Performance Analysis of Haar wavelet-Based BPSK OFDM system with reduced PAPR

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is the most popular modulation technique in modern wireless communication systems. Even though there are many advantages of OFDM, it has two main drawbacks: High peak-to-average power ratio (PAPR) and frequency offset. In this paper the issue of PAPR in OFDM is discussed. Due to the large number of subcarriers used, OFDM systems have a large dynamic signal range with a very high peak-to-average-power ratio (PAPR) which makes OFDM sensitive to nonlinear effects. This paper proposes a novel Haar-Wavelet based BPSK OFDM system. The Haar Wavelet transformation decomposes the data symbol sequence like half of the data symbols are zeros and the rest are √2 or -√2 and we can have the PAPR reduced by 3dB at most, compared with the conventional OFDM system. In this paper we propose a novel decoding algorithm for the proposed OFDM system, derive the bit error rate performance in theory. We compare cumulative distribution function for PAPR for the proposed OFDM v/s conventional OFDM and finally we compare the BER performance of our proposed OFDM system with the conventional one.**

Keywords- Peak-to-average power ratio(PAPR); Bit error rate(BER); Haar Wavelet; Orthogonal frequency division multiplexing(OFDM).

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I. INTRODUCTION

After more than thirty years of research and developments carried out in different places, orthogonal frequency division multiplexing (OFDM) has been widely implemented in high speed digital communications. Due to the recent advancements in digital signal processing (DSP) and very large scale integrated circuits (VLSI) technologies, the initial obstacles of OFDM implementations do not exist anymore. Mean-while, the use of Fast Fourier transform (FFT) algorithms has eliminated arrays of sinusoidal generators and coherent demodulation required in parallel data systems and made the implementation of the technology cost effective. In recent years OFDM has gained a lot of interest in diverse communication applications. This has been due to its favorable properties like high spectral efficiency and robustness to channel fading. Recently OFDM has become the technique of choice for many broadband applications, such as asymmetric digital subscriber line (ADSL) modems, digital audio broadcasting (DAB)[1], digital video broadcasting(DVB)[2] and wireless local area networks (WLAN) systems (IEEE 802.11a [3], IEEE 802.11g[4]. In the conventional serial data transmission system, the information symbols are transmitted sequentially where each symbol occupies the entire available spectrum bandwidth. But in the OFDM system, the information is converted to N parallel sub channels and sent at lower rates using frequency division multiplexing. The subcarrier frequency spacing is selected carefully such that each subcarrier is located on the other sub-carrier zero crossing points. This implies that there is overlapping among the subcarriers but will not interfere with each other, if they are sampled at the subcarrier frequencies. This means that all subcarriers are orthogonal.

The frequency offset of the subcarriers and the high PAPR are the major drawbacks of OFDM [5]. However, due to the large number of subcarriers used, OFDM system has a large dynamic signal range with a very high Peak-to-average power ratio (PAPR). As a result OFDM signal will be clipped when passed through a nonlinear power amplifier at the transmitter end. Clipping degrades the bit-error-rate (BER) performance and causes spectral spreading. One way to solve this problem is to force the amplifier to work in its linear region [6]. In high speed digital wireless applications, the inter-symbol interference (ISI) channel may have spectral nulls, which may degrade the performance of the existing OFDM system. Hence PAPR and spectral null channels need to be handled properly in implementation of OFDM systems

The Wavelet-OFDM system was widely studied in [7-9]. Wavelet-OFDM system just substitutes the DFT and IDFT with DWT and IDWT, respectively. In this paper we propose a novel Haar wavelet based BPSK OFDM system. Since the data sequence produced by the BPSK modulator is either +1 or -1, the Haar wavelet transformation decomposes the data symbol sequence like half of the data symbols are zeros and the rest are √2 or-√2. Then the proposed BPSK OFDM system will have the PAPR reduced by 3dB at most, compared with the conventional OFDM system. In this paper we also propose a novel decoding algorithm for the proposed OFDM system to show robustness to spectral null channels.

The rest of the paper is organized as follows. The next section provides an introduction to Haar wavelet transformation, followed by the principle of the proposed OFDM system, a novel decoding algorithm and the simulation results.

II. HAAR WAVELET TRANSFORMATION

Wavelets are a special kind of functions which exhibits oscillatory behavior for a short period of time and then die out. The oldest and most basic wavelet system is named Haar wavelet that is a group of square waves with magnitude of ±1 in the interval [0, 1) [10, 11]. Haar wavelet has Haar two functions namely Haar scaling equation and wavelet function, they are represented as,

$$\varphi_0(t) = \begin{cases} 1, \; for\, 0 \le t < 1 \\ 0, otherwise \end{cases} \quad (1)$$

$$\varphi_1(t) = \begin{cases} 1, \; for\, 0 \le t < \frac{1}{2} \\ -1, \; for\, \frac{1}{2} \le t < 1 \\ 0, otherwise \end{cases} \quad (2)$$

All the other subsequent functions are generated from

$$\varphi_1(t) = \varphi_1(2^j t - k) \quad (3)$$

Where

$i = 2^j + k, j \ge 0$ And $0 \le k < 2^j$. All the Haar wavelets are orthogonal to each other. From the Haar functions, the scale equation and wavelet equations are obtained as follows

$$\varphi(t) = \sqrt{2}\left(\tfrac{1}{\sqrt{2}}\varphi(2t) + \tfrac{1}{\sqrt{2}}\varphi(2t-1)\right) \quad (4)$$

$$\psi(t) = \sqrt{2}\left(\tfrac{-1}{\sqrt{2}}\varphi(2t) + \tfrac{1}{\sqrt{2}}\varphi(2t-1)\right) \quad (5)$$

From the equations 4 and 5, we get low pass components as $h_0 = h_1 = 1/\sqrt{2}$ and high pass components as $g_0 = -1/\sqrt{2}$, $g_1 = 1/\sqrt{2}$. Using these filter components Haar wavelet decomposition over one-dimension digital signals can be expressed as

$$\begin{pmatrix} C(j) \\ D(j) \end{pmatrix} = T.C(j+1), \quad (6)$$

And

$$T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & . & . & . & 0 & 0 \\ & & & . & & & & & & \\ & & & . & & & & & & \\ 0 & 0 & 0 & 0 & 0 & . & . & . & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & . & . & . & 0 & 0 \\ & & & . & & & & & & \\ & & & . & & & & & & \\ 0 & 0 & 0 & 0 & 0 & . & . & . & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}_{N \times N} \quad F$$

rom the equation (6), we can compute the approximation coefficients vector C(j) and detail coefficients vector D(j), through operating Haar wavelet decomposition over the vector C(j+1).

III. PROPOSED OFDM SYSTEM

The proposed Haar wavelet based BPSK OFDM system structure is shown in figure 1. From figure 1 we can observe that the proposed OFDM system only increases Haar wavelet transformation at the transmitter compared with the conventional OFDM system.

Some literatures use wavelet transformation to substitute the Fourier transform unit [7-9], which is different from our paper. In this paper we use both wavelet transformation and Fourier transform unit. The proposed OFDM system has many advantages over the conventional OFDM system. The advantages of our proposed OFDM system are obtained from the Haar wavelet transformation.

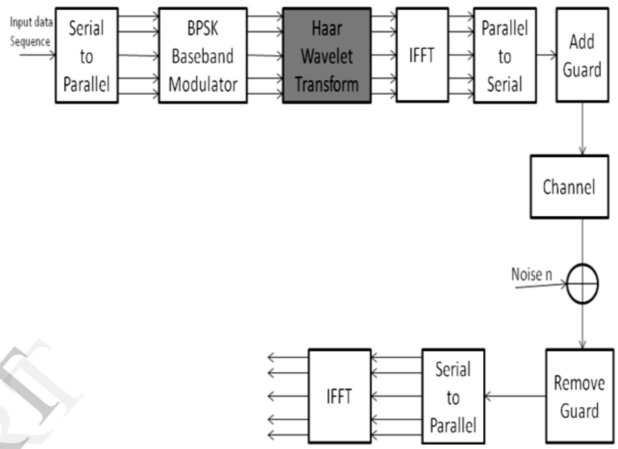

Figure 1: The proposed Haar wavelet based BPSK OFDM system.

A. *Principle of the proposed OFDM sysytem.*

The Haar wavelet transform unit in figure 1 operates decomposition over the input data sequence. We denote the input data sequence as $\bar{x}(n) = [x(0), x(1), ..., x(N-1)]^T$, and the decomposition result as $\bar{x}'(n) = [x'(0), x'(1), ..., x'(N-1)]^T$. Since the input N by 1 data sequence is generated by BPSK modulator, each component of $\bar{x}(n)$ is either 1 or -1. The detail process of Haar wavelet decomposition over input vector sequence $\bar{x}(n)$ is

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$$\begin{bmatrix} x'(0) \\ x'(1) \\ \cdot \\ x'\left(\frac{N}{2}-1\right) \\ x'\left(\frac{N}{2}\right) \\ x'\left(\frac{N}{2}+1\right) \\ \cdot \\ \cdot \\ x'(N-1) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & . & . & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & . & . & 0 & 0 \\ & & & \cdot & & & & & \\ & & & \cdot & & & & & \\ 0 & 0 & 0 & 0 & 0 & . & . & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & . & . & 0 & 0 \\ 0 & 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & . & . & 0 & 0 \\ & & & \cdot & & & & & \\ & & & \cdot & & & & & \\ 0 & 0 & 0 & 0 & 0 & . & . & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}_{N\times N} \begin{bmatrix} x(0) \\ x(1) \\ \cdot \\ \cdot \\ x(N-1) \end{bmatrix} \quad (7)$$

From the equation (7), we can conclude that:

If $x'(i) \neq 0, i = 0,1,....,N/2 - 1$, then

$x'(i) = \sqrt{2}$ Or $-\sqrt{2}$

$$x'\left(i + \frac{N}{2}\right) = 0 \quad (8)$$

Similarly if $x'(i) = 0, i = 0,1,.....,N/2 - 1$, then

$$x'\left(i + \frac{N}{2}\right) = \sqrt{2} \text{ Or } -\sqrt{2} \quad (9)$$

After Haar wavelet transformation half of the coefficients of $\bar{x}'(n)$ are zeros and the rest are $\sqrt{2}$ or $-\sqrt{2}$.

Since Haar wavelet transform is added to the conventional OFDM system, the computational complexity may increase. But the Haar wavelet transformation is simple to operate at the transmitter as follows

$$\begin{bmatrix} x'(0) \\ x'(1) \\ \cdot \\ x'\left(\frac{N}{2}-1\right) \\ x'\left(\frac{N}{2}\right) \\ x'\left(\frac{N}{2}+1\right) \\ \cdot \\ \cdot \\ x'(N-1) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}\left[x(0)+x(1)\right] \\ \frac{1}{\sqrt{2}}\left[x(2)+x(3)\right] \\ \cdot \\ \cdot \\ \frac{1}{\sqrt{2}}\left[x(N-2)+x(N-1)\right] \\ \frac{1}{\sqrt{2}}\left[-x(0)+x(1)\right] \\ \frac{1}{\sqrt{2}}\left[-x(2)+x(3)\right] \\ \cdot \\ \cdot \\ \frac{1}{\sqrt{2}}\left[-x(N-2)+x(N-1)\right] \end{bmatrix} \quad (10)$$

From equation (10) we can say that the proposed OFDM system don't increase too much computational complexity, compared with the conventional OFDM system.

IV. PAPR PERFORMANCE

In this section, we study the PAPR benefits of the proposed OFDM system. In OFDM system, PAPR is the peak power per OFDM symbol versus the average power in the same symbol [12, 13], i.e., mathematically

$$PAPR = \frac{\max\left|x(t)\right|^2}{E\left\{\left|x(t)\right|^2\right\}} \quad (11)$$

Suppose that the average power of the conventional OFDM system is $\bar{P}_{OFDM}$ with the signal peak value A_{OFDM} and the average power of the Haar wavelet-based BPSK OFDM system is $\bar{P}_{DWT/OFDM}$ with the signal peak value $A_{DWT/OFDM}$. The average power of the conventional OFDM and the proposed OFDM system is equal for every frame, which can be derived from Parseval theorem, which means

$$\bar{P}_{DWT/OFDM} = \bar{P}_{OFDM} \quad (12)$$

Since the Haar wavelet transformation used in our proposed OFDM system, half of the information symbols are zeros and the rest are either √2 or -√2 in each OFDM symbol. Hence non-zero symbols in Haar wavelet-based BPSK OFDM system only occupies half of the subcarriers and the magnitude of each symbol is √2 times compared with conventional OFDM system. Considering the worst situation (all subcarriers appear the maximal value at the same time), we get

$$\frac{A_{DWT/OFDM}}{A_{OFDM}} = \frac{\sqrt{2} \times \frac{N}{2}}{N} = \frac{\sqrt{2}}{2}$$

$$\left(\frac{A_{DWT/OFDM}}{A_{OFDM}}\right)^2 = \frac{1}{2} \quad (13)$$

N denotes number of subcarriers in the OFDM system.

Equation (13) says that peak power of the proposed OFDM system is reduced by half, compared with the conventional OFDM system. So PAPR of the Haar wavelet based BPSK OFDM system is reduced by 3dB. Which means proposed OFDM system is able to overcome the main drawback of conventional OFDM system.

V. NOVEL DECODING ALGORITHM.

As we know, the output data sequence y_l has the following relationship with the input data sequence x'_l at the l^{th} subcarrier.

$$y_l = H_l x'_l + n_l \quad (14)$$

Where $H_l = H(z)\Big|_{z=\exp\left(j t\pi/N\right)}$ and n_l is Additional Gaussian White Noise (AWGN). At the decoder, we decode the output sequence y_l and $y_{l+\frac{N}{2}}$ simultaneously, that is

$$\begin{bmatrix} y_l \\ y_{l+\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} H_l & 0 \\ 0 & H_{l+\frac{N}{2}} \end{bmatrix} \begin{bmatrix} x'_l \\ x'_{l+\frac{N}{2}} \end{bmatrix} + \begin{bmatrix} n_l \\ n_{l+\frac{N}{2}} \end{bmatrix} \quad (15)$$

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Where $l = 0,1,\ldots, N/2$, n_l and $n_{l+\frac{N}{2}}$ are independently, identically distributed Gaussian random variables. Since the input data sequence $\left[x'_l . x'_{l+\frac{N}{2}}\right]^T$ only be one of the following four possible symbols, which is

$$\begin{bmatrix} x'_l \\ x'_{l+\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}, or \begin{bmatrix} 0 \\ -\sqrt{2} \end{bmatrix}$$

Corresponding to

$$\begin{bmatrix} s_l \\ s_{l+\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, or \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (16)$$

Where $\left[s_l . s_{l+\frac{N}{2}}\right]^T$ represents the decoding result over l^{th} and $l + \frac{N}{2}th$ subcarrier. At the receiver, we assume maximum-likelihood (ML) estimation. The ML estimation is used to find out which $\left[s_l . s_{l+\frac{N}{2}}\right]^T$ is sent from the transmitter, which means

$$\left[s_l . s_{l+\frac{N}{2}}\right]^T = \arg\left(\left[y_l . y_{l+\frac{N}{2}}\right]^T\right)$$

$$= \arg\left(\arg\left(\min\left\{\left[\begin{bmatrix} y_l \\ y_{l+\frac{N}{2}} \end{bmatrix} - \begin{bmatrix} H_l & 0 \\ 0 & H_{l+\frac{N}{2}} \end{bmatrix}\begin{bmatrix} x'_l \\ x'_{l+\frac{N}{2}} \end{bmatrix}\right]^H \cdot \left[\begin{bmatrix} y_l \\ y_{l+\frac{N}{2}} \end{bmatrix} - \begin{bmatrix} H_l & 0 \\ 0 & H_{l+\frac{N}{2}} \end{bmatrix}\right]\right\}\right)\right)$$

The decoding algorithm consists of the correlation of the transmitted symbols sufficiently.

VI. SIMULATION RESULTS

In this section, we provide the simulation results of the cumulative distribution functions for PAPR for the proposed OFDM versus conventional OFDM, simulation results of the BER versus SNR for conventional OFDM system versus proposed OFDM system.

We assume 256 subcarriers, i.e., N=256, and assume BPSK modulation is used in conventional and proposed OFDM systems. For simplicity, we assume that the maximum-likelihood (ML) estimation method is used at the receiver. one can clearly see that the BER performance of proposed OFDM system is better than the conventional system.

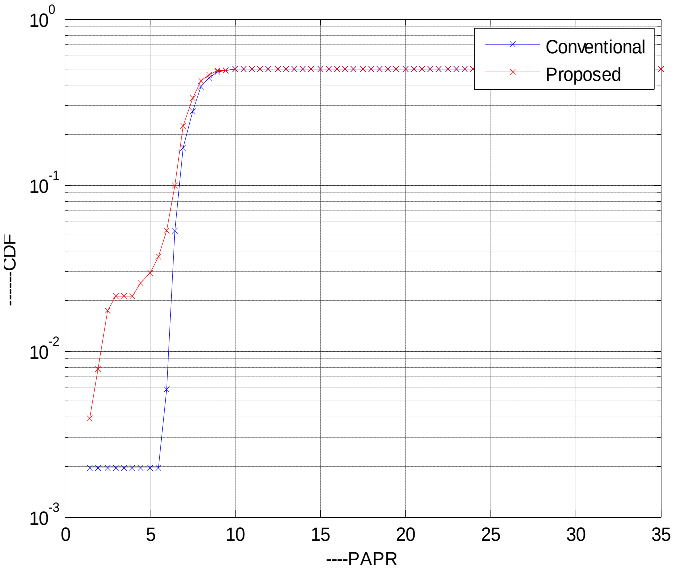

Figure 2: Cumulative distribution functions for PAPR for the proposed OFDM system versus conventional OFDM system.

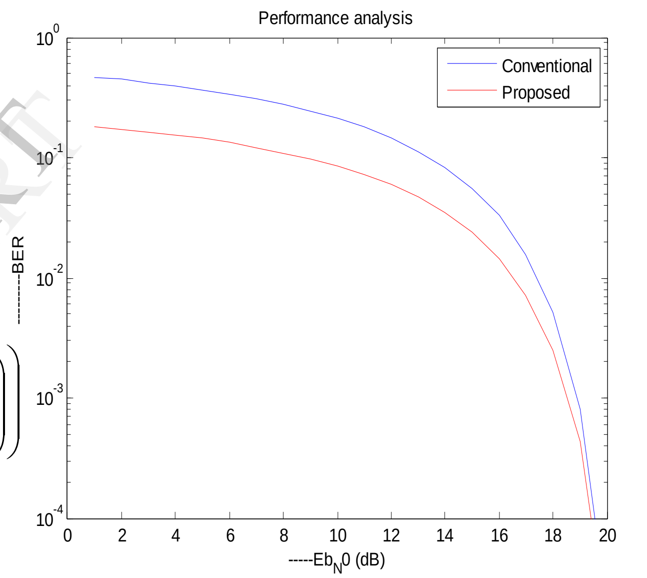

Figure 3: BER performance comparision for Conventional OFDM system versus Conventional OFDM system.

VII. CONCLUSION

In this paper, we proposed a novel Haar wavelet-based BPSK OFDM system. The simulation results prove that the proposed system has two advantages compared with conventional OFDM system: Reduces the PAPR by 3dB at most, BER performance improves 3dB at most. Analysis also shows that proposed system does not increase too much computational complexity at the transmitter.

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