Performance Analysis of CFO Compensation Procedure under Two Diverse Channel Conditions

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Abstract*—OFDM is a widely used multicarrier modulation and multiplexing technology which has become the basis of many telecommunication standards. Here in this paper we are presenting a Carrier frequency Offset(CFO) correction method, by considering the OFDM model. Two different modulation schemes Quadrature Amplitude Modulation(QAM) and Quadrature Phase Shift Keying(QPSK) under two different channel fading conditions flat and frequency selective fading are considered. In this Doppler effect is also considered for finding CFO which is used in testing performance. CFO correction is carried out by Cramer's rule method, which uses a single pilot data in an OFDM frame. Using this a characteristic function can be derived and this is considered as an error term which shows minimum errors when the CFO present in the system equals to the one of the CFO among the range of CFO's given. By this method the CFO can be estimated and corrected which eliminates the Inter-carrier Interference(ICI) which arises as a result of CFO. The performance of this method is tested by plotting different properties and also tested by transmitting an image and retrieving it without any error.

Keywords— OFDM, CFO, Doppler Effect, Fading Channels, Cramer's Rule, Characteristic Function, ICI

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing(OFDM) is a method of encoding digital data on multiple carrier frequencies. OFDM is a Frequency Division Multiplexing(FDM) scheme used as a digital multi-carrier modulation method. A large number of closely spaced orthogonal sub-carrier signals are used to carry data on several parallel data streams or channels. Each subcarrier is modulated with a conventional modulation schemes (such as Quadrature amplitude modulation or Phase-shift keying) at a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth. When modulation of any form- voice, data etc is applied to a carrier, then sidebands spread out either side. It is necessary for a receiver to be able to receive the whole signal to be able to successfully demodulate the data. As a result when signals are transmitted close to one another they must be spread so that the receiver can separate them using a filter and there must be a guard band between them. This is not the case with OFDM. Although the sidebands from each carrier overlap, they can still be received without the interference that might be expected because they are orthogonal to each another. This is achieved by having the carrier spacing equal to the reciprocal of the symbol period. Orthogonality gives the carriers a valid reason to be closely spaced with overlapping without Inter-carrier Interference(ICI).

The orthogonality of the OFDM relies on the condition that transmitter and receiver operate with exactly the same frequency reference. If this is not the case, the perfect orthogonality of the subcarriers will be lost, which can result to subcarrier leakage, this phenomenon is also known as the Inter-carrier Interference(ICI). In another word, the OFDM systems are sensitive to the frequency synchronization errors in form of CFO. CFO can lead to the Inter-carrier Interference; therefore CFO plays a key role in frequency synchronization. Basically for getting a good performance of OFDM, the CFO should be estimated and compensated. Lack of the synchronization of the local oscillator signal; for down conversion in the receiver with the carrier signal contained in the received signal causes carrier frequency offset(CFO) which can create the following factors;

(i) Frequency mismatched in the transmitter and receiver oscillator.

(ii) Inter-carrier Interference(ICI)

(iii) Doppler effect

Mainly CFO estimation techniques are based on Maximum Likelihood Estimation. In that we can consider either Channel Side Information (CSI) is available or not available. In both cases CFO can be estimated by using Training Sequence, Null Subcarriers or using Cyclic prefix property. J. Hoseyni[1] introduced a carrier frequency offset (CFO) correction technique for Orthogonal Frequency Division Multiplexing (OFDM) by exploiting baseband characteristics of the Nyquist sampled received signal Fourier transform**.** Some of the earlier works were done considering that the CSI is not available. P.H Moose[2] proposed a technique involves repetition of a data symbol and comparison of the phases of each of the carriers between the successive symbols. Since the modulation phase value are not changed, the phase shift of each of the carriers between successive repeated symbols is due to the frequency offset. Then we will use the Maximum likelihood Estimation(MLE) algorithm. T.M. Schmidl and D.C. Cox[3], proposed a method which avoids extra overhead of using a null symbol, while allowing a large acquisition range for the CFO. M. Morelli and U. Mengali [4] introduced an algorithm which is an extension of Schmidl and Cox Algorithm(SCA) where only one training symbol is employed.

Some methods are introduced which uses null subcarrier for CFO estimation. H. Liu and U. Tureli [5] proposed a technique where reference symbols are not used. It provides a high accuracy carrier estimator by taking advantage of the inherent orthogonality among OFDM sub channels. Even when the OFDM signal is distorted by an unknown carrier offset, the

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received signal posses an algebraic structure which is a direct function of the carrier offset. Another method was proposed by D. Huang and K.B. Letaief [6], where CFO is divided into the fractional part and the integer part, with the fractional CFO estimation followed by the integer CFO estimation; here complexity is lower, due to the fact that the main component required is a simple correlator. Here null subcarriers are used for estimation. . Yan Wu [7] showed that the SNR of the CFO estimation using null subcarriers is a function of the null subcarrier placement. Then formulated the CFO-SNR optimization for the null subcarrier placement.

CFO estimation can also be carried out using the cyclic prefix property of OFDM symbols. J.J. Beek et al [8] presented a joint maximum likelihood symbol-time and carrier frequency offset estimator in orthogonal frequency division multiplexing systems. Redundant information contained within the cyclic prefix enables this estimation without additional pilots. The method introduced by N. Lashkarian et al [9], is used for CFO estimation and symbol-timing error estimation of OFDM. This method exploits the properties of the cyclic prefix subset to reveal the synchronization parameters in the likelihood function of the received vector. D. Huang et al[10] used channel side information for CFO estimation. They used Training OFDM symbols, pilot tones, null subcarriers or combination of these for CFO estimation.

In this paper an algorithm is proposed which is used to capture the inter-carrier interference (ICI) effects using a function called CFO characteristic function (CF).This is obtained by representing the symbols in a matrix form. Finally roots of this specialized function is estimated which is the estimated CFO and is cancelled. Doppler effect is also used for analyzing the performance. Performance is measured mainly in terms of Symbol Error Rate. Different channel conditions are also considered.

This paper is organized as follows. Section II deals with Signal representation. In Section III Simulation results are shown. Finally, conclusion remarks are given in Section IV.

II. SIGNAL REPRESENTATION

In this paper a method to estimate the CFO that is, present in the OFDM system and hence to correct it is presented. The OFDM signal consists of N subcarriers which are orthogonal to each other. In frequency domain, the transmitted symbols can be represented by modulated symbols A_k, where modulation used here is Quadrature Amplitude Modulation (QAM) and Quadrature Phase-shift Keying(QPSK) and received symbols by B_k, where k=0,1,2,...,N-1; N is the number of subcarriers. *Fig. 1.* shows the OFDM signal representation generated using Sinc function.

At the transmitter the message signal is converted into pass band using carrier frequency and the same carrier frequency is used at the receiver to get back the message signal. But unfortunately there will always be a small difference between the frequencies generated at the receiver and transmitter due to the inherent nature of the oscillator (δf_c) and Doppler effect (f_d).

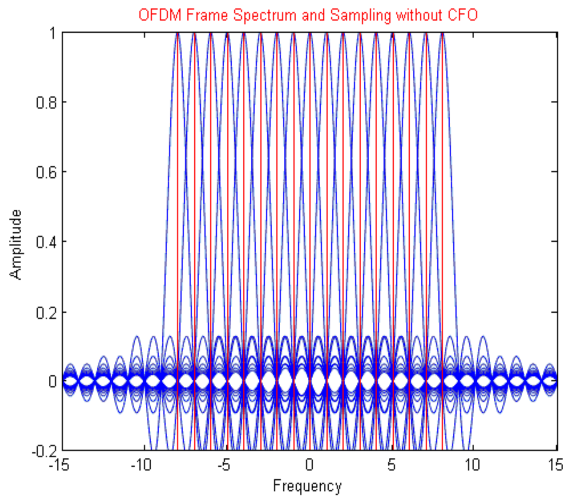

Fig. 1. OFDM frame spectrum representation without CFO

This difference in the frequency is called frequency offset. The Doppler frequency is determined from carrier frequency (f_c) and relative velocity (v) between transmitter and receiver as,

$$f_d = v f_c / c \quad (1)$$

where c is the speed of light.

Normalized CFO δ can be defined as the ratio of effective frequency offset (δf) and subcarrier spacing Δf.

$$\delta = \delta f / \Delta f. \quad (2)$$

where $\delta f = \delta f_c + f_d$

Data-aided or blind CFO estimation can be used, but here we are dealing with blind CFO estimation, in the sense that CFO values are not manually given, it is obtained by using the previous equation. Fig 2 shows the OFDM signal with CFO, which is calculated using the above equation.

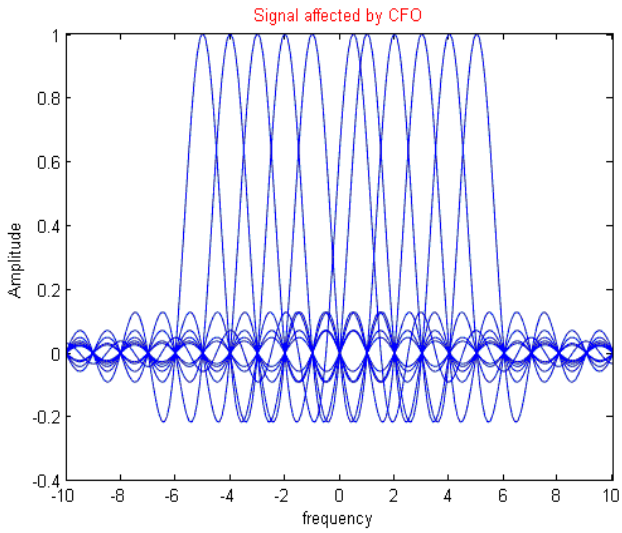

Fig. 2. OFDM frame spectrum representation with CFO

A. *OFDM System*

The Discrete time baseband OFDM system model with N subcarriers is shown in Fig. 3. It consists of transmitter, Channel and receiver blocks. The complex data symbols are obtained by encoding input bits using modulation techniques

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like M-PSK, M-QAM, etc. The transmitted signal is the inverse discrete Fourier transform(IDFT) of the complex input data symbols, and hence it can be easily and efficiently generated using inverse fast Fourier transform(IFFT). Similarly, the fast Fourier transform(FFT) can be used at the receiver side for demodulation. A technique of cyclic prefix(CP) is suggested a s a solution of maintaining the orthogonality when signal is passed through multipath fading channel. The OFDM system is generally used in wireless environment therefore the multipath fading channel has been considered in present study. At the receiver after finding the start of frame, the cyclic prefix is removed and the sample of OFDM symbol is converted from serial to parallel. Then these parallel samples are applied for FFT operation. Then demodulated to get data out.

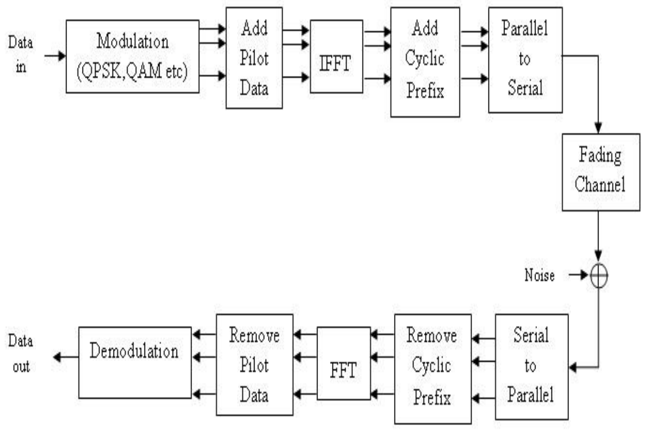


Fig.3. OFDM System

Using the difference in data out and data in Bit error rate(BER) for each Signal to noise ratio(SNR) can be calculated. Fig. 4. shows the BER plotting for different SNR values in an OFDM system. When image is send through OFDM system it must retrieve correctly at the receiver side. A 256x256 cameraman image is used to verify this. When the Signal to Noise ratio is very low the image cannot be recovered effectively. As SNR value increases the error is minimized and image is recovered at receiver. Binary image is given as input.

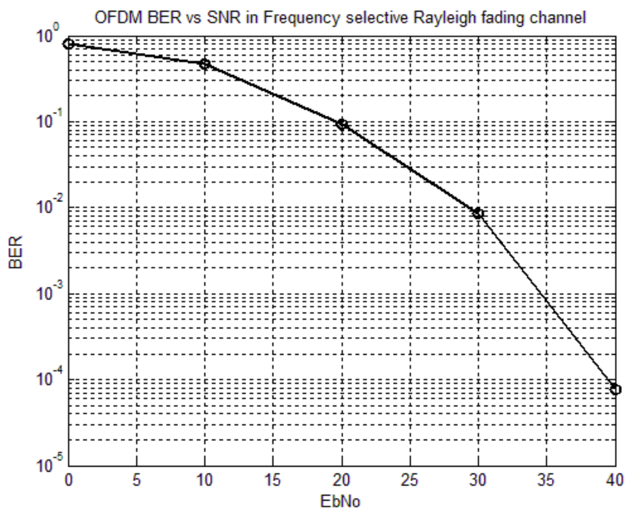


Fig.4. BER vs. SNR in OFDM System

TABLE I

Retrieved Image for each SNR values

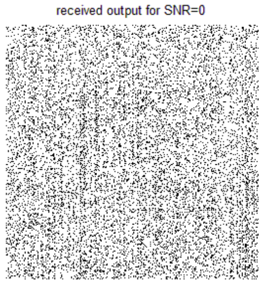
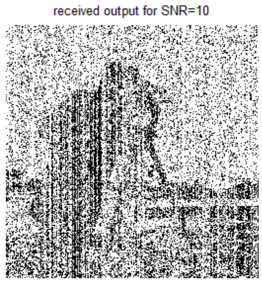
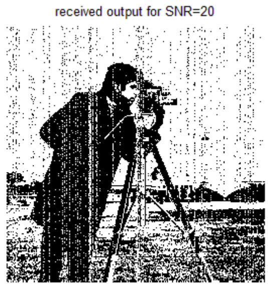
Image 256x256	Total Mismatch Bits	BER
 **SNR=0dB**	**51923**	**0.7293**
 **SNR=10dB**	**31657**	**0.4830**
 **SNR=20dB**	**8092**	**0.1235**
 **SNR=30dB**	**937**	**0.0143**

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Image 256x256	Total Mismatch Bits	BER
 **SNR=40dB**	**4**	**.000061**
 **SNR=50dB**	**0**	**0**

B. Two Channels

A fading channel is a communication channel that experiences fading. Here fading channels under consideration are flat and Rayleigh Channels. Fig.5. shows the BER plots of the OFDM signal after passing through each channels after adding AWGN noise.

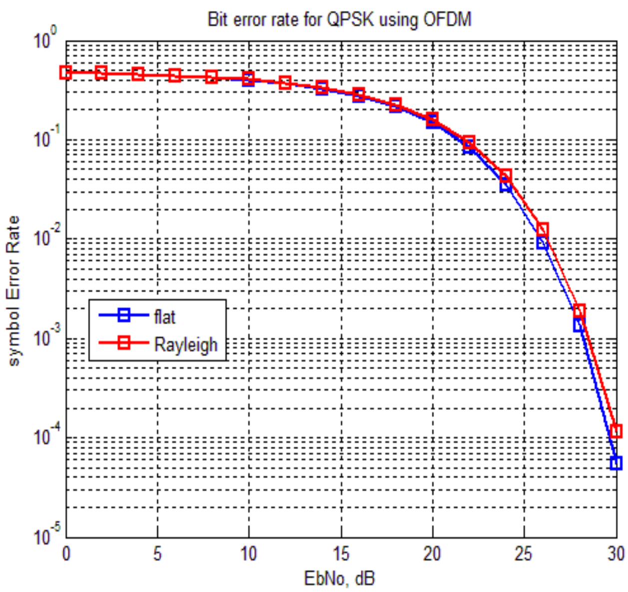

Fig.5. BER for flat & Rayleigh channels

C. CFO Estimaton and Correction

When the transmitted signals is affected by Carrier frequency offset, then the orthogonality of the subcarriers will be affected. This causes Inter-carrier Interference(ICI), which must be avoided to retrieve the original signal transmitted. In this a method to overcome this problem is discussed. Interaction between the signals(for 4 subcarriers) in the frequency domain can be represented by an equation;

$$S(f) = A_0 \text{sinc}\,(f-f_0) + \ldots + A_3 \text{sinc}\,(f-f_3) \qquad (3)$$

in which f_i is the i^{th} subcarrier frequency with $f_i - f_{i-1} = N/T_s = \Delta f$ considering the received samples B_k; the 1^{st} sample can be represented as

$$B_0 = A_0.\ \text{sinc}(x) + \sum_{k=1}^{3} A_k.\ \text{sinc}(x-k) \qquad (4)$$

$$B_0 = A_0.\ \text{sinc}(x) + \text{ICI}\ (0) \qquad (5)$$

In frequency domain, we can represent the OFDM signal given by,

$$S_r(f) = \sum_{k=0}^{N-1} A_k\ \text{sinc}(f - f_k) \qquad (6)$$

The received samples for N subcarriers can written as a system of linear equations as follows.

$$B_0 = A_0\ \text{sinc}(0) + \ldots + A_{N-1}\ \text{sinc}(N-1)$$

$$B_1 = A_0\ \text{sinc}(1) + \ldots + A_{N-1}\text{sinc}(N-2)$$

.
.
.

$$B_k = A_0\ \text{sinc}(k) + \ldots + A_{N-1}\ \text{sinc}(N-1-k) \qquad (7)$$

.
.
.

$$B_{N-1} = A_0\ \text{sinc}(N-1) + \ldots + A_{N-1}\ \text{sinc}(0)$$

The matrix form of (7) is

$$\mathbf{B = S\ .\ A} \qquad (8)$$

With the normalized CFO,
$x = \delta f\ /\ \Delta f$, $B_m = S_r(f)\,|_{f=f_m+x}$, the (7) can be modified to include effect of CFO as;

$$B_0 = A_0\ \text{sinc}(x) + \ldots + A_{N-1}\ \text{sinc}([N-1]-x)$$

$$B_1 = A_0\ \text{sinc}(1+x) + \ldots + A_{N-1}\text{sinc}([N-2]-x)$$

.
.
.

$$B_k = A_0\ \text{sinc}(k+x) + \ldots + A_{N-1}\ \text{sinc}([N-1-k]-x) \qquad (9)$$

.
.
.

$$B_{N-1} = A_0\ \text{sinc}([N-1]+x) + \ldots + A_{N-1}\ \text{sinc}(x)$$

The matrix representation is

$$\mathbf{B} = \mathbf{S_x} \cdot \mathbf{A} \qquad (10)$$

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where,

$$S_x = \begin{pmatrix} \mathrm{sinc}(x) & \cdots & \mathrm{sinc}(N-1+x) \\ \mathrm{sinc}(1+x) & \cdots & \mathrm{sinc}(N-2+x) \\ & \vdots & \\ \mathrm{sinc}(k+x) & \mathrm{sinc}(x) & \mathrm{sinc}(N-1-k+x) \\ & \vdots & \\ \mathrm{sinc}(N-1+x) & \cdots & \mathrm{sinc}(x) \end{pmatrix} \quad (11)$$

The transmitted and received symbols can be represented as a matrix form given by (11). To proceed in the case when A_k is used for finding the CFO , we try to use matrix relation (11) to express A_0, assuming that we know the data on the $k = 0$ subcarrier. Using Cramer's rule to solve for one (A_0) of the variables in **A** without solving the whole system of equations. Cramer's rule is an explicit formula for the solution of a system of linear equations with as many equations as unknowns, valid whenever the system has a unique solution. It expresses the solutions in terms of the determinants of the (square) coefficient matrix and of matrices obtained from it by replacing one column by the vector of right hand sides of the equations.

In this approach, A_0 is expressed as:

$$A_0 = \frac{det|S_x^1|}{det\,|S_x|} \quad (12)$$

where det | . | is the determinant of the matrix and the matrix $\mathbf{S_x^1}$ is $\mathbf{S_x}$ with the first column of $\mathbf{S_x}$ replaced by the received symbol vector **B**. Here optimizing the placement of pilot symbols by minimizing the maximum steady-state MSE of the channel estimator, single pilot periodic placement achieves the minimum MSE. Here we are considering only one pilot tone; let A_0 represents the pilot tone. The determinant det $|\mathbf{S_x}|$ is a deterministic function of an unknown x and the determinant det $|\mathbf{S_x^1}|$ is a function of x and **B** is the known data at the receiver.

So, we can write the relation as

$$A_0 \det|\mathbf{S_x}| - \det|\mathbf{S_x^1}| = 0 \quad (13)$$

is used to find x, which is the CFO. The Cramer's rule can be used to form a function which when solved gives the CFO.

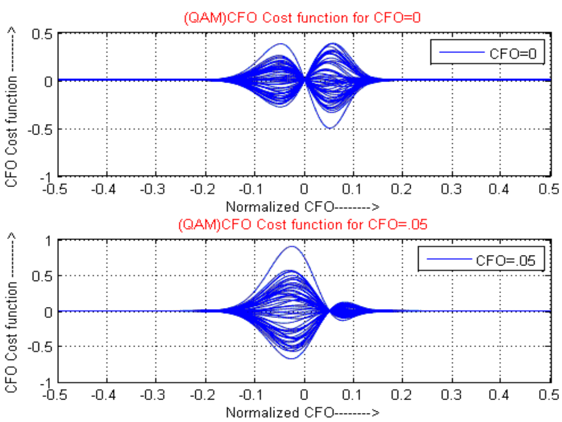

Fig. 6. CFO Characteristic function(QAM).

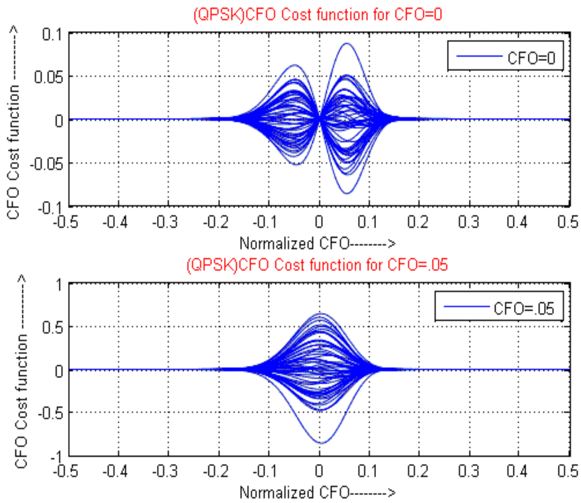

Fig. 7. CFO Characteristic function(QPSK).

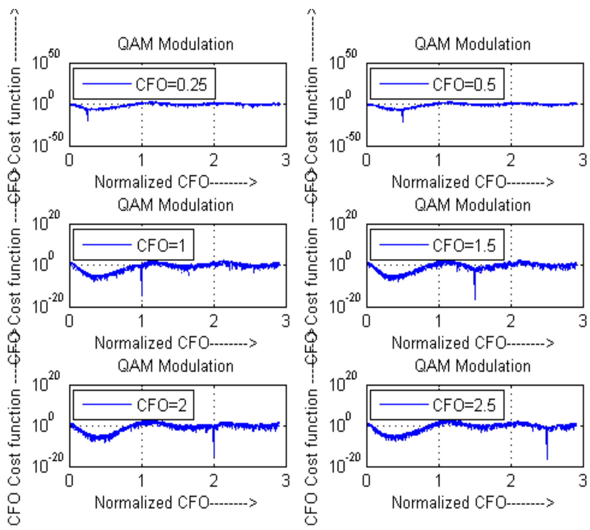

Fig. 8. CFO Nulls representation for different CFO values.

This function is called CFO characteristic function. Fig. 6. shows the plot for 50 OFDM symbols with CFO set to zero and 5% CFO. The CFO values are determined using (2). It can be determined in both QAM(Fig.6.) and QPSK(Fig.7.). For a range of CFO values the specialized function can be considered as an error function which gives minimum value when the CFO estimated and actual error are equal. This is shown in the Fig. 8. CFO Nulls means the roots of the characteristic equation that gives the idea about the CFO introduced in to the signal and by knowing it we can correct the CFO. In this the CFO are estimated by a trial and error method and can be done for both QAM and QPSK modulations.

III. SIMULATION RESULTS

Using the methods proposed the CFO is estimated and corrected. The performance of the method can be shown by plotting some functions. Mainly we are concentrating on plotting the constellation diagram, since QAM and QPSK modulations are checked in this work and also in plotting Bit Error Rate(BER).

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A. Constellation Diagram

In OFDM system one of the main block is the modulation, we are plotting the constellation diagram of transmitted and

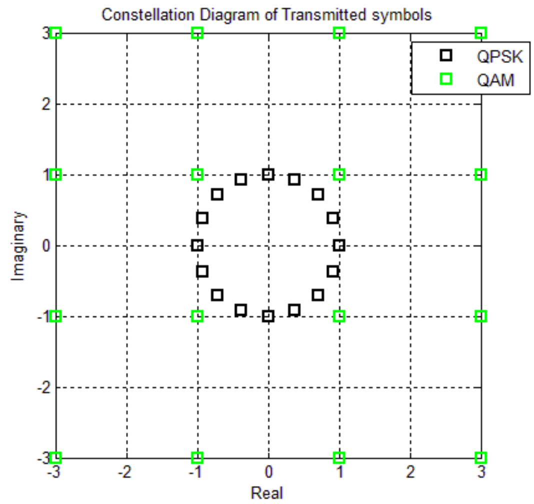

Fig. 9. Transmitted symbols

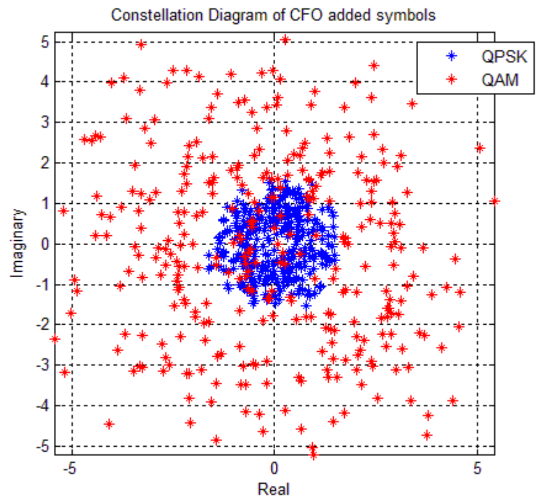

Fig. 10. Received symbols without compensation for both flat and frequency selective channels

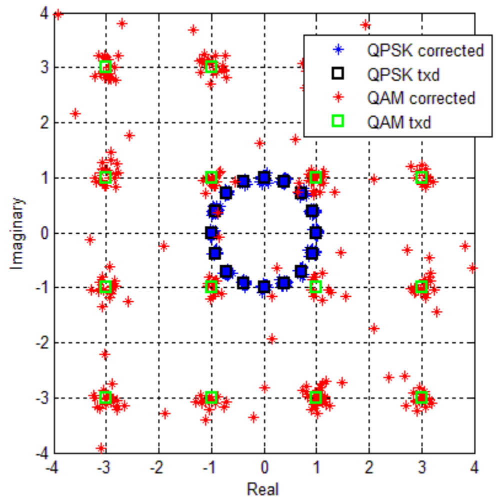

Fig. 11. CFO Corrected symbols for flat channel

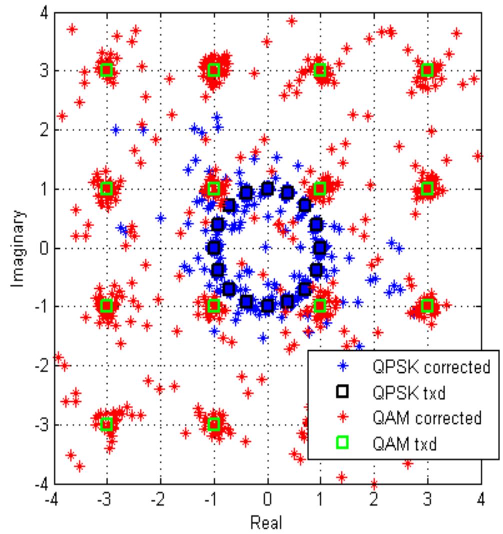

Fig. 12. CFO Corrected symbols for frequency selective fading channel

received symbols. Here both the cases of 16-QAM and 16-QPSK modulations are shown. Our method works best for both flat and frequency selective channels. Fig 9. shows the plot of transmitted symbols. Fig 10. shows the distorted received signal constellations without compensation, while Fig 11. and Fig. 12. shows the received signal constellations after compensation by the proposed method based compensator using single pilot data in case of flat fading and frequency selective fading channels respectively. The result shows that the CFO can be identified and compensated, and it can be efficiently shown using constellation diagram.

B. Error Rate

The performance of a system can be best analyzed by finding its error rate. Here the symbol error rate is found out for each CFO values. The error rate is found out by considering the characteristic function as an error function. Fig. 13. shows the plot of error rate for different CFO values. It shows that the error rate is reduced to a minimum value by using our method.

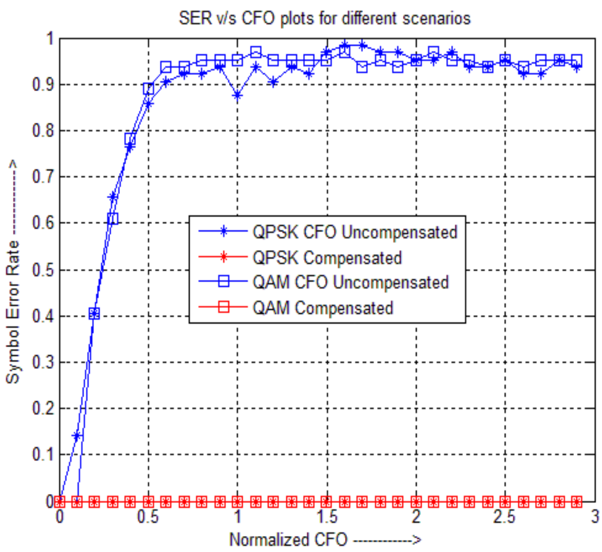

Fig. 13. SER vs. CFO for two modulation schemes

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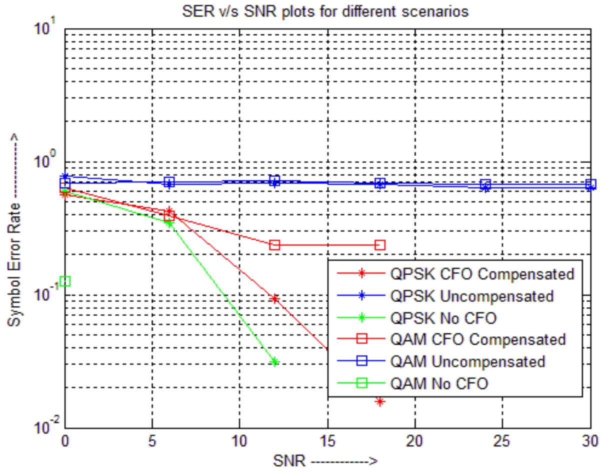

Fig. 14. SER vs. SNR for flat fading channel

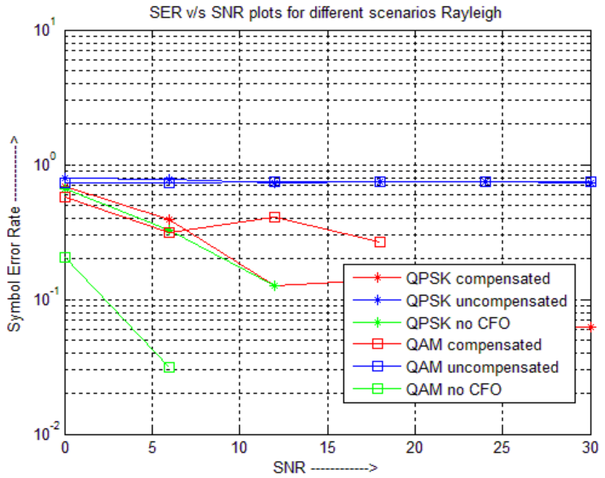

Fig. 15. SER vs. SNR for frequency selective fading channel

When considering the noisy channel, the error rate can be plotted for each signal to noise ratio values. Here also both modulations are studied. It shows a slight variation as the fading channel changes. Fig. 14. shows the plot of symbol error rate for flat fading channel, where Fig. 15. shows the plot for frequency selective fading channel. SER in QAM and QPSK modulations are studied for both channels.(Case1-CFO uncompensated and Case2-CFO Compensated).

TABLE II
SER for flat channel (CFO=45%)

SNR	SER for QPSK (M=8)		SER for QAM (M=8)	
	Case 1	Case 2	Case 1	Case 2
0dB	0.7813	0.5625	0.7031	0.6406
6dB	0.6875	0.4219	0.6875	0.3906
12dB	0.6719	0.0938	0.7188	0.2344
18dB	0.6719	0.0156	0.6875	0.2344
24dB	0.6406	0	0.6719	0
30dB	0.6406	0	0.6719	0

TABLE III
SER for Rayleigh channel (CFO=45%)

SNR	SER for QPSK (M=8)		SER for QAM (M=8)	
	Case 1	Case 2	Case 1	Case 2
0dB	0.7969	0.6875	0.7344	0.5781
6dB	0.7813	0.3906	0.7344	0.3125
12dB	0.7344	0.1250	0.7500	0.4063
18dB	0.7500	0.1406	0.7500	0.2656
24dB	0.7500	0.0625	0.7500	0
30dB	0.7344	0.0625	0.7500	0

A image can be transmitted through the system to verify whether the effect of CFO is cancelled or not. Here we considered the cameraman image(256x256) resized to 8x8 for calculation simplicity. For a particular CFO value, the image is retrieved from both modulations and total number of mismatch between corresponding bits in received and transmitted image is calculated. It shows that as Signal to noise ratio increases, the CFO is corrected effectively, which reduces the mismatch to zero.

TABLE IV
Total number of mismatch bits out of 64 Bits(8x8 image)

SNR	Flat Channel		Rayleigh Channel	
(CFO=35%) (M=4)	QPSK (Bits)	QAM (Bits)	QPSK (Bits)	QAM (Bits)
0dB	20	6	39	3
6dB	5	0	30	0
12dB	0	0	12	0
18dB	0	0	3	0
24dB	0	0	0	0
30dB	0	0	0	0

IV. CONCLUSION

In this paper a method which corrects the CFO present in the OFDM systems is developed and analyzed. It is verified for both QAM and QPSK modulation schemes and also the performance is evaluated under both flat and frequency selective fading channels. The Doppler effect is also considered in this work. The error performance of the proposed algorithm is mathematically analyzed and the theoretical results are verified with simulations. Simulations shows that the performance of the method agrees with the mathematical results. Most notably, its performance is not

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significantly degraded by noisy, frequency selective channels. The image transmitted through the OFDM system in which CFO is present can be recovered efficiently at the receiver side by the CFO compensation method discussed in this paper. This can be extended to the case of successful recovery of voice message also.

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