among subcarriers and leads to significant performance degradation. Therefore CFO estimation and compensation is one of the most important functions of OFDM receiver. In this paper, we investigate the performance of four different CFO estimation techniques in OFDM system in AWGN and multipath fading channels of four paths. The mean square error (MSE) criteria is considered in the comparison. We examined two methods in time domain (pre-FFT) and two methods in frequency domain (post-FFT) that can estimate CFO. The simulation results show that CFO estimators in frequency domain more accurate than the CFO estimators in time domain. The pilot based estimator has best performance in terms of MSE in AWGN multipath fading channels. The performance CP-based estimator improves with increasing the CP-length but there is a tradeoff between the CP-length and the bandwidth efficiency in addition to small range estimation. The training sequence-based estimator is good for estimating a wide range of CFO but with large MSE. The training symbol based estimator close to pilot tone based estimator.

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