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Performance Analysis of Linear Frequency Modulated Pulse Compression Radars under Pulsed Noise Jamming

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Abstract

Pulsed noise jamming is a common anti-radar jamming technique. It creates a noise pulse when radar signal is received, thus concealing any aircraft flying behind it with a block of noise. Modern Linear Frequency Modulated Pulse Compression (LFM-PC) radar, which is characterized by its high processing gain, is considered as one of the challenges to jammer systems. In this paper, the performance of such radar is evaluated analytically, which has not been exploited in any other literature before, under the effect of pulsed noise jamming. Mathematical models of the LFM-PC matched filter response in clear environment as well as pulsed noise jamming are derived. Receiver Operating Characteristic (ROC) is derived and used as a performance measurer. The Derived analytical results agreed with simulation results.

1. Introduction

Pulse compression techniques are used to provide radar systems with high resolution without affecting the maximum detection range [1]. Modern LFM-PC radar, whose receiver signal processor is shown in Figure 1, supports high Doppler shifts with excellent time sidelobe levels [2]. Moreover; pulse compression provides radar receiver with a processing gain equals the time bandwidth product of the transmitted pulse [3]. The coherent integration process in modern LFM PC radar gives an additional processing gain proportional to the length of the Coherent Pulse Interval (CPI) [4]. Using Constant False Alarm Rate (CFAR) processing along with pulse compression and coherent integration enhance the immunity of LFM-PC search radar against jamming [4, 5].
Pulsed noise jamming is one of the early used jamming techniques against radars [6]. It is located in front of the target. When it receives the victim radar pulses, it generates a noise pulse with the same radar pulse length. It is also called cover pulse jamming [7]. This noise pulse causes saturation to the victim radar receiver in this sector, consequently, preventing the target from being detected by the victim radar [8].

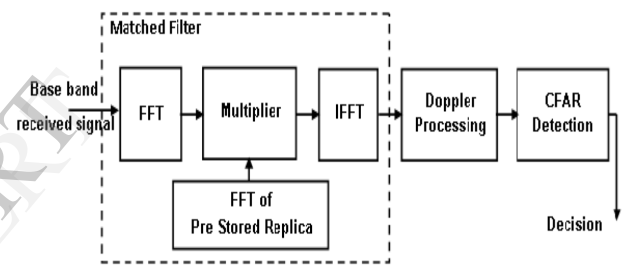

Figure 1 Block diagram of LFM-PC radar receiver signal processor

Literature lacks neither a mathematical model of Linear Frequency Modulation (LFM) matched filter response to pulsed noise jamming nor a simulation model for the effect of pulsed-noise jamming on the detection performance of modern LFM-PC radars. In this paper, a derived mathematical model for the matched filter response of the LFM-PC radar against pulsed noise jamming is proposed. The detection performance of the LFM-PC search radar under the effect of pulsed-noise jamming is evaluated analytically through the ROC curves. A simulation model for the LFM-PC search radar is built to calculate the ROC and compare it with the derived results.
After the introduction, the rest of this paper is organized as follows; section 2 introduces the mathematical model of LFM PC radar waveform and matched filter response without jamming. A mathematical model for pulsed noise jamming and the corresponding LFM-PC matched filter response has been derived in section 3. In section 4,

a Matlab-based simulation model for the LFM-PC search radar is introduced and verified in both quantitative and qualitative point of view with the theoretical results in case of no jamming. Based on the verification of the LFM-PC search radar simulation model in clear environment (jamming free), the effect of pulsed noise jamming on the detection performance of LFM-PC search radar is tested and compared to the theoretical results which can be found in section 5. Finally, conclusion comes in section 6.

2. Mathematical Modeling of LFM-PC radar under Clear Environment

The idea of LFM signal is to sweep a bandwidth, B, linearly in a time duration equals the pulse width, T. The complex envelop of saw tooth LFM pulsed signal can be expressed as follows [9]:

$$s(t) = \frac{1}{\sqrt{T}} rect\left(\frac{t}{T}\right) exp(j\pi k t^2) \quad , k = \frac{B}{T} \tag{1}$$

The instantaneous phase, $\varphi(t)$, and instantaneous frequency, $f(t)$, of this complex envelop are:

$$\varphi(t) = \pi k t^2 \tag{2}$$

$$f(t) = \frac{1}{2\pi}\frac{d(\pi k t^2)}{dt} = kt \tag{3}$$

The impulse response of the matched filter for the LFM signal of equation (1) can be expressed as [10]:

$$h(t) = s^*(T-t)$$

$$= \frac{1}{\sqrt{T}} rect\left(\frac{T-t}{T}\right) \exp(-j\pi k (T-t)^2) \tag{4}$$

The matched filter output magnitude response due to the LFM signal of equation (1) can be calculated by performing a convolution process between this signal and the matched filter impulse response as follows [3, 11]:

$$y(t) = h(t) * s(t) \tag{5}$$

$$|y(t)| = sinc(\pi B(t-T)) \tag{6}$$

If the target is located at a range, R_t, corresponding to a time delay, t_{dt}, such that, $t_{dt} = \frac{2R_t}{c}$, where, c is the speed of light, then:

$$|y(t)| = sinc(\pi B(t-T-t_{dt})) \tag{7}$$

For conventional pulsed radar, based on Nyman-Pearson criteria, the probability of detection, P_d, is given by the Marcum Q function as follows [10]:

$$P_d = Q\left(\sqrt{2SNR} \; , \; \sqrt{2ln\left(\frac{1}{P_{fa}}\right)}\right) \tag{8}$$

Where, SNR is the peak signal power to average-noise power ratio and P_{fa} is the probability of false alarm.
For LFM-PC radar, the additional processing gain due to pulse compression and coherent integration shall be added to the term SNR [4, 10] giving a new SNR, designated as SNR_2, which is given by:

$$SNR_2 = SNR \cdot N \cdot B \cdot T \tag{9}$$

Where, N is the number of pulses in one CPI and $(B \cdot T)$ is the compression gain. Hence, the detection probability of the LFM-PC radar can be expressed as:

$$P_{d2} = Q\left(\sqrt{2SNR_2} \; , \; \sqrt{2ln\left(\frac{1}{P_{fa}}\right)}\right) \tag{10}$$

3. Mathematical Modeling of LFM-PC radar under Pulsed Noise Jamming

Pulsed noise jamming is a technique which depends on creating a noise pulse, $j(t)$, which can be expressed as:

$$j(t) = n(t) \qquad 0 \le t \le T \tag{11}$$

Where, $n(t)$ is a zero mean, unity variance White Gaussian Noise (WGN), and T is the radar pulse width. It is assumed that the jammer is a self-screening repeater that responds to radar pulse with a noise like signal.
The matched filter output response, $y_j(t)$, of the LFM-PC radar to the pulsed noise jamming, $j(t)$ can be obtained by convoluting $j(t)$ with the matched filter impulse response, $h(t)$, as follows:

$$y_j(t) = \int_{-\infty}^{\infty} j(t-\tau).h(\tau)\, d\tau \tag{12}$$

Since $n(t)$ is a stationary process, time shift does not change its mean or variance. Consequently, $j(t\text{-}\tau)$ can be simply written as $n(t)$. Moreover, it can be put outside the integral and equation (12) can be rewritten as:

$$y_j(t) = \frac{1}{\sqrt{T}}.n(t) \int_{-\infty}^{\infty} \exp(-j\pi k(T-\tau)^2)\, d\tau \tag{13}$$

For a self-screening jammer located at a range, R_j, corresponding to a time delay, t_d, (the jammer processing time is considered) and performing the convolution process, shown in Figure 2, on equation (13), then:

$$y_j(t) = n(t)\,.\begin{cases} \dfrac{1}{\sqrt{T}}.\displaystyle\int_0^{t-t_d} \exp(-j\pi k(T-\tau)^2)\,d\tau & ,t_d \le t \le t_d + T \\ \dfrac{1}{\sqrt{T}}.\displaystyle\int_{t-t_d-T}^{T} \exp(-j\pi k(T-\tau)^2)\,d\tau & ,t_d + T \le t \le t_d + 2T \end{cases} \quad (14)$$

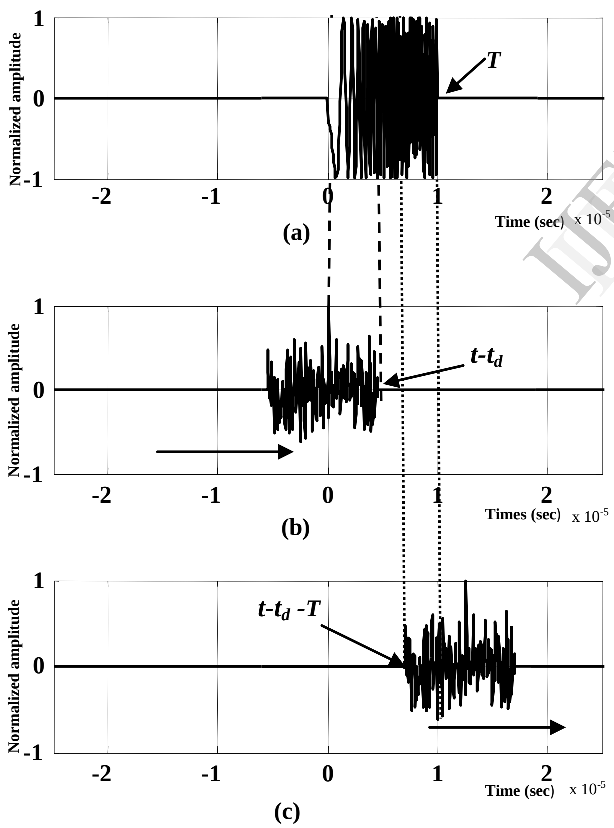

Figure 2. Graphical representation of the convolution of equation (12) (a) matched filter impulse response, h(τ), (b) noise jamming pulse, *j(t-τ)*, when $0 \le t \le t_d + T$, and (c) noise jamming pulse, *j(t-τ)*, when $t_d + T \le t \le t_d + 2T$

Let $x = \sqrt{\pi k}(T-\tau)$, and substituting in equation (14), taking into account the corresponding changes in the integral limits and the integral variable, $d\tau$, then:

$$y_j(t) = n(t)\,.\begin{cases} \dfrac{-1}{\sqrt{\pi B}}.\displaystyle\int_{x_1}^{x_2} \exp(-jx^2)\,dx & ,t_d \le t \le t_d + T \\ \dfrac{-1}{\sqrt{\pi B}}.\displaystyle\int_{x_3}^{0} \exp(-jx^2)\,dx & ,t_d + T \le t \le t_d + 2T \end{cases} \quad (15)$$

Where, $x_1 = \sqrt{\pi k}T$, $x_2 = \sqrt{\pi k}(T + t_d - t)$, $d\tau = \frac{-dx}{\sqrt{\pi k}}$, and $x_3 = \sqrt{\pi k}(2T + t_d - t)$.

A closed mathematical form for the matched filter output response, $y_j(t)$, of the LFM-PC radar to the pulsed noise jamming, $j(t)$, can be obtained:

$$y_j(t) = n(t)\,.\begin{cases} \dfrac{1}{\sqrt{\pi B}}\left[C(x_1) - C(x_2) + j\big(S(x_2) - S(x_1)\big)\right] & ,t_d \le t \le t_d + T \\ \dfrac{1}{\sqrt{\pi B}}\left[C(x_3) - jS(x_3)\right] & ,\ t_d + T \le t \le t_d + 2T \end{cases} \quad (16)$$

Where, $C(x)$ and $S(x)$ are the Fresnel integrals which defined as [4]:

$$C(x) = \int_0^x \cos(t^2)\,dt$$

$$S(x) = \int_0^x \sin(t^2)\,dt$$

Theoretically, using pulsed noise jamming, at the same jammer average power, has an advantage of increasing the effective jamming average power at the radar front end over conventional noise jamming with a factor equals the inverse of the duty cycle of the pulsed waveform [12]. So that, the probability of detection, P_{d3}, in presence of pulsed noise jamming can be expressed as:

$$P_{d3} = Q\left(\sqrt{2\frac{S_p}{N_{av} + J_{av}/\sigma}} \quad , \quad \sqrt{2ln\left(\frac{1}{P_{fa}}\right)}\right) \qquad (17)$$

Where, S_p is the peak signal power, N_{av} is the average noise power, J_{av} is the average noise jamming power, and σ is the pulsed waveform duty cycle.

4. Simulation Modeling of LFM-PC Radar in Clear Environment

A simulation model of LFM-PC radar is built using MATLAB. The assumed simulated radar parameters are shown in Table 1. The simulated radar performs coherent integration with an assumed CPI of *N=16* pulses. So, the radar model has two sources of processing gain. The first is the compression gain (*10log(B.T)=18.5 dB*), and the second is the coherent integration gain (*10log(N)=12 dB*) resulting in a total processing gain of *30.5 dB*. The purpose of choosing the radar parameters to provide the radar with this high processing gain is to give it a full advantage in presence of jamming.

Table 1. Radar and target simulated parameters

Parameter	Value	Unit
Pulse Width	*10*	*µs*
Pulse Repetition Interval	*1.6*	*ms*
Carrier Frequency	*3*	*GHz*
Chirp Bandwidth	*7*	*MHz*
Target Range	*3.576*	*Km*
Target Doppler	*312*	*Hz*
CFAR Type	*Cell Average*	
CFAR Window size	*16*	*Range cells*

The simulated target range and Doppler are chosen such that the target is totally located in one range cell and one Doppler cell. This prevents the occurrence of range or Doppler straddle [10].

The model is verified in both quantitative and qualitative methods in clear environment. To verify the model qualitatively, the output of the radar processor for the assumed parameters is plotted to ensure the resulting pulse width and the precision in both range and Doppler measurements. Signals at the output of different nodes of the simulated LFM-PC radar receiver are shown in Figure 3. To verify the model quantitatively, the simulated and the theoretically derived detection curves are calculated in clear environment and shown in Figure 4. The simulated and the theoretical results agreed very well to each other.

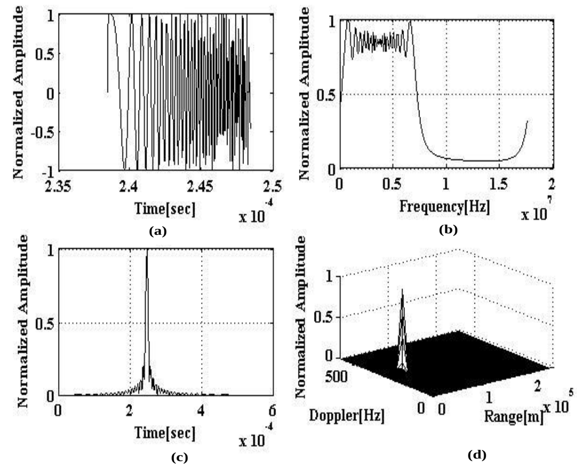

Figure 3 Simulation results at different LFM-PC radar receiver nodes:(a) base band received signal in time domain, (b) spectrum of received signal, (c) time domain matched filter output, and (d) final output after coherent integration and CFAR.

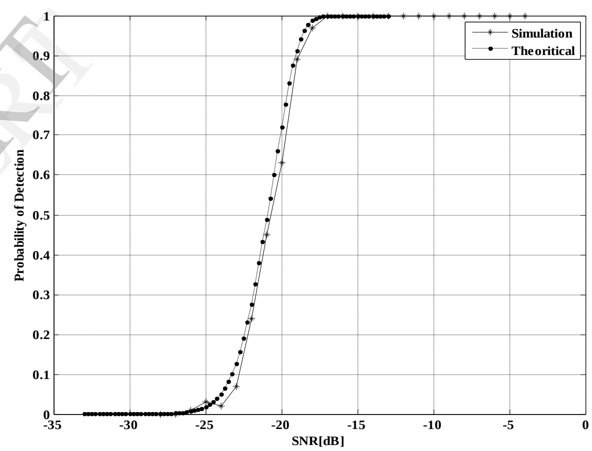

Figure 4. Simulated and theoretically derived ROC curves for LFM-PC radar in clear environment at $P_{fa}=10^{-7}$

5. Simulation Modeling of LFM-PC Radar under Pulsed Noise Jamming

After the verification of LFM-PC radar simulation model in clear environment, the effect of pulsed noise jamming is to be studied. To compare the simulation results with the derived mathematical expression of the LFM-PC matched filter of equation (12), a pulsed waveform as a jamming signal without noise is fed to the radar model. As shown in Figure 5, the output of the mathematically derived expression gives, nearly, the same results of the simulated one.

To verify the effect of pulsed noise jamming on the detection capability of the LFM-PC radar quantitatively, the simulated and the theoretically derived ROC curves in presence of pulsed noise jamming are calculated at different Jamming to Signal Ratios (JSRs). Results shown in Figure 6 demonstrate the agreement between theoretical and simulated models. The reason of the slightly deviation between simulated and theoretical results comes from the limited number of simulation trials.

It is clear from Figure 6 that, the factor controls the effectiveness of pulsed noise jamming on LFM-PC radar is the JSR. For JSR of 0 dB, the detection capability of the LFM-PC radar decreases about 80% of its performance in clear environment. To completely jam the LFM-PC radar, only 5 dB of JSR is required.

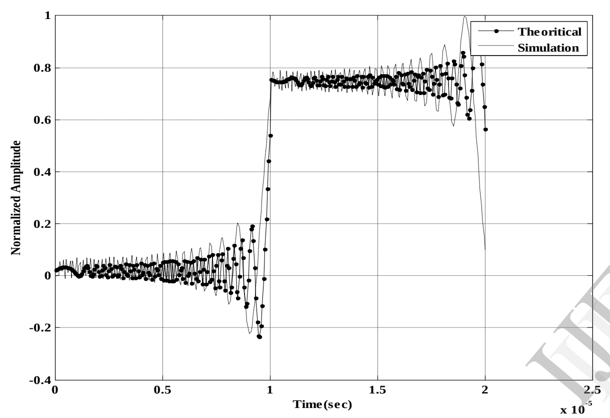

Figure 5. Simulated and theoretically derived outputs of the LFM-PC matched filter in the presence of pulsed jamming

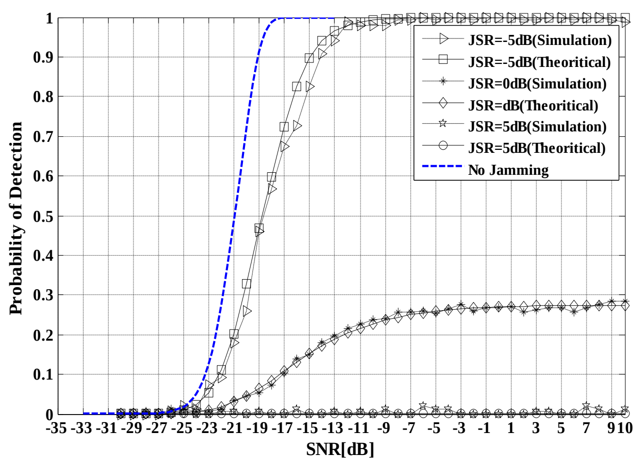

Figure 6. Simulated and theoretically derived ROC curves of the LFM-PC radar at P_{fa} =10^{-7} under pulsed noise jamming at different JSRs.

6. Conclusion

In this paper, a derived mathematical model for the LFM-PC radar matched filter response under pulsed noise jamming was proposed. The performance of the LFM-PC radar under clear environment and pulsed noise jamming was evaluated analytically through the ROC curves. A complete simulation model for the LFM-PC radar was built. The validity of the derived equations was verified with the simulation model in both clear and jamming environments. It was found that, for P_{fa}=10^{-7}, a JSR value of 0 dB is capable of decreasing the LFM-PC detection performance by about 80%, while a value of 5 dB could achieve a complete radar blinding.

7. References

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